Reachability/Unsafe Region Analysis for Drifters

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Presentation Outline

- Introduction
- Motivation and Problem Statement
- Mathematical Tools
- Implementation
- Results

Introduction

http://float.berkeley.edu/drifters

Motivation

- A network of drifters in a river can provide realtime updates on river conditions.
- Long term operation requires avoiding obstacles and collision with the shore.

Problem Statement

- Regions of unsafe operation must be determined in order to prevent collisions between the drifter and the shore.

Mathematical Tools

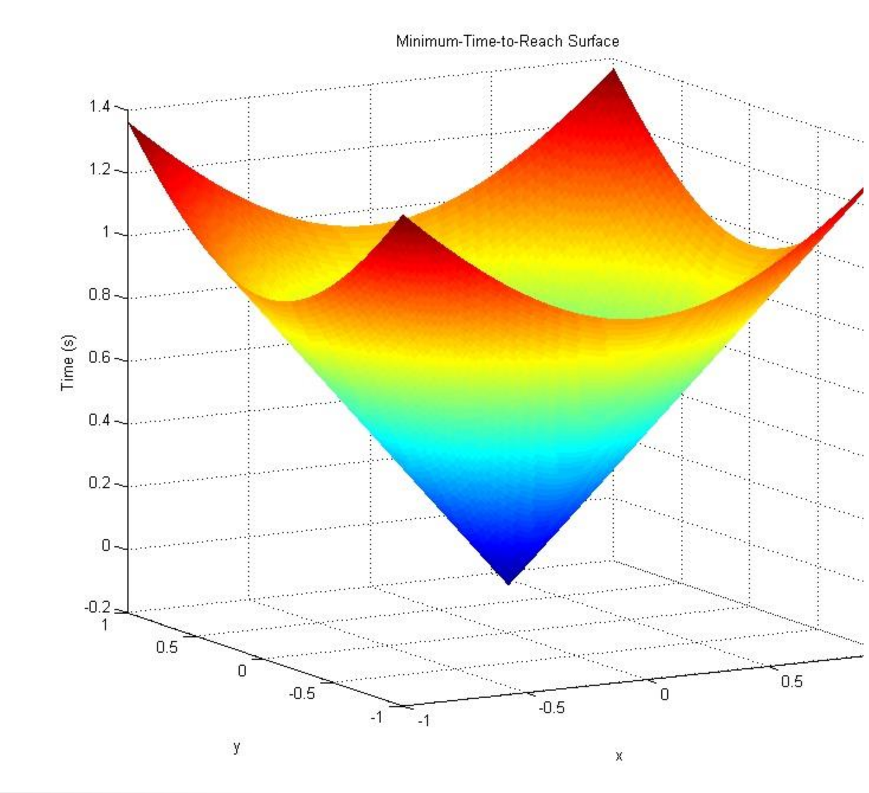
- Consider a simple dynamical system with bounded input

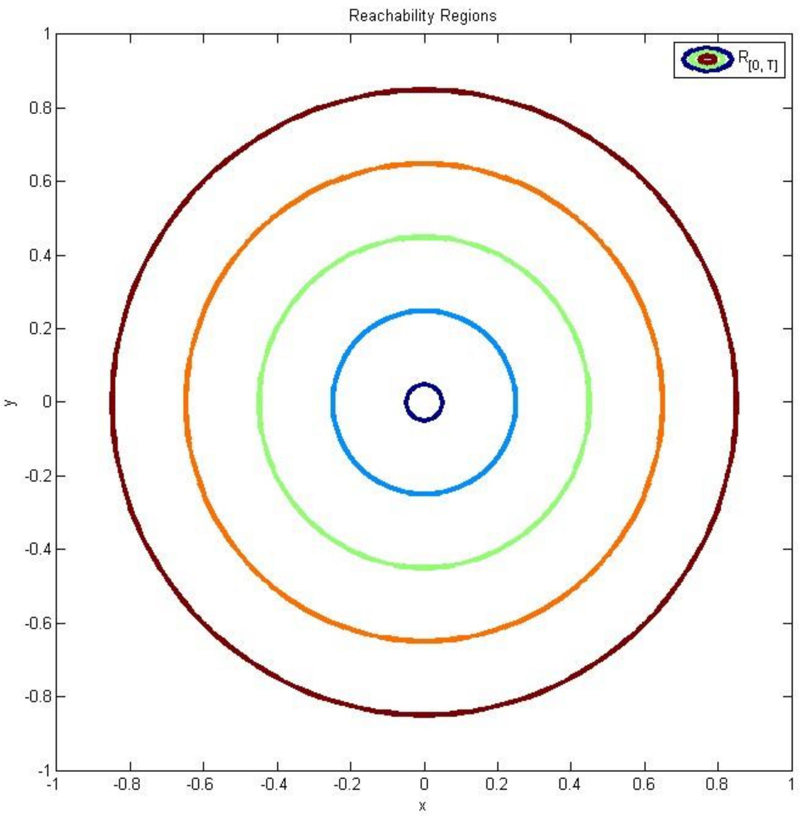
$$\dot{x} = a_x(t)$$
$$\dot{y} = a_y(t)$$
$$\|a\|_2 \leq \bar{a}$$

- How do we define reachability?
  - Assume $x_0 = (0,0)$, then reachable set is the set of states such that there exists a sequence of actions a(t) to maneuver the system to that set
  - For this system, R^2 is reachable

Mathematical Tools

- Minimum-Time-to-Reach is an extension upon this idea
- By the time $t = \tau$, what states are reachable?





Mathematical Tools

- Backward Reachability – Modify model to include constant disturbance

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} a_x(t) \\ a_y(t) \end{bmatrix} + \begin{bmatrix} 0.75 \\ 0.5 \end{bmatrix}$$

- Define a target set, T

$$T = \{(x, y) |\ x^2 + y^2 \leq 1\}$$

- From what states can we guarantee controllability into T?

Mathematical Tools

- From what states can we guarantee controllability into T?

Implementation

- We start with a dynamics model

$$\dot{x} = f(x, a, b)$$

- x is the state
- a defines drifter inputs
- b defines disturbance inputs

$$\dot{x} = w_x(x, y) + a_x(t) + b_x(t)$$
$$\dot{y} = w_y(x, y) + a_y(t) + b_y(t)$$
$$\|a\|_2 \le \bar{a}, \qquad \|b\|_2 \le \bar{b}$$

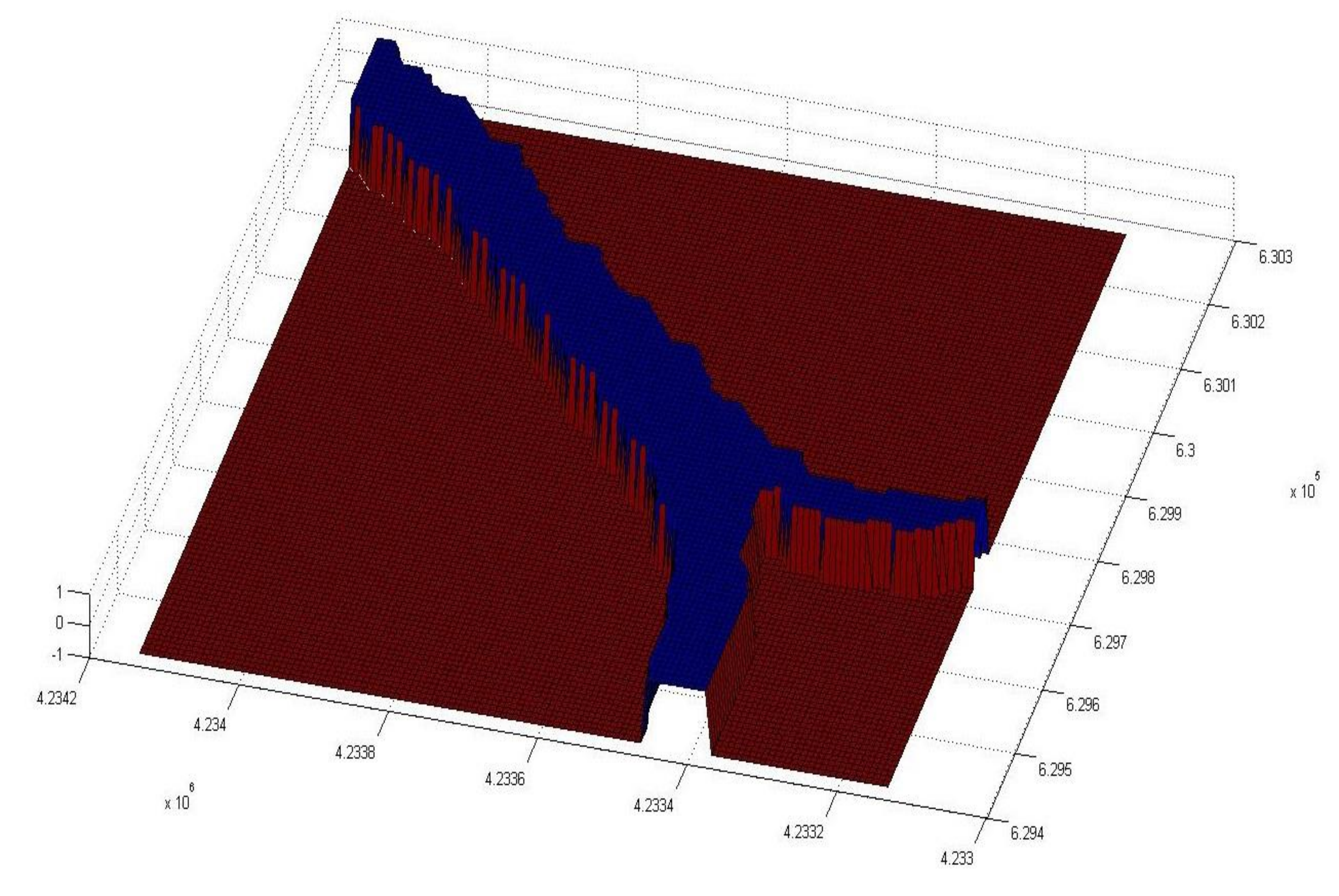
Hamilton-Jacobi-Isaacs PDE

- Use numerical solver (Level Set Toolbox) to solve HJI PDE for $v(x,t)$:

$$v_t(x,t) + \min\left[0, H\left(x, v_x(x,t)\right)\right] = 0$$

- Need to define a target set, T, and Hamiltonian
  - $v(x,0) = v_0(x) = \begin{cases} -1, & for\ x \in T \\ +1, & otherwise \end{cases}$
  - $H(x,p) = \max_{\|a\|_2 \leq \bar{a}} \min_{\|b\|_2 \leq \bar{b}} p^T f(x,a,b)$

Target Set Definition

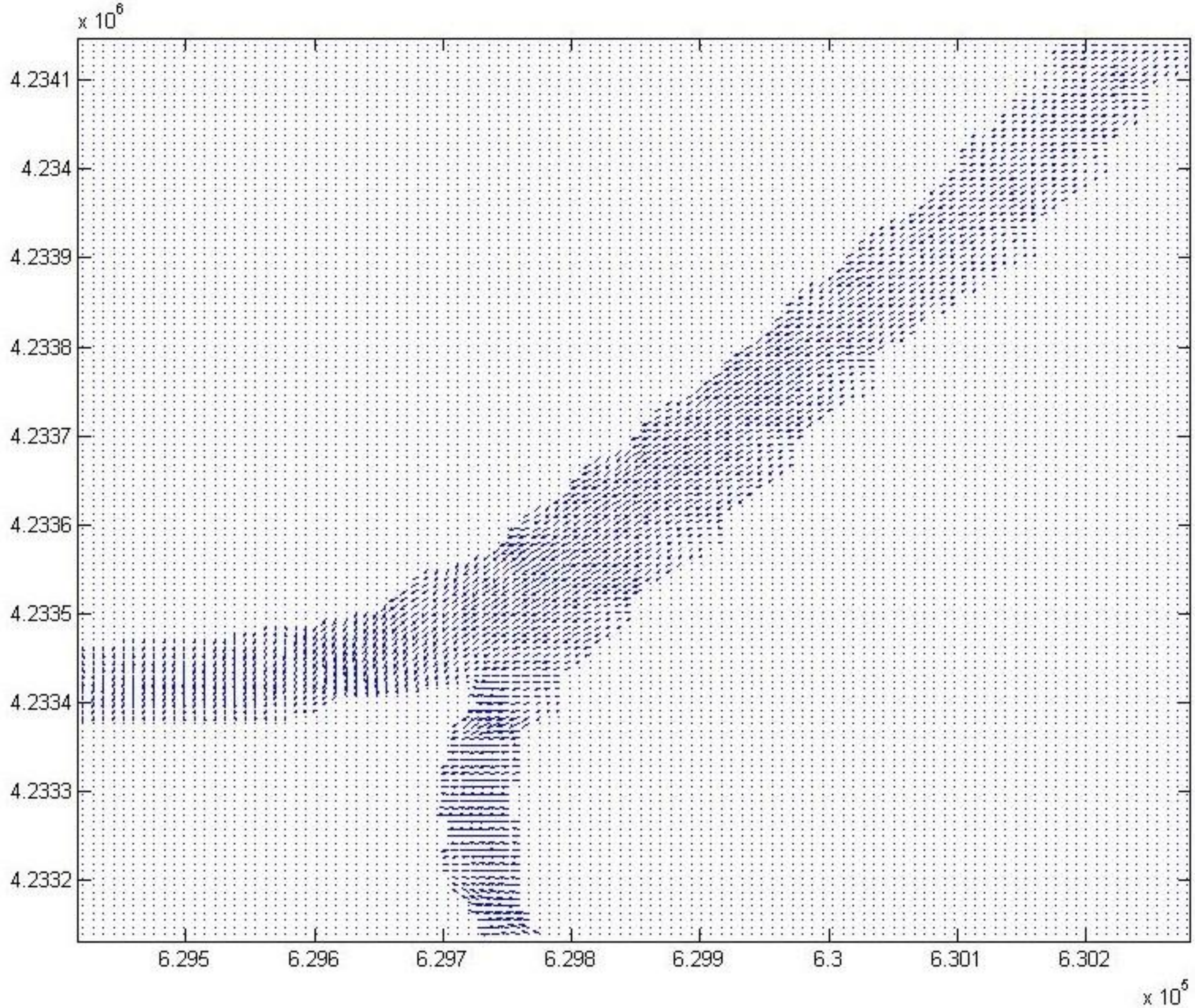


Hamiltonian

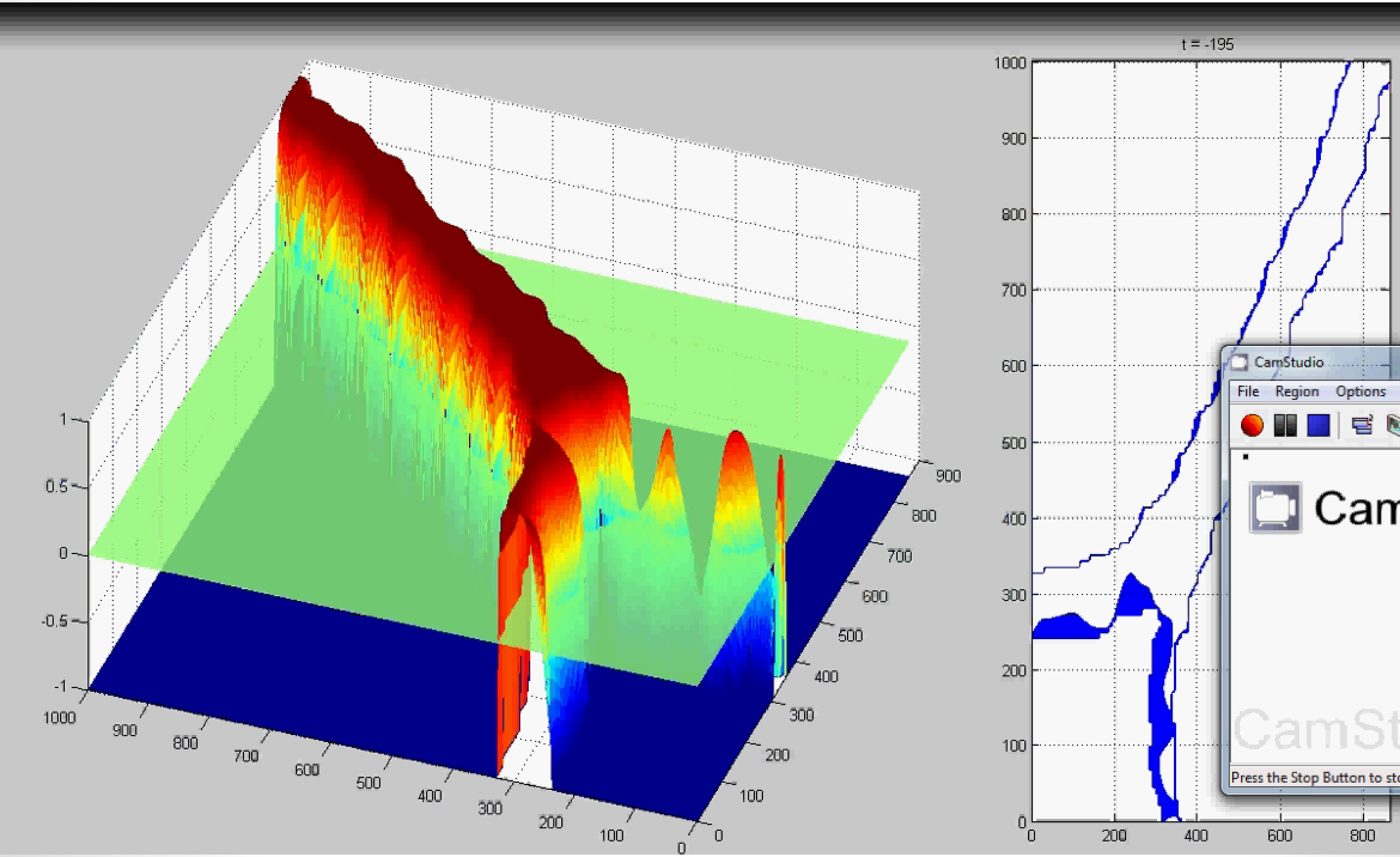
- $H(x,p) = \max_{\|a\|_2 \le \bar{a}} \min_{\|b\|_2 \le \bar{b}} \; p_1[w_x + a_x + b_x] + p_2[w_y + a_y + b_y]$

$$= p_1 w_x + p_2 w_y + [\bar{a} - \bar{b}]\sqrt{}(p_1^2 + p_2^2)$$

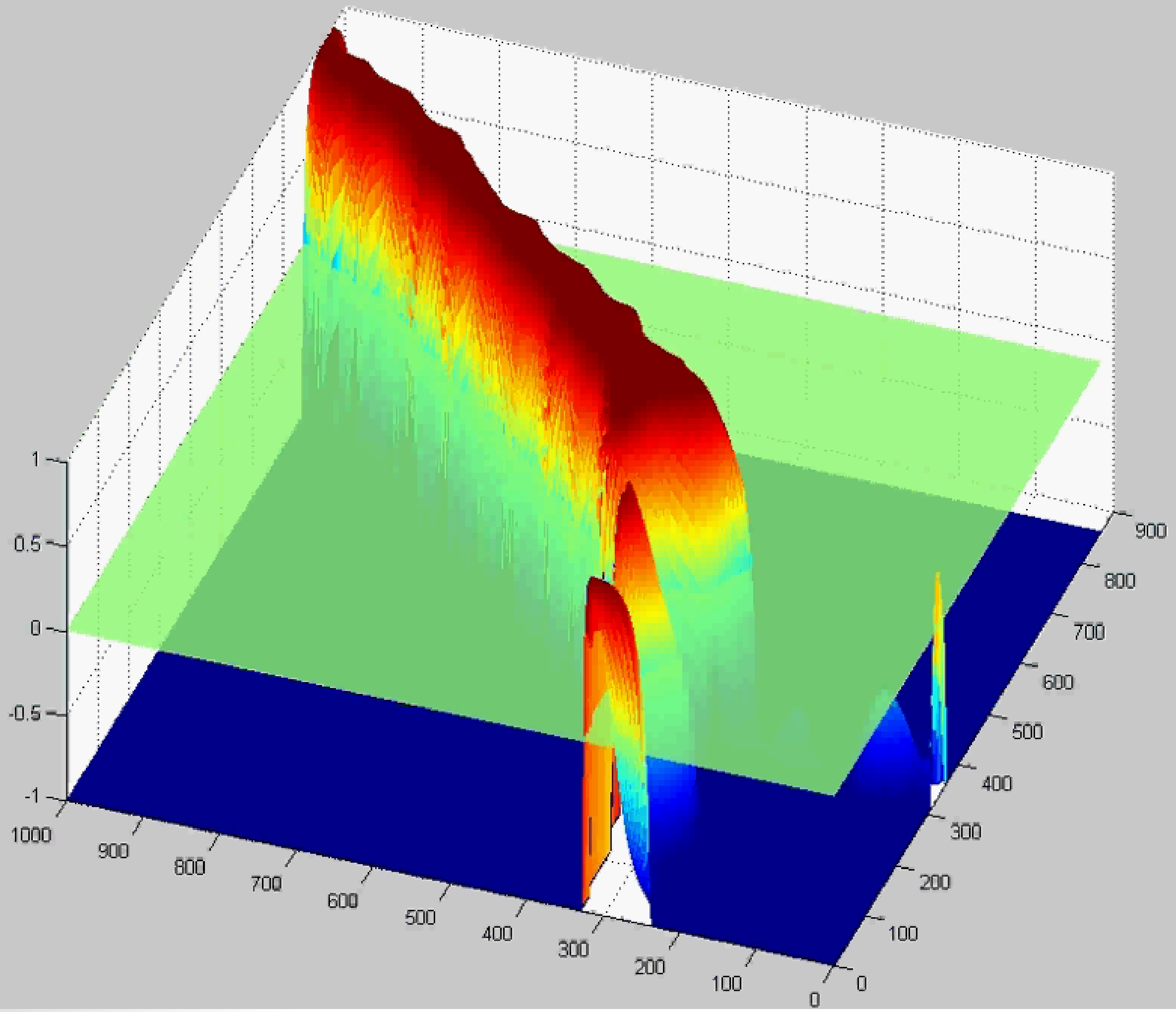
Flowfield Data

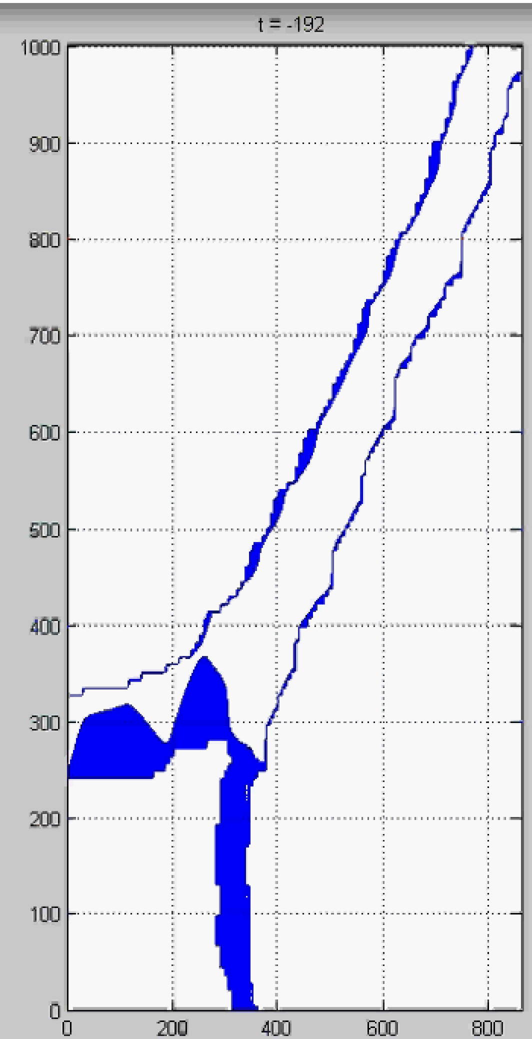


Visualization of $v(x, t)$

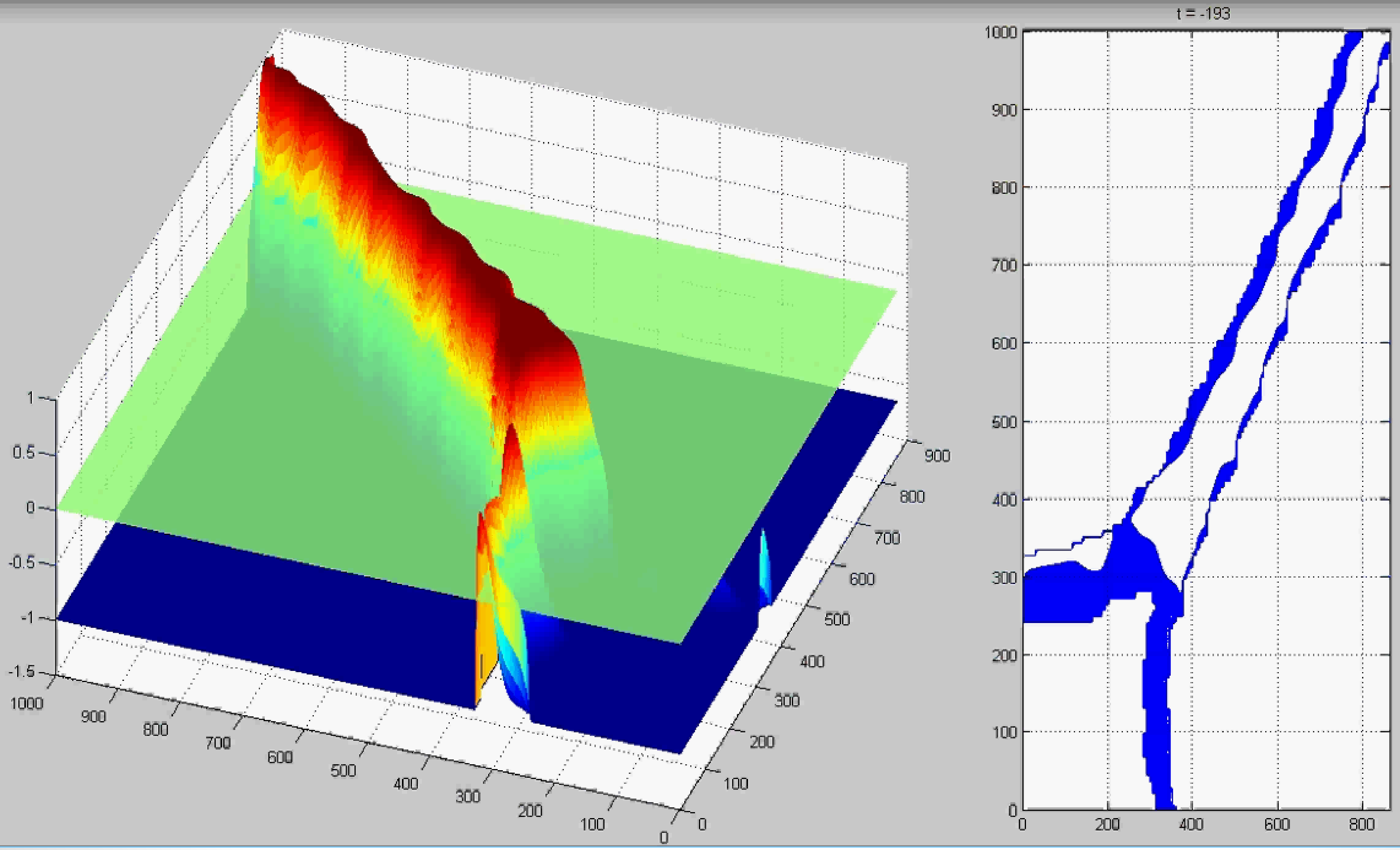


Time-varying flowfield

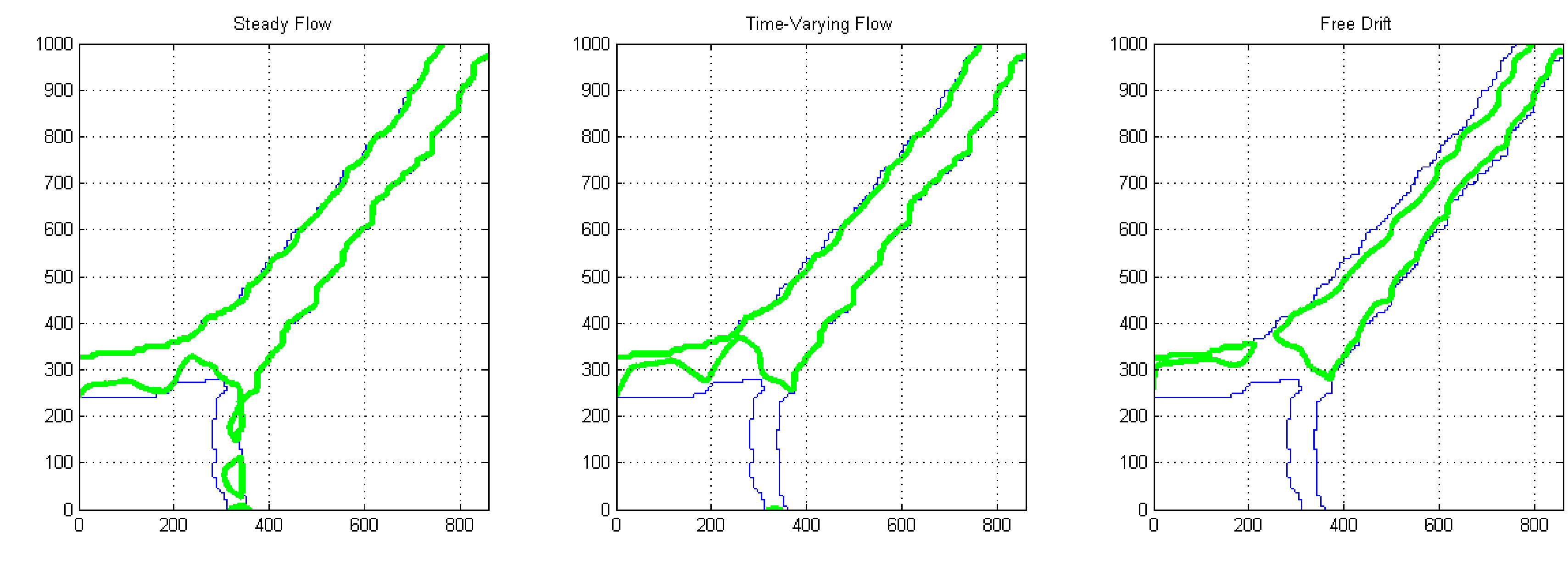




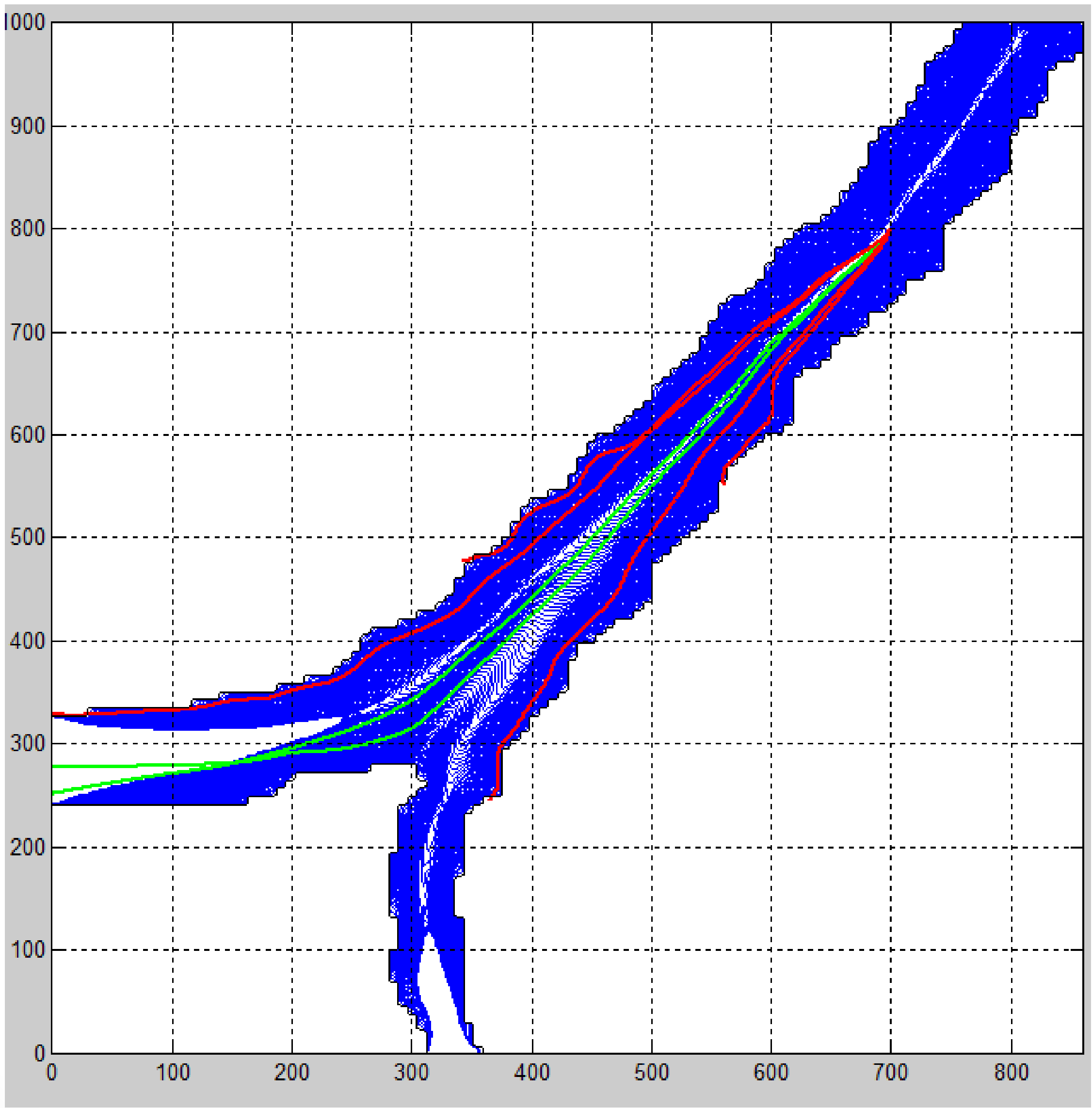
No Inputs – Free Drift



Comparisons



Simulated Trajectories



Conclusion

- This method has seen actual use in determining safe/unsafe regions for determining control strategies
- Modeling Assumptions
  - Unrealistic model
- Visualize safe and unsafe trajectories

References

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- REALM, River Estuary and Land Model.

Thanks!