Using PDEs to Calculate Optimal Foot Motion for Walking

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CE C291F: Control and optimization of distributed systems

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Outline

- Research project background
- Approach for distributed system
  - Hybrid systems
- PDE model
  - Compass gait biped
- Implementation of control
  - Human motion data
- Refinements of model

Project Background

- Motivation
  - Quantitative Parkinson's disease diagnosis
- Inertial measurement units for human movement analysis
  - Wireless Lagrangian sensors attached to feet
  - Triaxial accelerometers and gyroscopes
  - Gait analysis

Problem Statement

- Distributed system
  - Data collection along trajectory
  - Multiple parameters (position, velocity, time, etc.)
- PDE to calculate optimal foot motion to take a step during walking

Approach

- Walking: continuous and discrete dynamics
  - Hybrid systems
  - State consists of vector signals
  - Flow equation f(x) and jump equation g(x)

Hybrid Systems

- Simple hybrid system: a *4*-tuple

$$\mathrm{H} = (D, G, R, f)$$

where

$D \subseteq \mathbf{R}^n$ domain

$G \subset D$ guard

$R : G \rightarrow D$ reset map, from impact equations

f vector field on D, i.e. $\dot{x} = f(x)$

- Simple hybrid control system: a *6*-tuple

$$\mathrm{H} = (D, U, G, R, f, g)$$

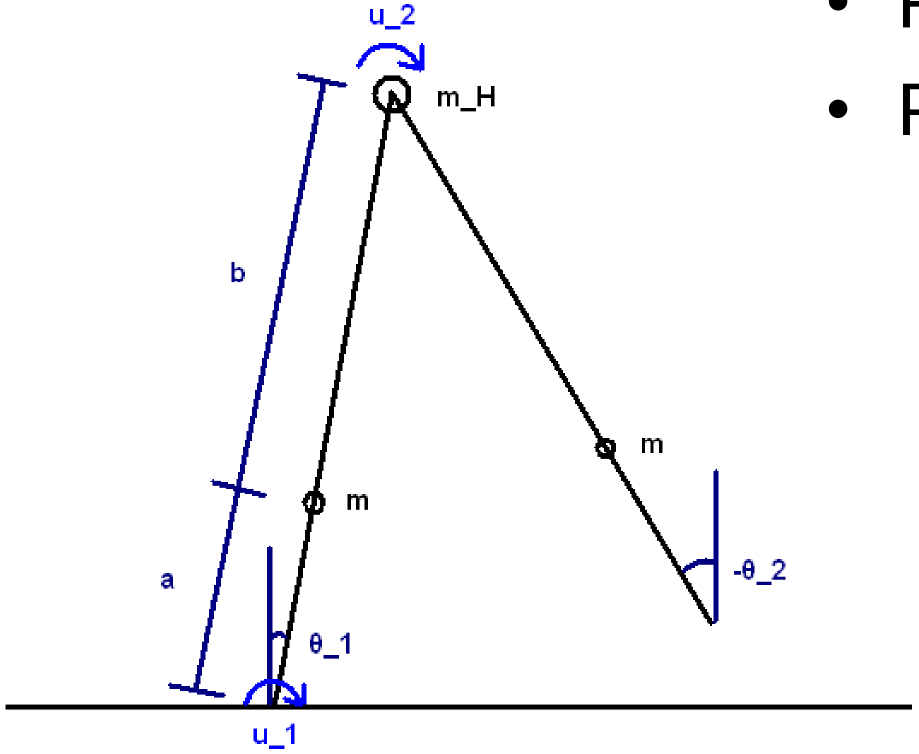
where

D, G, R domain, guard, reset map

$U \subseteq \mathbf{R}^k$ set of admissable controls

(f, g) control system, i.e. $\dot{x} = f(x) + g(x)u$

Compass Gait Biped Model



- From robotics and control
- Parameters of model

m, m_H	masses of limbs, hip
a, b	length of limbs
$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$	angle vector
$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$	control torque vector

Langrangian Dynamics

- Langrangian of compass gait biped model

$$L(\theta,\dot{\theta}) = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} - P(\theta)$$

where

$M(\theta)$ inertial matrix, i.e. $\frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta}$ kinetic energy

$P(\theta)$ potential energy

- Controlled Euler-Lagrange equations

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = Su$$

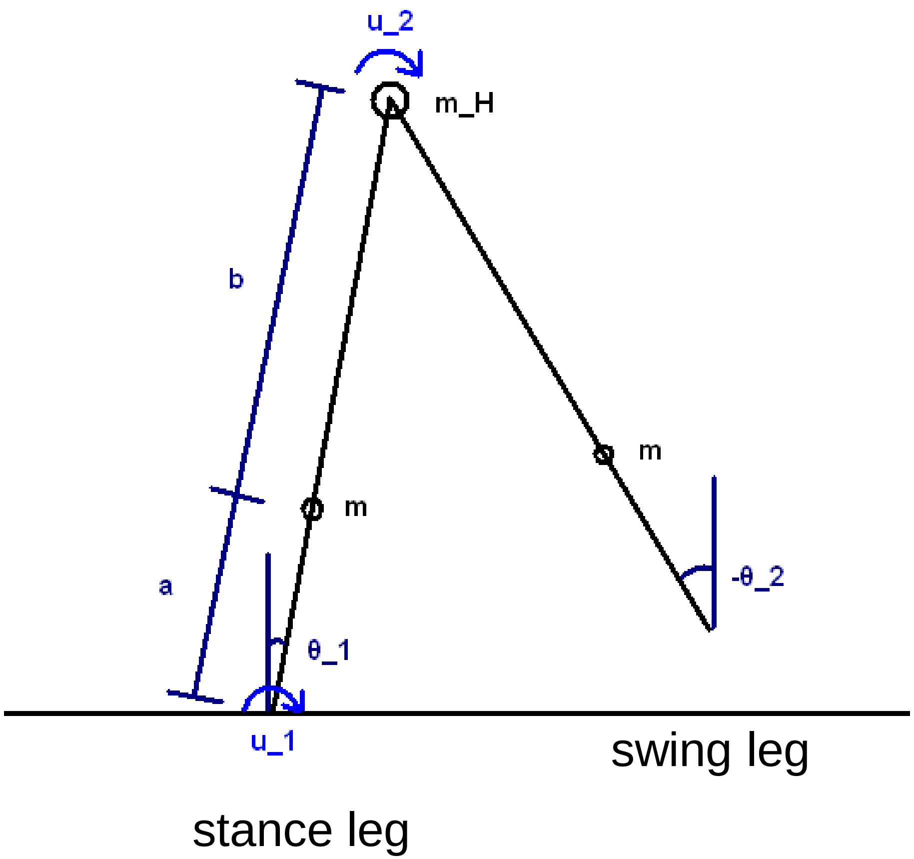
where

$M(\theta)$ inertial matrix

$C(\theta,\dot{\theta})$ coriolis matrix

$$G(\theta) = \frac{\partial P}{\partial \theta}(\theta)$$

Simple Hybrid Mechanical Control System



constraint function $V(\theta) = \cos\theta_1 - \cos\theta_2$

SHMCS a 6-tuple

$\mathrm{H} = (D, U, G, R, f, g)$

where

$$D = \left\{ \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \in \mathbf{R}^4 : V(\theta) \geq 0 \right\}$$

$$G = \left\{ \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \in \mathbf{R}^4 : V(\theta) = 0, \left[\frac{\partial V}{\partial \theta}(\theta) \right]^T \dot{\theta} < 0 \right\}$$

R from kinematic constraint function

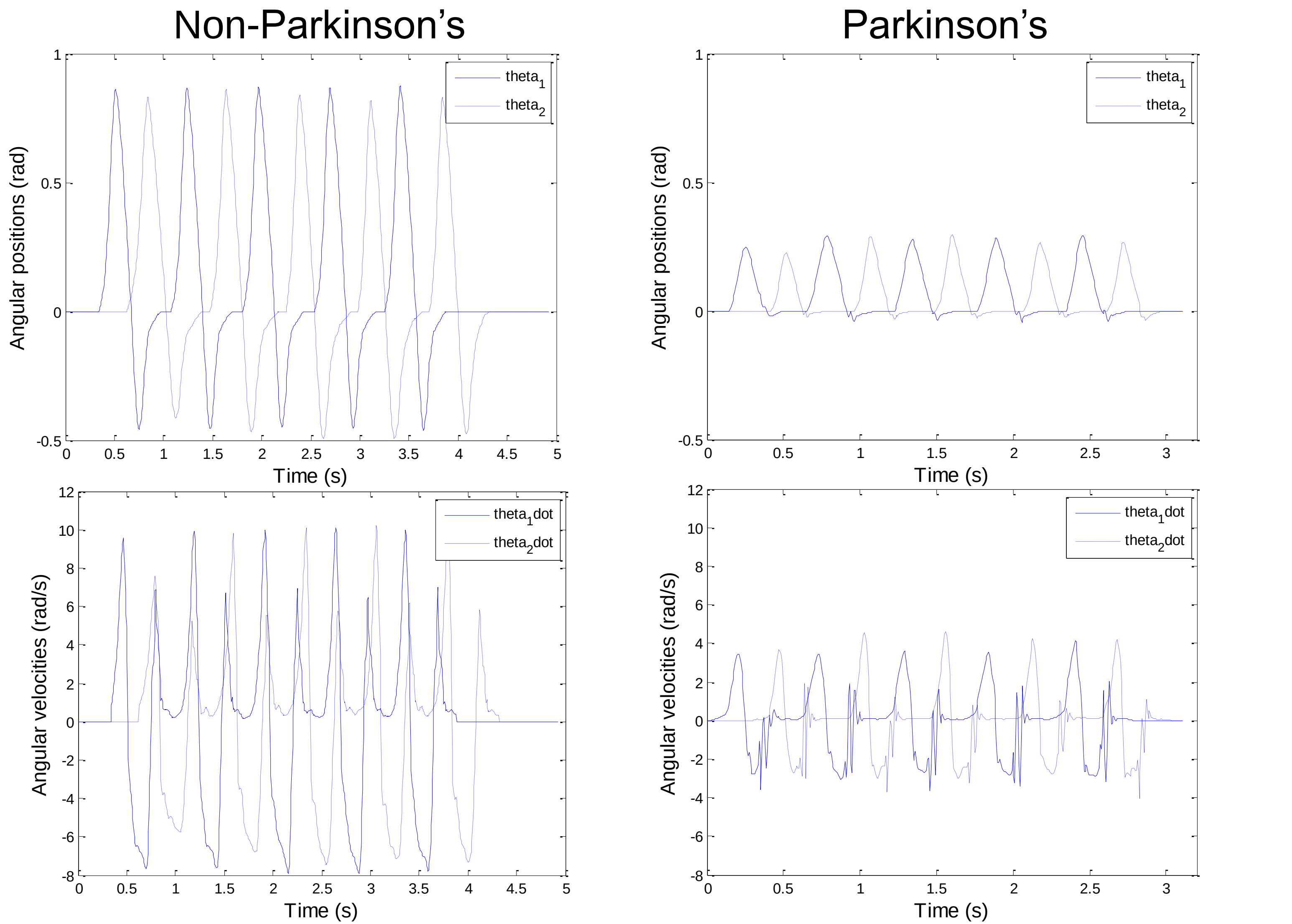
$U = \mathbf{R}^2$

$$f(\theta, \dot{\theta}) = \begin{bmatrix} \dot{\theta} \\ M(\theta)^{-1}(-C(\theta,\dot{\theta})\dot{\theta} - G(\theta)) \end{bmatrix}$$

$$g(\theta, \dot{\theta}) = \begin{bmatrix} 0_{2x2} \\ M(\theta)^{-1} S \end{bmatrix}$$

Implementation of Control: θ and $\dot{\theta}$

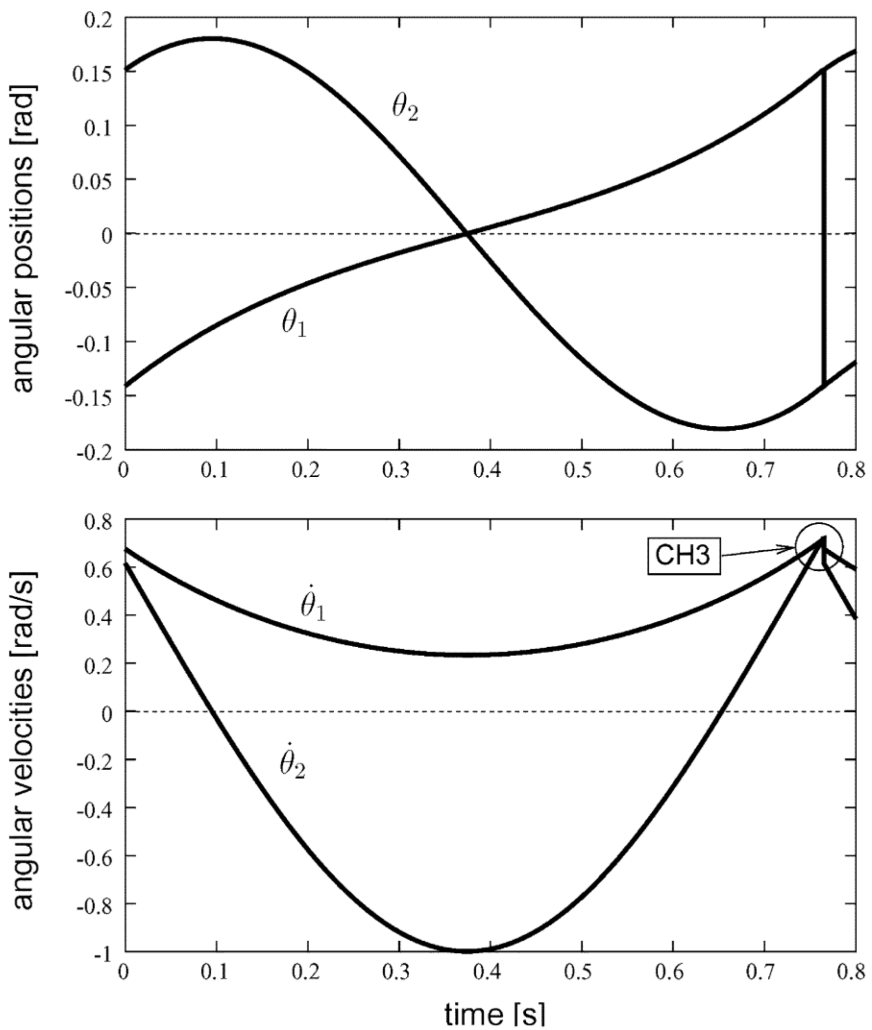
- Experimental human motion data (five cycles)



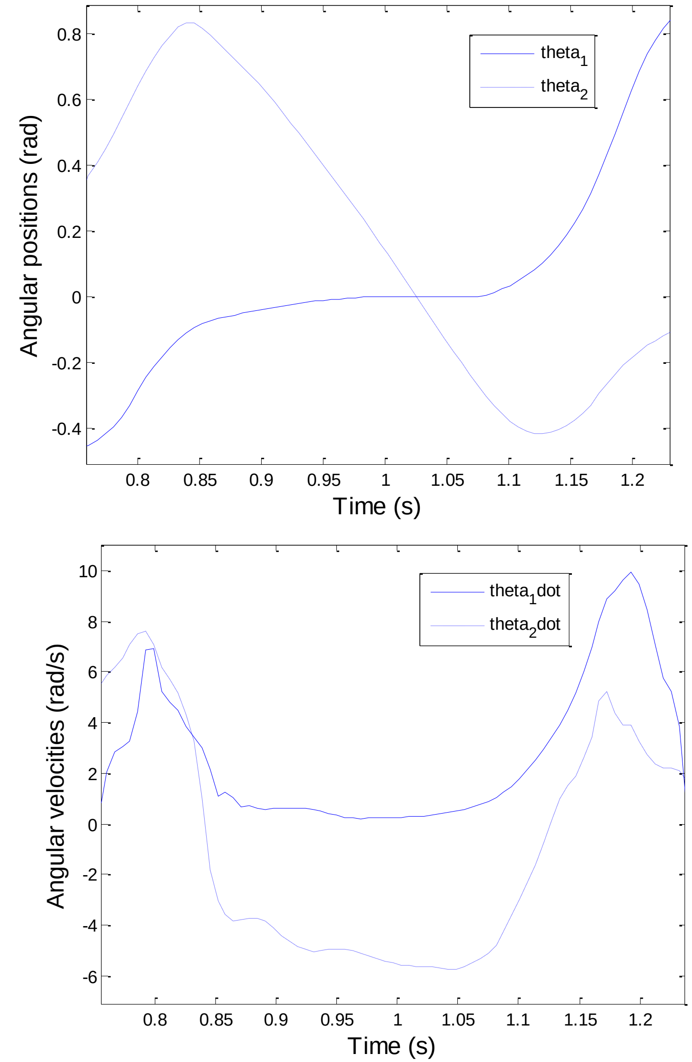
Comparison with Biped Model: θ and $\dot{\theta}$

- One cycle (reference for left figures: Asano, 2004)

Biped Model



Non-Parkinson's



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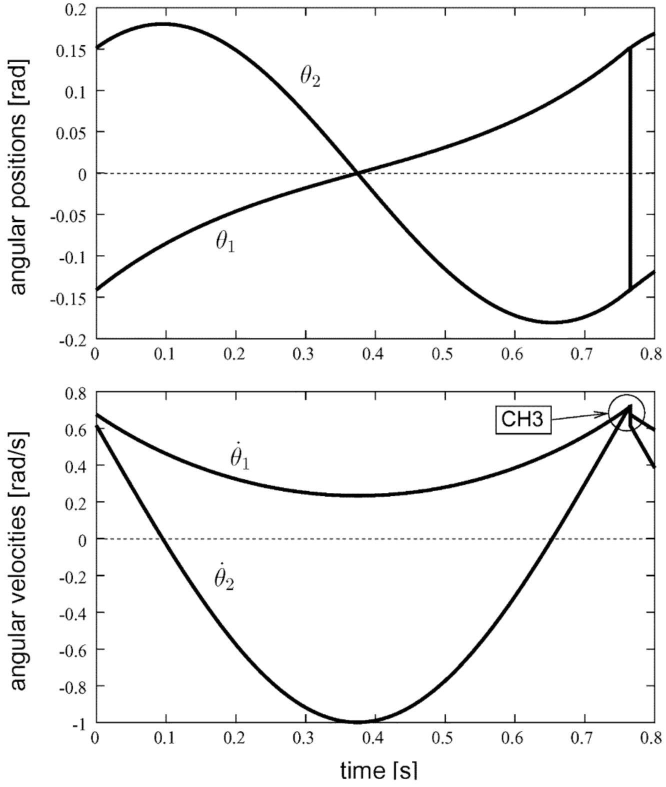
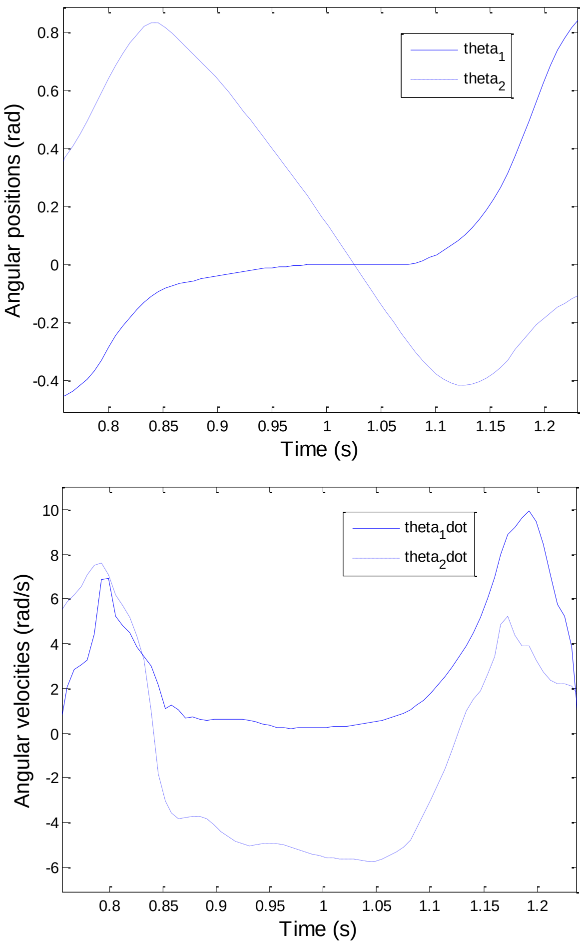
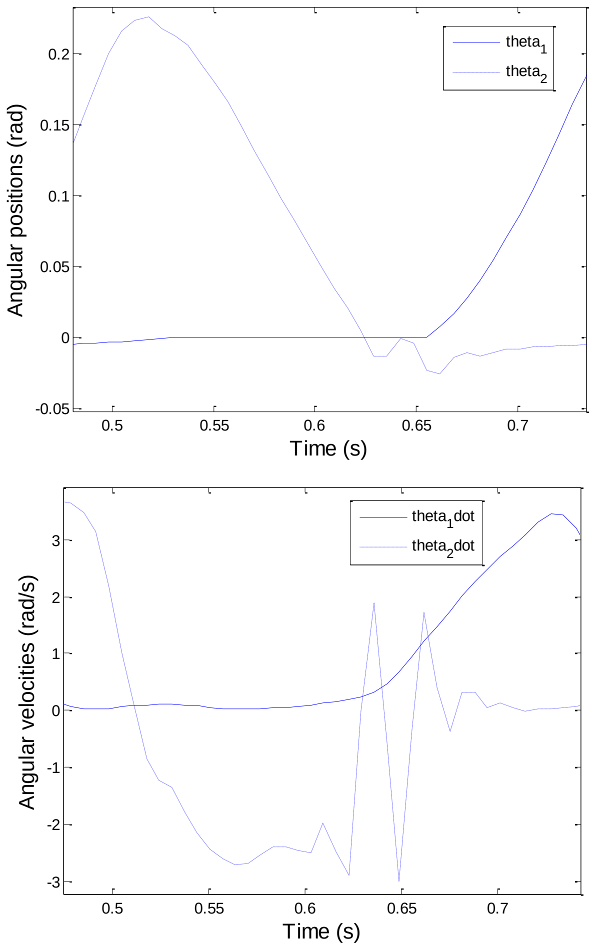
Biped Model



Non-Parkinson's



Parkinson's

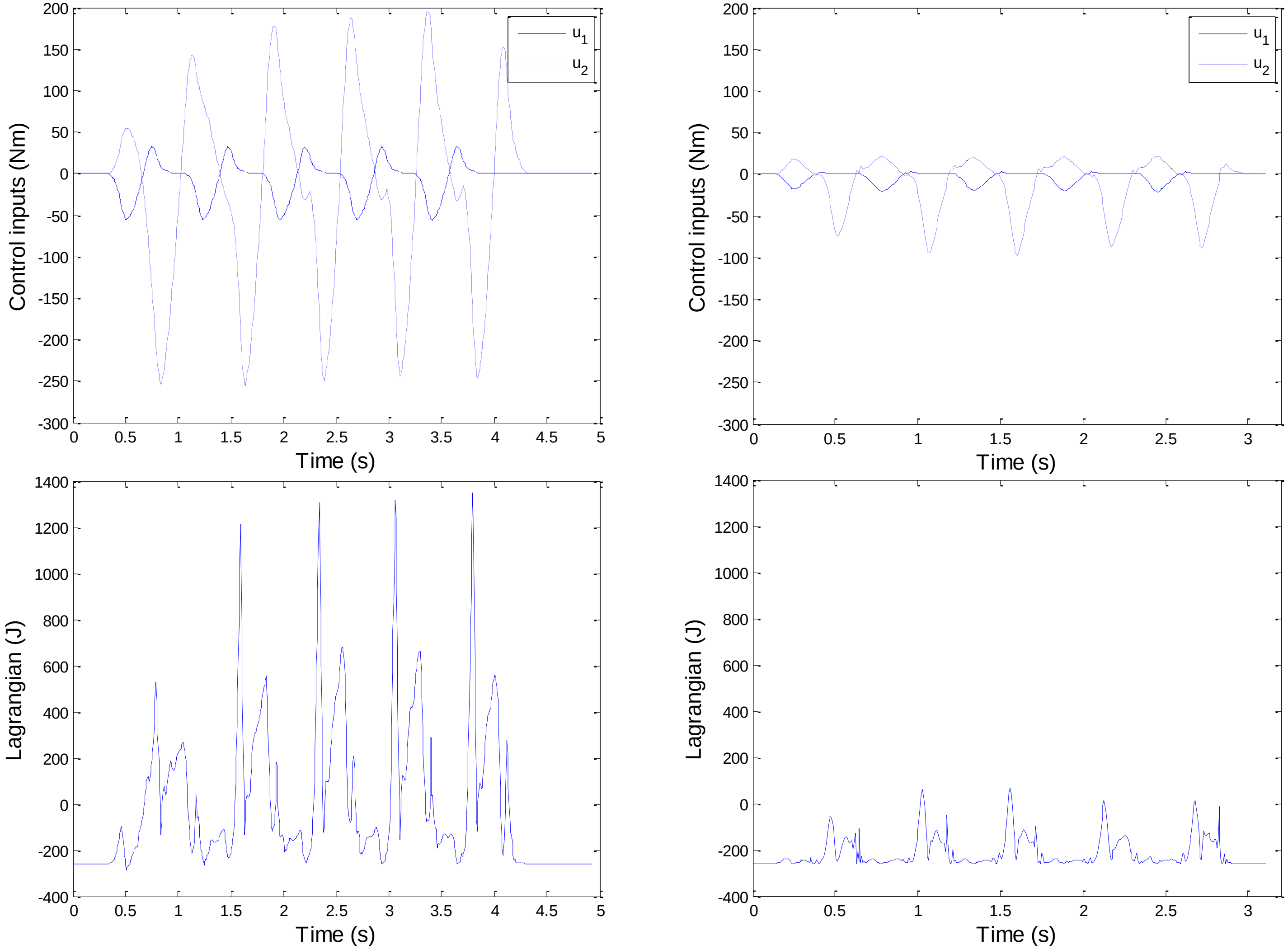


Implementation of Control: u and L

- Experimental human motion data (five cycles)

Non-Parkinson's

Parkinson's

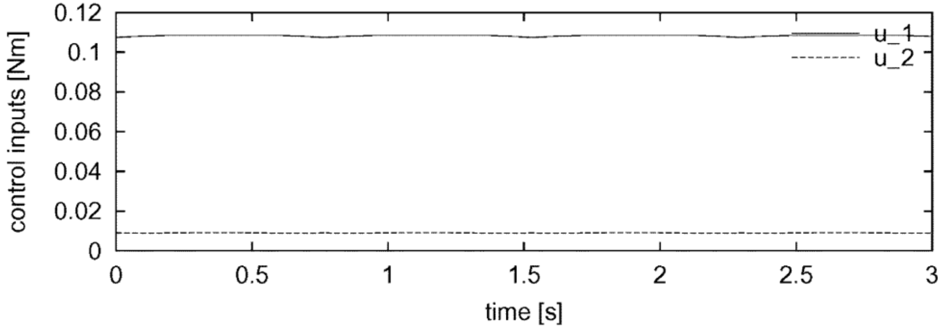


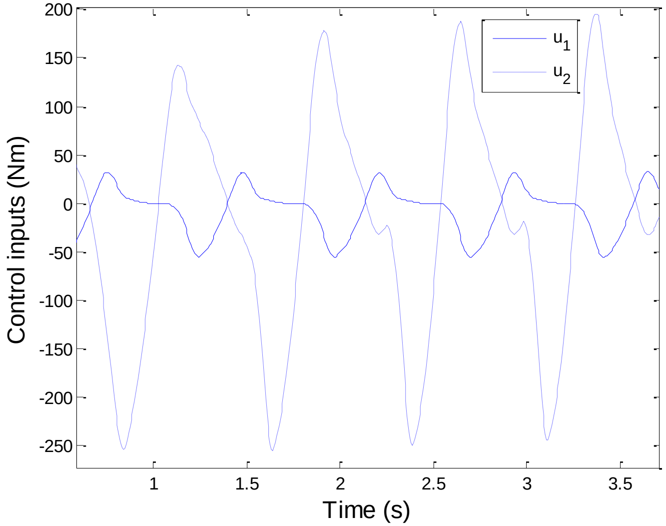
Comparison with Biped Model: u and L

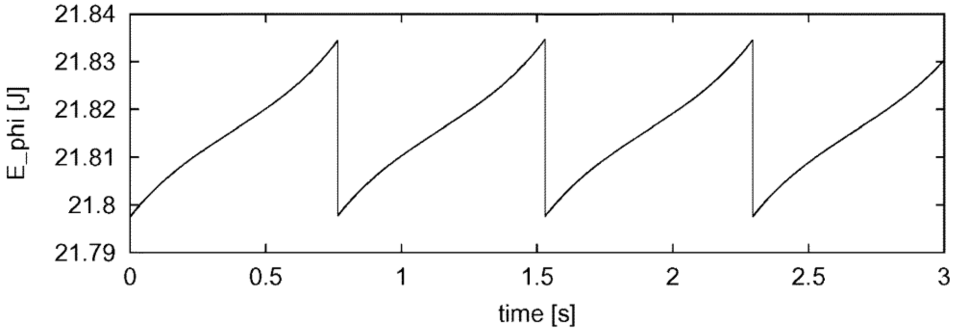
- Four cycles (reference for left figures: Asano, 2004)

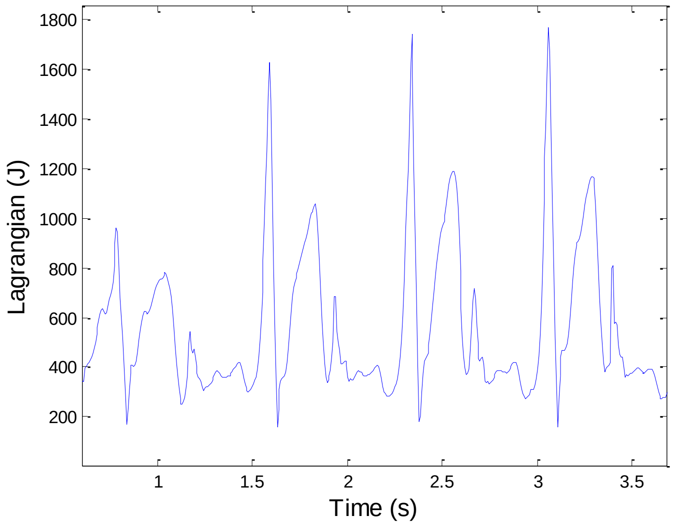
Biped Model

Non-Parkinson's









Comparison with Biped Model: u and L

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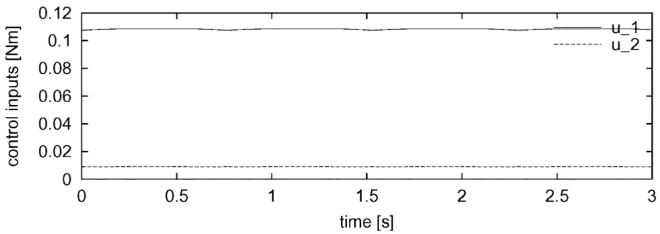
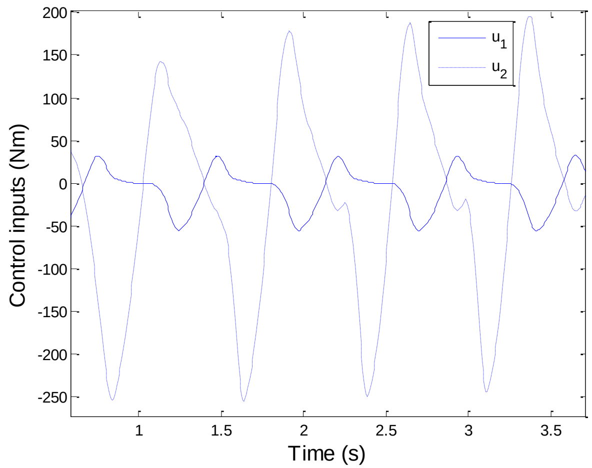
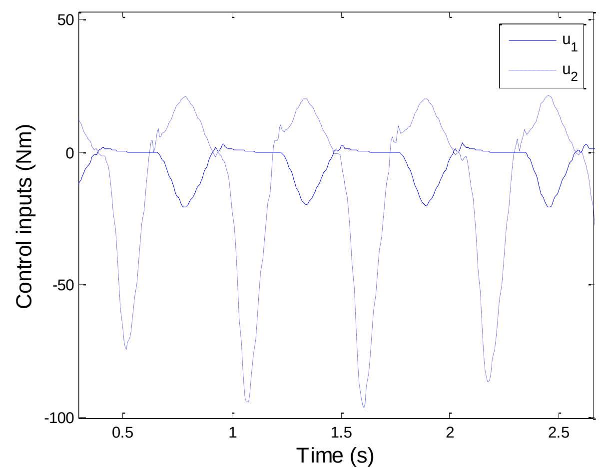
Biped Model

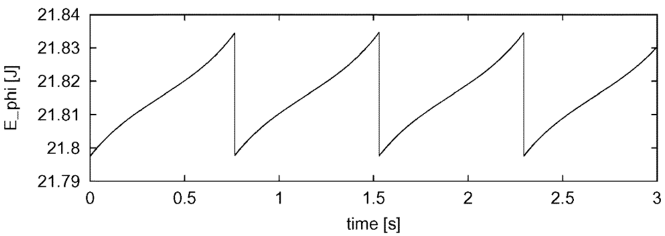
Non-Parkinson's

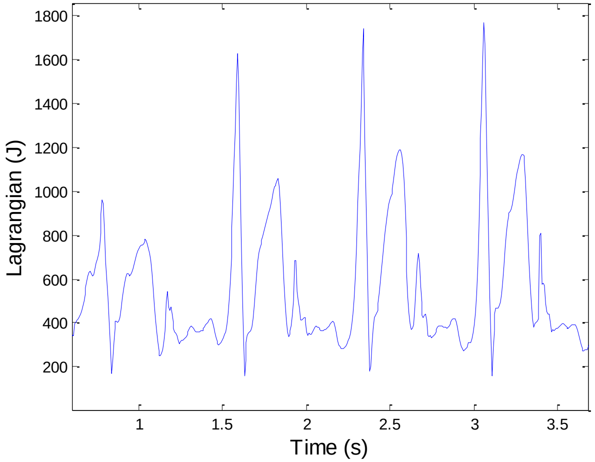
Parkinson's

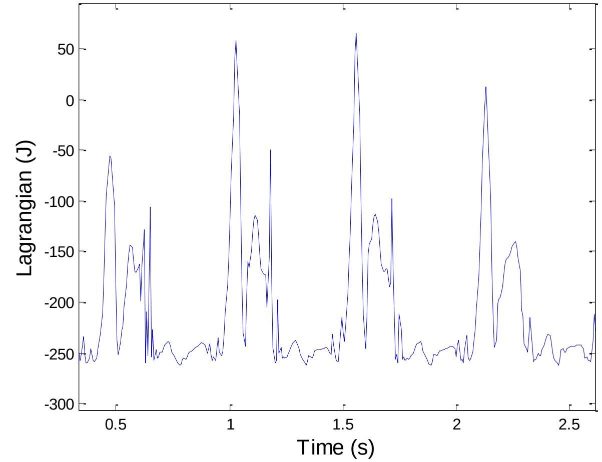










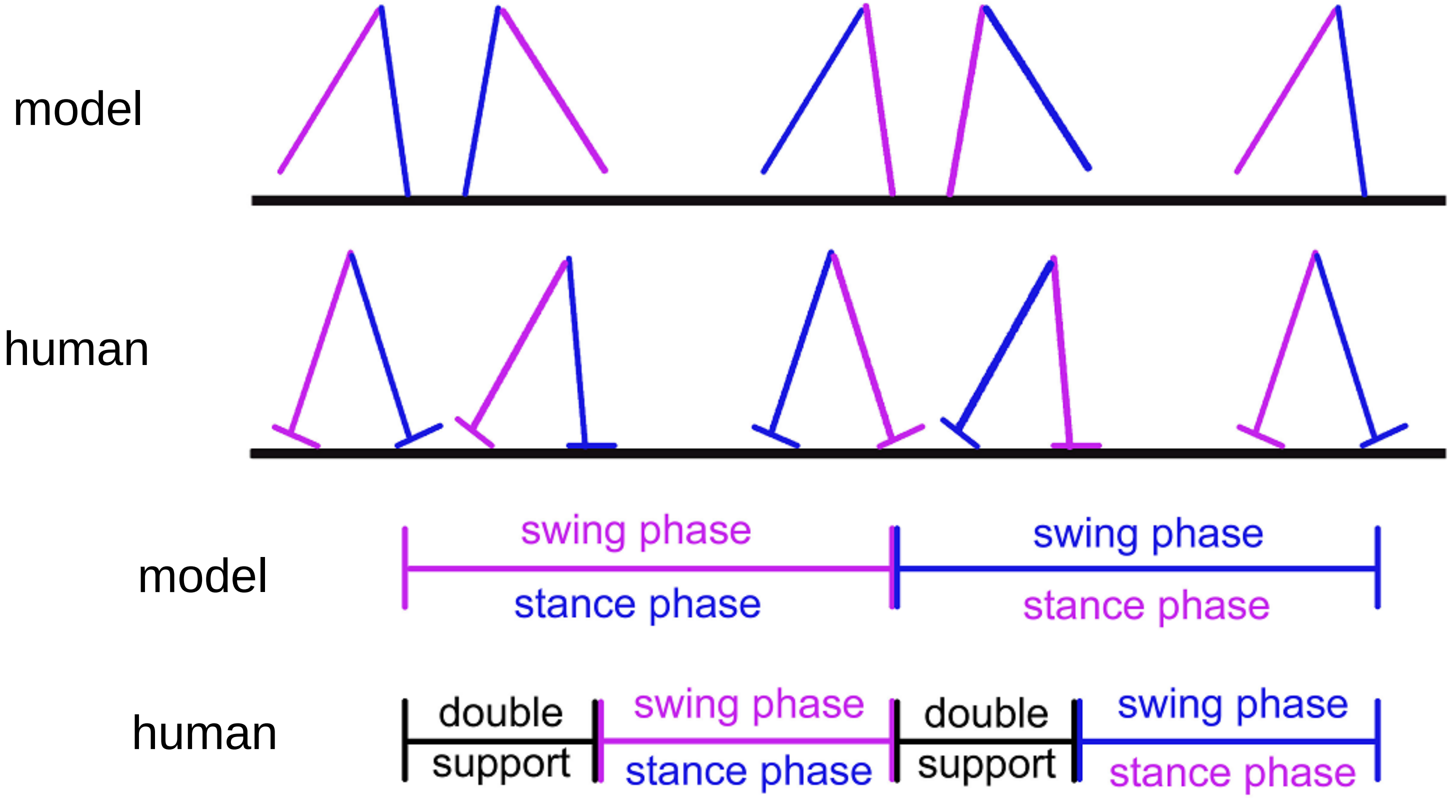


Refinements of Model

- Assumptions of biped model:
  - Impacts are perfectly inelastic (no bounce)
  - No slipping of the stance leg at ground contact
  - Transfer of support between swing and stance legs is instantaneous, i.e. negligible double support phase

Refinements of Model Cont'd

– Transfer of support between swing and stance legs is instantaneous, i.e. negligible double support phase



Summary

- Research project background
  - Quantitative diagnosis of Parkinson’s disease
- Approach for distributed system
  - Simple hybrid mechanical control system
- PDE model
  - Compass gait biped
- Implementation of control
  - Human motion data for non-Parkinson’s and Parkinson’s subjects
- Refinements of model
  - Incorporate double support phase

References

- Ames, A. D., Gregg, R. D., Wendel, E. D. B., and Sastry, S., “Towards the geometric reduction of controlled three-dimensional bipedal robotic walkers,” in *3rd Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control*, Nagoya, Japan, 2006.
- Ames, A. D., Gregg, R. D., “Stably extending two-dimensional bipedal walking to three dimensions,” in *Proc. of the American Control Conference*, 2007, pp. 2848-2854.
- Asano, F., Yamakita, M., Kamamichi, N., and Luo, Z., “A novel gait generation for biped walking robots based on mechanical energy constraint,” *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, 2004, pp. 565-573.
- Choi, J. H. and Grizzle, J. W., “Planar bipedal walking with foot rotation,” in *Proc. of the American Control Conference*, 2005, pp. 4909-4916.
- Grizzle, J. W., Abba, G., and Plestan, F., “Asymptotically stable walking for biped robots: Analysis via systems with impulse effect,” *IEEE Transactions on Automatic Control*, vol. 46, no. 1, 2001, pp. 51-64.
- Pekarek, D., Ames, A. D., Marsden, J. E., “Discrete mechanics and optimal control applied to the compass gait biped,” in 46th *IEEE Conference on Decision and Control*, 2007, pp. 5376-5382.
- Spong, M. W., “Passivity based control of the compass gait biped,” in *Proc. of IFAC World Congress*, Beijing, China, 1999.
- Spong, M. W. and Bullo, F., “Controlled symmetries and passive walking,” *IEEE Transactions on Automatic Control*, vol. 50, no. 7, 2005, pp. 1025-1031.