Water Flow Estimation in One Open Channel with Data Assimilation

ME236
Presenters: Mohamed Hariri Nokob, Meng Cai
mhariri@berkeley.edu, mengcai@berkeley.edu
Instructor: Professor Alex Bayen

April 24th, 2012

Outline

- Introduction
- Notations
- Special Methods with PDE
- Preliminary Results
- Conclusion
- Future work

Introduction

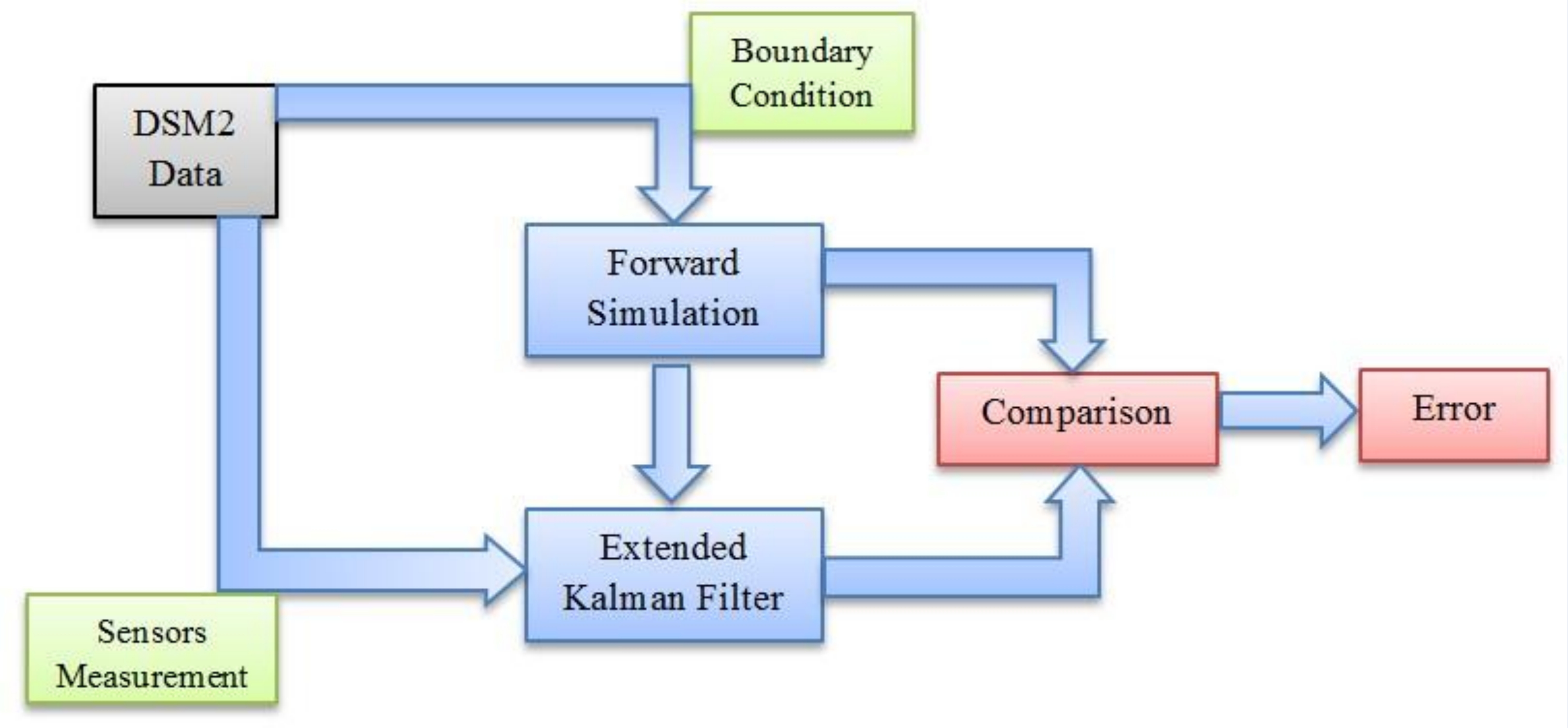
Problem Description:
Real-time estimation of 1-D state in an open channel water flow networks with streaming data

- ❑ Assumptions
- 1-D State: Discharge flow and Average depth
- Streaming Data: Noisy measurements at the boundaries and internal locations
- ❑ Delta Simulation Model II (DSM2)

  -one-dimensional mathematical model for dynamic simulation

Procedure

Process of data assimilation in open channel



Notations

Main Variables

H: Stage (ft)

V: Velocity (ft/s)

B: Channel Width (ft)

D: Hydraulic Diameter (ft)

S_0: Bed Slope (ft/ft)

S_f: Friction Slope (ft/ft)

Auxiliary Variables

Q: Flow Rate (ft^3/s)

A: Cross Sectional Area (ft^2)

P: Parameter (ft)

m: Manning Coefficient $(s/\sqrt[3]{ft})$

Δt: Time Step Size (s)

Δx: Spatial Step Size (ft)

Governing Equations – Saint Venant Equations

$$\frac{\partial H}{\partial t} + \frac{\partial VH}{\partial x} = 0$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial H}{\partial x} = g\left(S_0 - S_f\right)$$

Boundary Conditions

$$Q(0,t) = Q_u(t)$$

$$H(L,t) = H_d(t)$$

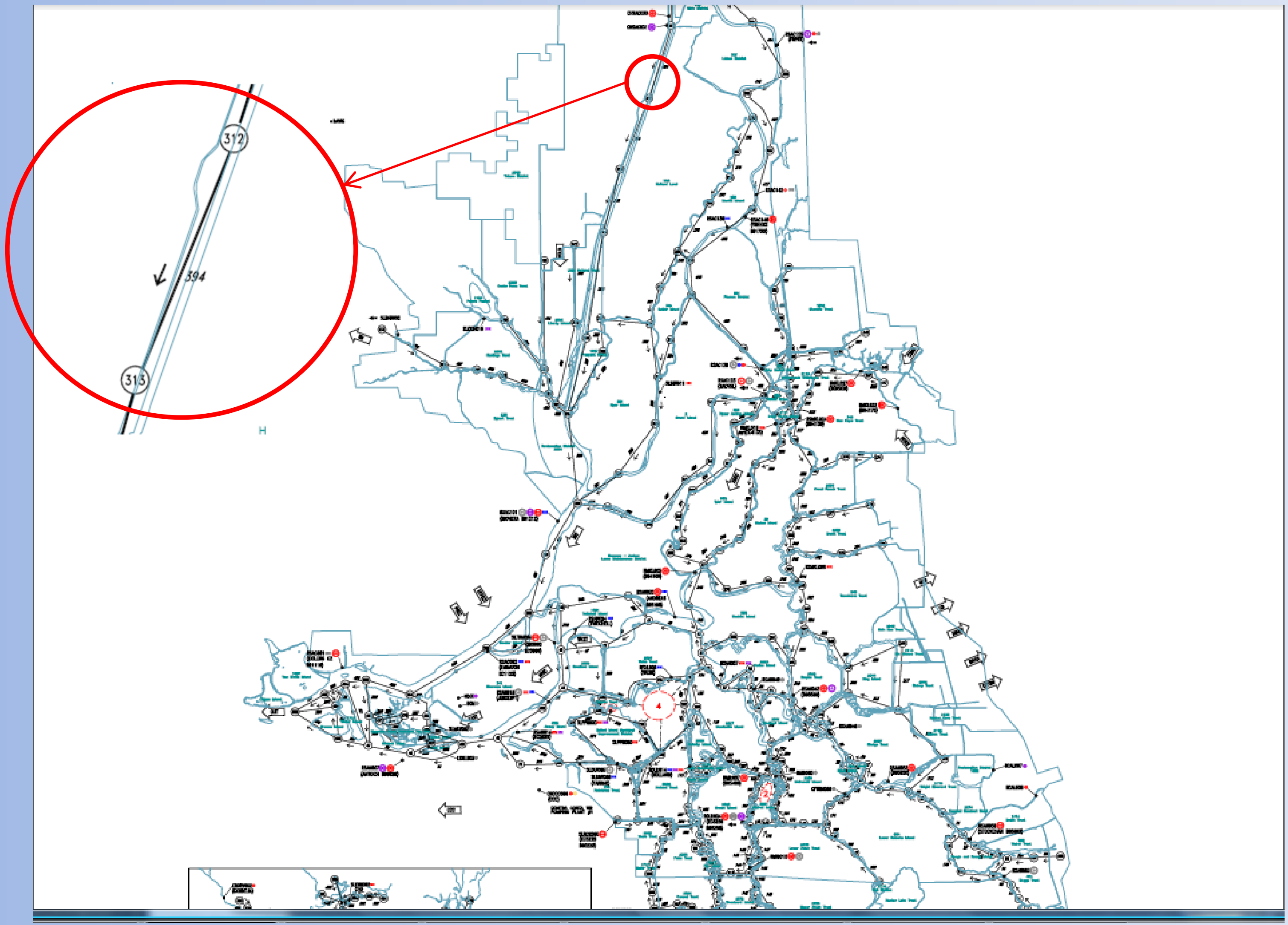
Initial Conditions

$$Q(x,0) = Q_i(x)$$

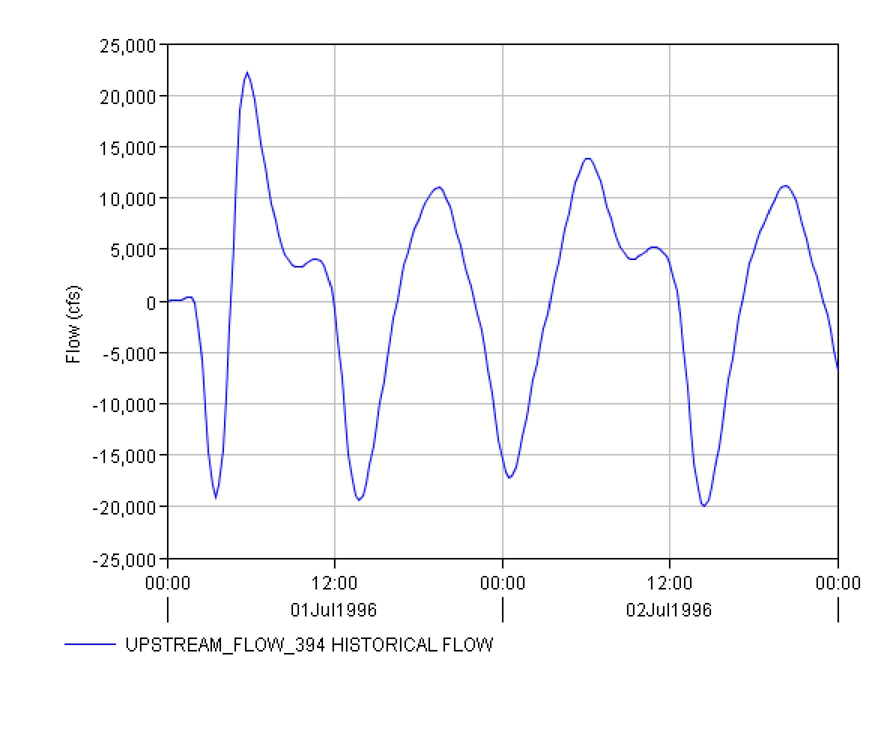
$$H(x,0) = H_i(x)$$

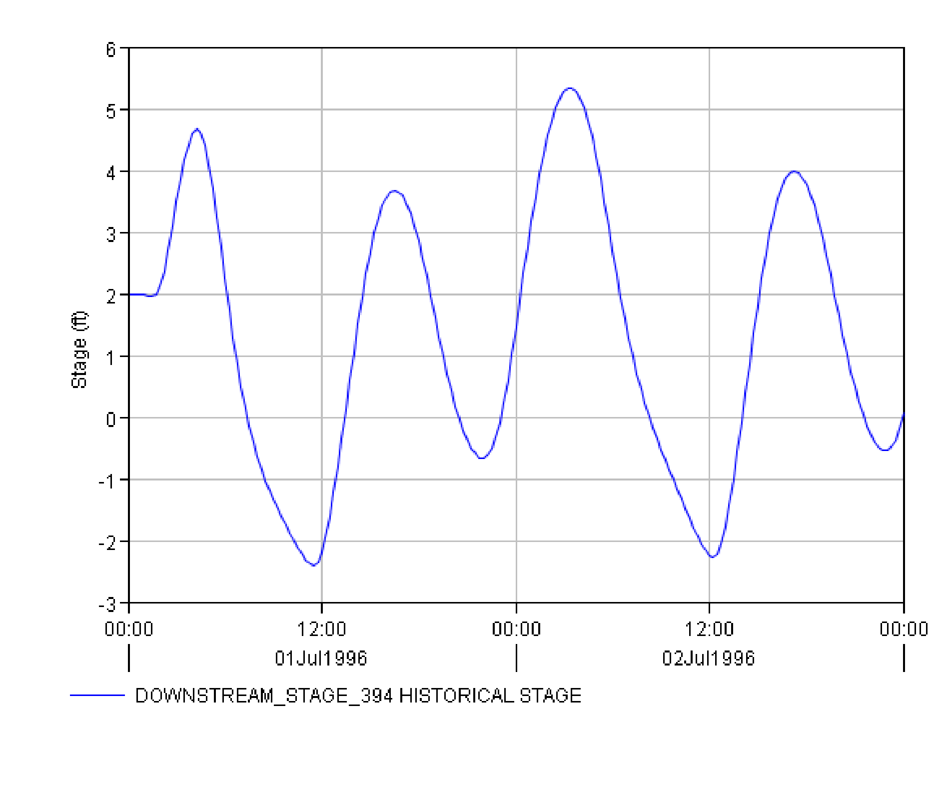
The Sacramento Delta

Channel 394



Delta Simulation Model 2 (DSM2)



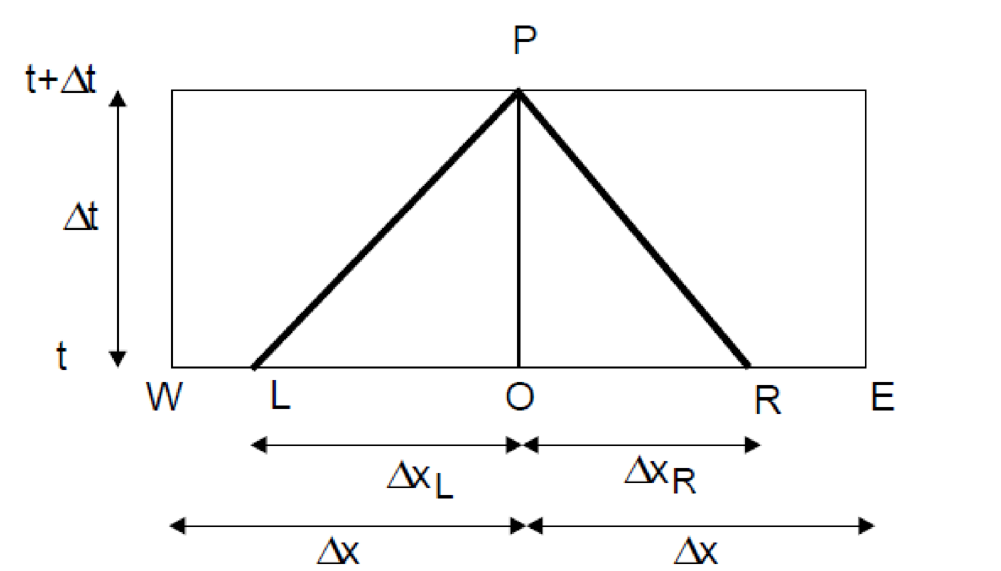


Discretization – Inner Nodes (Lax Diffusive Scheme)

$$H_i^{k+1} = \frac{1}{2}\left(H_{i+1}^k + H_{i-1}^k\right) - \frac{\Delta t}{4\Delta x}\left(V_{i+1}^k + V_{i-1}^k\right)\left(H_{i+1}^k - H_{i-1}^k\right) - \frac{\Delta t}{4\Delta x}\left(D_{i+1}^k + D_{i-1}^k\right)\left(V_{i+1}^k - V_{i-1}^k\right)$$

$$V_i^{k+1} = \frac{1}{2}\left(V_{i+1}^k + V_{i-1}^k\right)\left(1 - \frac{\Delta t}{2\Delta x}\left(V_{i+1}^k - V_{i-1}^k\right)\right) - \frac{g\Delta t}{2\Delta x}\left(H_{i+1}^k + H_{i-1}^k\right) + g\Delta t\left(S_0 - \frac{S_{f,i+1}^k + S_{f,i-1}^k}{2}\right)$$

Discretization – Boundary Nodes (Characteristics)



$$H_1^{k+1} = H_R^k + C_R^k\left(V_1^{k+1} - V_R^k\right) + C_R^k \Delta t\left(S_{fR}^k - S_0\right)$$

$$V_{N+1}^{k+1} = V_L^k + \frac{g}{C_L^k}\left(H_L^k - H_{N+1}^{k+1}\right) - g\Delta t\left(S_{fL}^k - S_0\right)$$

PA Sleigh, I M Goodwill, "*The St Venant Equations*", School of Civil Engineering, University of Leeds, March 2000

Special Methods with PDE

Extended Kalman Filter

- Linearize the state equations about an estimation of the current mean and covariance
- Optimize the simulation results and improve the accuracy
- Time Update:

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_{k-1}, 0)$$

$$P_{k|k-1} = \phi_{k-1} P_{k-1|k-1} \phi_{k-1}^T + \varphi_{k-1} Q_{k-1|k-1} \varphi_{k-1}^T$$

- Measurement Update:

$$K_k = P_{k|k-1} G_k^T (G_k P_{k|k-1} G_k^T + D_{k-1} R_k D_k^T)^{-1}$$

$$\hat{y}_k = G_k \hat{x}_{k|k-1}$$

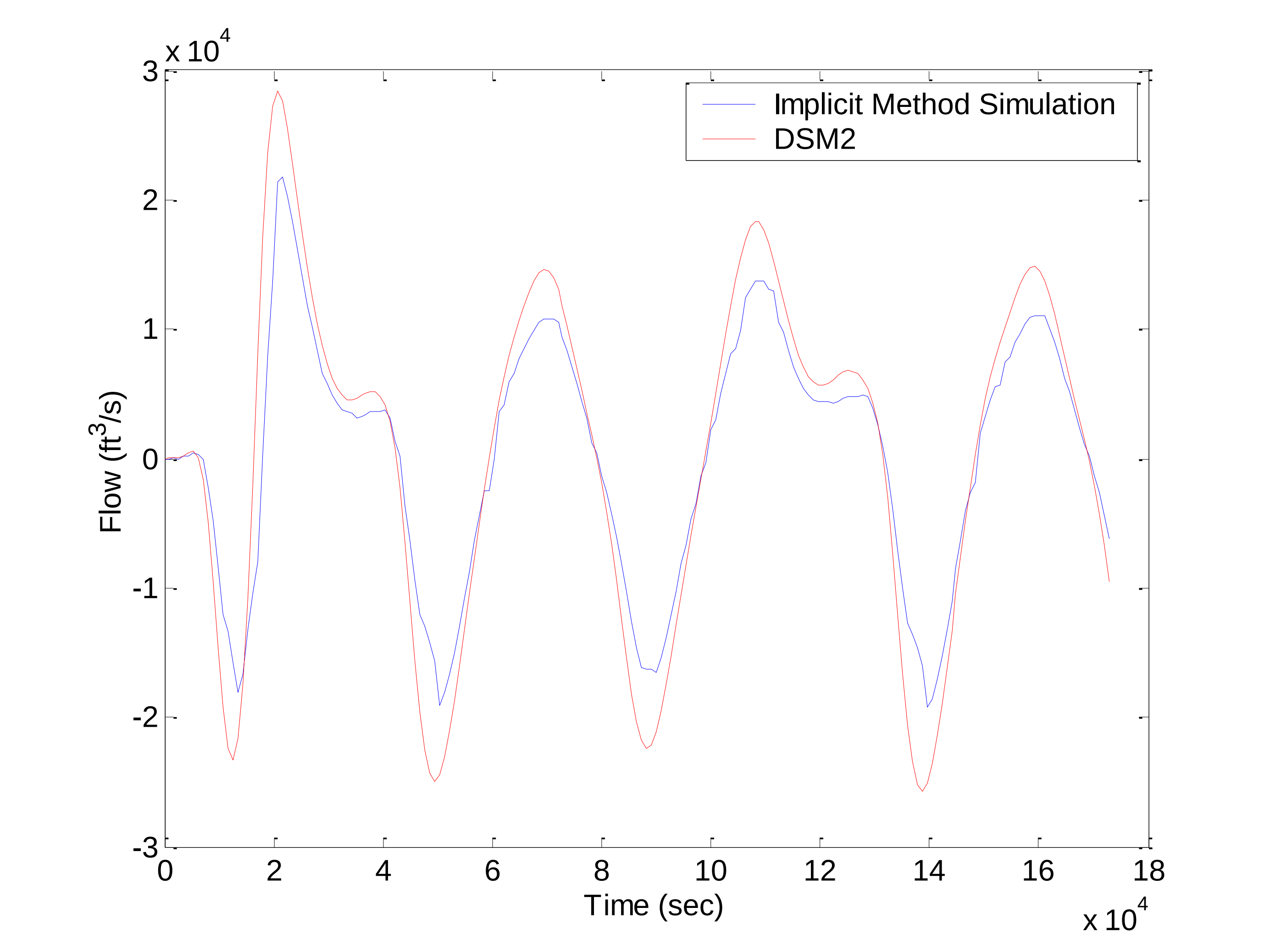
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_k)$$

$$P_{k|k} = (I - K_k G_k) P_{k|k-1}$$

$$\phi_{k-1} = \frac{\partial f}{\partial x} \Big|_{\hat{x}_{k|k-1}, u_{k-1}}$$

$$\varphi_{k-1} = \frac{\partial f}{\partial \omega} \Big|_{\hat{x}_{k|k-1}, u_{k-1}}$$

Preliminary Results



Future Work

We need to get this stuff fixed in time!!

Thank you !