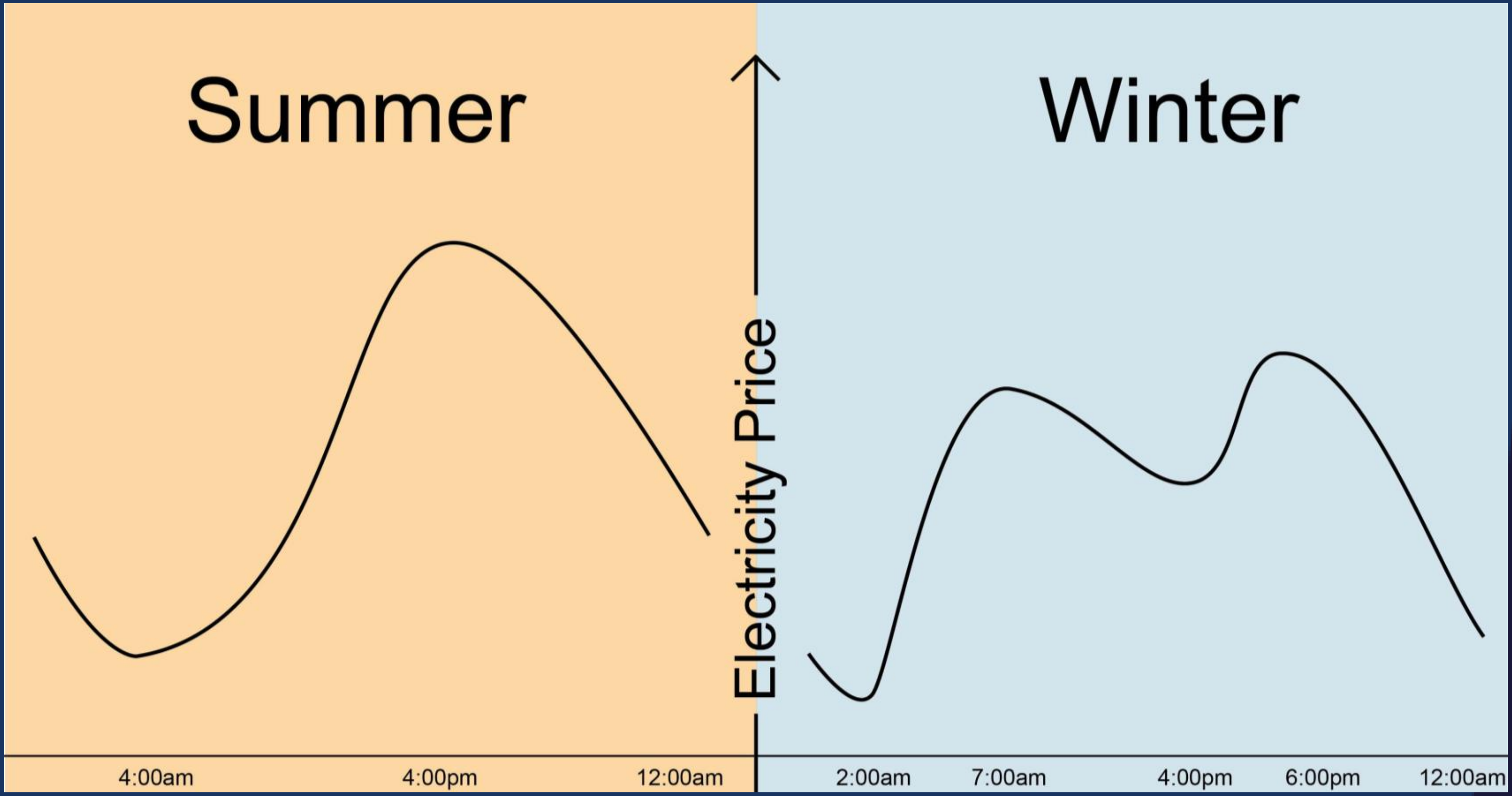
Optimal charging of a V2G aggregator system

François Debaillon – Caroline Le Floch

May 6, 2014

http://www.energystream-solucom.fr/2013/03/vehicle-to-grid-le-vehicule-electrique-au-servic-du-reseau/



http://research.cbei.psu.edu/research-digest-reports/electricity-pricing-a-primer#prettyPhoto/1/

Battery capacity $\approx 50\ \mathrm{kWh}$
charge/discharge power: 1 – 100 kW
200 000 PEV in the US

Berkeley
UNIVERSITY OF CALIFORNIA

Objective

Minimize charging cost under constraint of supplying a minimum required power to the grid.

Literature review

- QP

  $\min \sum_{k=0}^{N-1} c(k)P(k)$

  where c(k) is cost of electricity and P(k) power supplied

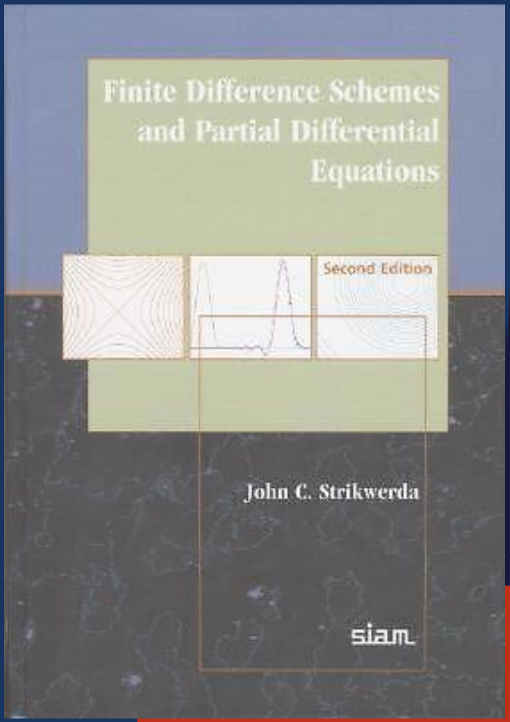
$$\min J(x) = \frac{1}{2}x^T H x + c^T x$$
$$A_{eq}x = B_{eq}$$
$$Ax \leq b$$

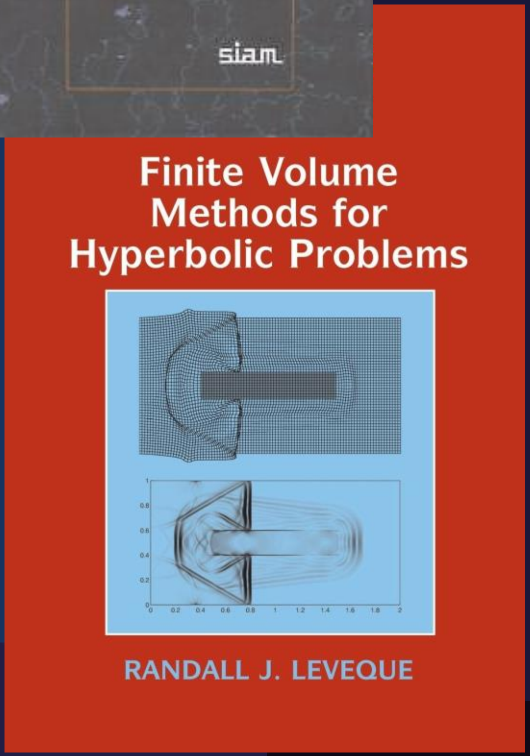
- Han et al: DP

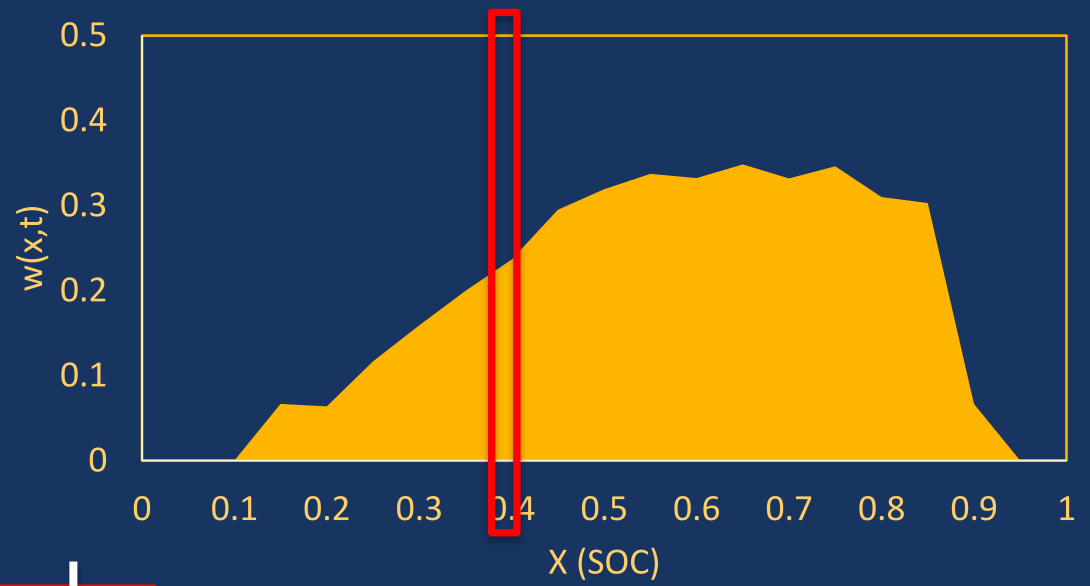
  Maximize profit for a fleet of vehicles that can supply power back to the grid. Vehicles always plugged in.

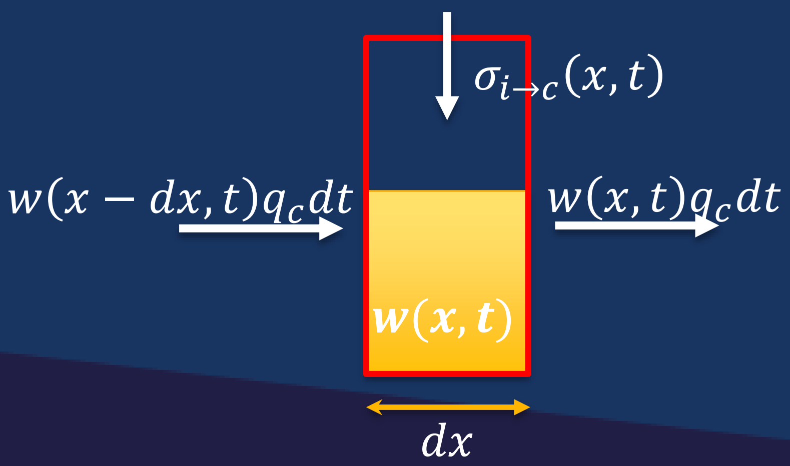
- Bashash & Fathy: first-order bilinear transport PDE, representing the charging dynamics of a G2V fleet

  Lyapunov function $V(t) = \frac{1}{2}e(t)^2$



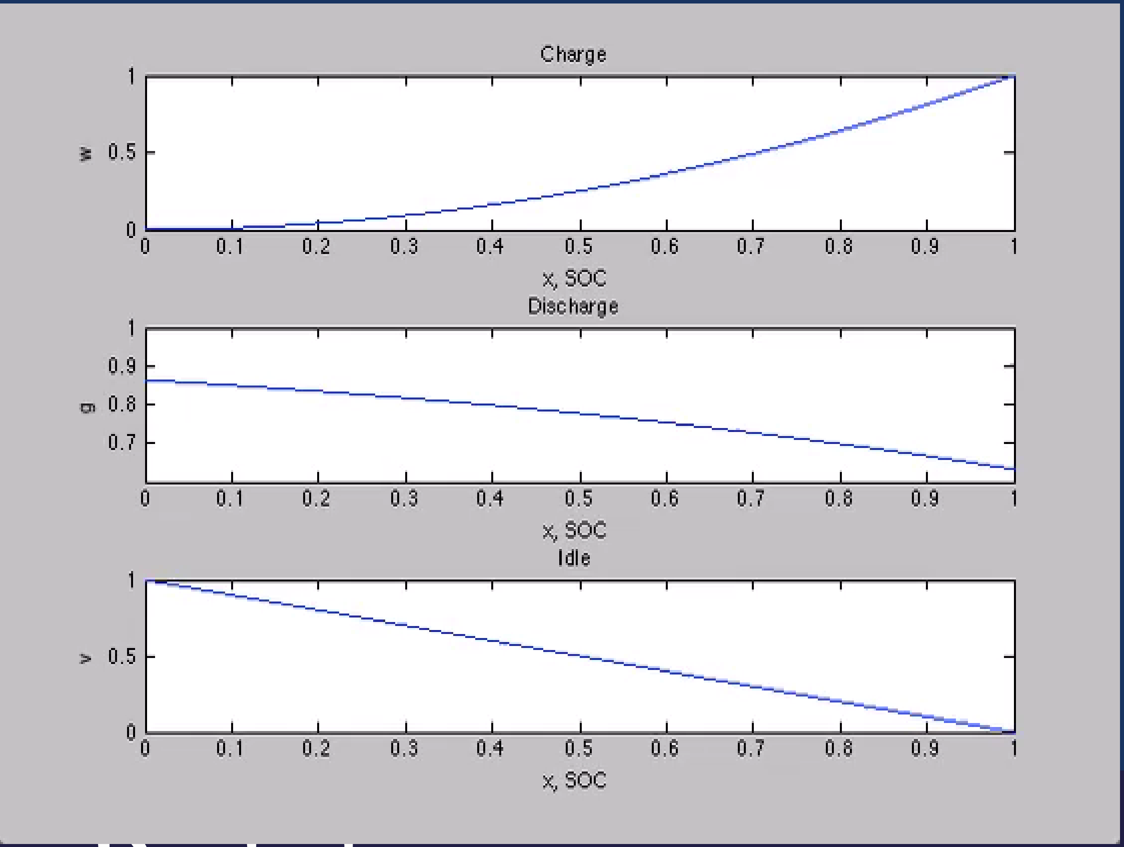






$$\frac{\partial w}{\partial t}(x,t) = -q_c \frac{\partial w}{\partial x}(x,t) + \sigma_{i \to c}(x,t)$$

- State of charge: x
- Number of vehicles charging: $w(x,t)$
- Number of vehicles discharging: $\mathrm{v}(x,t)$
- Number of idle vehicles: $\gamma(x,t)$
- Controllable inputs: $\sigma_{i\to c}(x,t), \sigma_{i\to d}(x,t)$
- Uncontrollable inputs: $\sigma_{i\to or}(x,t)$
  - Currently dertiministic
  - Improved to stochastic later on



$$\begin{cases} \dfrac{\partial w}{\partial t}(x,t) + q_c \dfrac{\partial w}{\partial x}(x,t) = \sigma_{i\to c}(x,t) \\ \dfrac{\partial \gamma}{\partial t}(x,t) - q_d \dfrac{\partial \gamma}{\partial x}(x,t) = \sigma_{i\to d}(x,t) \\ \dfrac{\partial \gamma}{\partial t}(x,t) = -[\sigma_{i\to c} + \sigma_{i\to d} + \sigma_{i\to or}] \end{cases}$$

Assumptions / Problem statement

Objective: Minimize the cost of charging the vehicles

Constraints:

- Power: supply enough power to the grid
- Demand : supply enough charged cars to customers
- Physics :
  - Dynamics of the charge (PDE formulation)
  - no overcharging / undercharging of batteries
- Initial conditions

Berkeley
UNIVERSITY OF CALIFORNIA

Assumptions / Problem statement

Objective: Minimize the cost of charging the vehicles

Assumptions:

Parameters known one day in advance:

- Price of electricity
- Arrivals and Demand of cars

Controls: flows of vehicles between different categories:

- From idle to charge / idle to discharge $\sigma_{i->d}$ $\sigma_{i->d}$
- Departure of vehicles: $Dep(x,t)$

Formulation

Cost function:

$$\min_{\sigma_{i\to d},\sigma_{i\to c},Dep} C = \int_{t=0}^{T_{max}} C_{elec}(t) \int_0^{x_{max}} w(x,t)q\,dx\,dt$$

subject to

Dynamics of the system

$$w_t(x,t) = -q_c w_x(x,t) + \sigma_{i\to c}(x,t)$$
$$\gamma_t(x,t) = q_d \gamma_x(x,t) + \sigma_{i\to d}(x,t)$$
$$v_t(x,t) = -[\sigma_{i\to c}(x,t) + \sigma_{i\to d}(x,t)] + Arr(x,t) - Dep(x,t)$$

Boundary conditions

$$w(x,t) = 0 \ \forall x \geq x_{Cmax}$$
$$\gamma(x,t) = 0 \ \forall x \leq x_{Dmin}$$
$$v(x,t) = 0 \ \forall x \leq x_{Dmin}$$

Initial conditions

$$w(x,0) = w_0(x), \ \gamma(x,0) = \gamma_0(x), \ v(x,0) = v_0(x)$$

Grid power constraint

$$P_{supply}(t) = \int_0^{x_{max}} \gamma(x,t)q\,dx = P^{des}_{regul}(t)$$

Demand constraint

$$\int_{X_{dep}}^{X^{max}} Dep(x,t)\,dx = Dem(t) \ \forall t$$

Final condition

$$\int_{X_{dep}}^{X^{max}} w(x,T_{max}) + \gamma(x,T_{max}) + v(x,T_{max}) \geq N_{min}$$

Solving the optimization problem

1. Discretization:
   - Numerical schemes and stability

2. LP formulation

Lax Wendroff discretization

Peter Lax

Define:

$$w_j^n = w(x_j, t_n)$$

Where

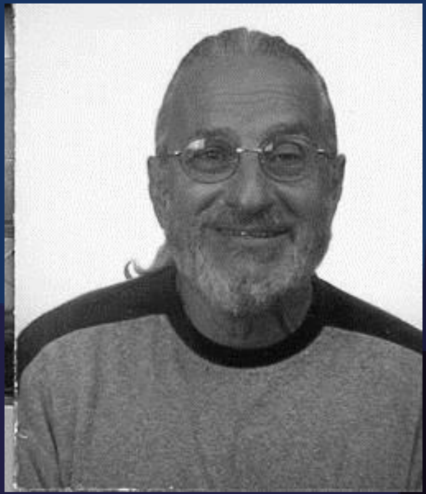
Taylor series: $w(x, t+k) = w(x,t) + k w_t(x,t) + \frac{k^2}{2} w_{tt}(x,t) + O(k^3)$

Since $w_t(x,t) + q_c w_x(x,t) = \sigma_{i\to c}(x,t)$

$$w(x, t+k) = w - q_c k w_x + \frac{q_c^2}{2} w_{xx} + k\sigma_{i\to c}(x,t) - \frac{q_c k^2}{2}\sigma_{i\to c}(x,t)_x + \frac{k^2}{2}\sigma_{i\to c}(x,t)_t + O(k^3)$$

Replacing derivatives: $w_x \approx \frac{w(x+dx)-w(x)}{dx} \approx \frac{w_{j+1}^n - w_j^n}{\Delta x}$

We obtain $w_j^{n+1} = M_c g(w_{j+1}^n, w_j^n, w_{j-1}^n)$



Burton Wendroff

Berkeley
UNIVERSITY OF CALIFORNIA

BC, IC, accuracy and stability

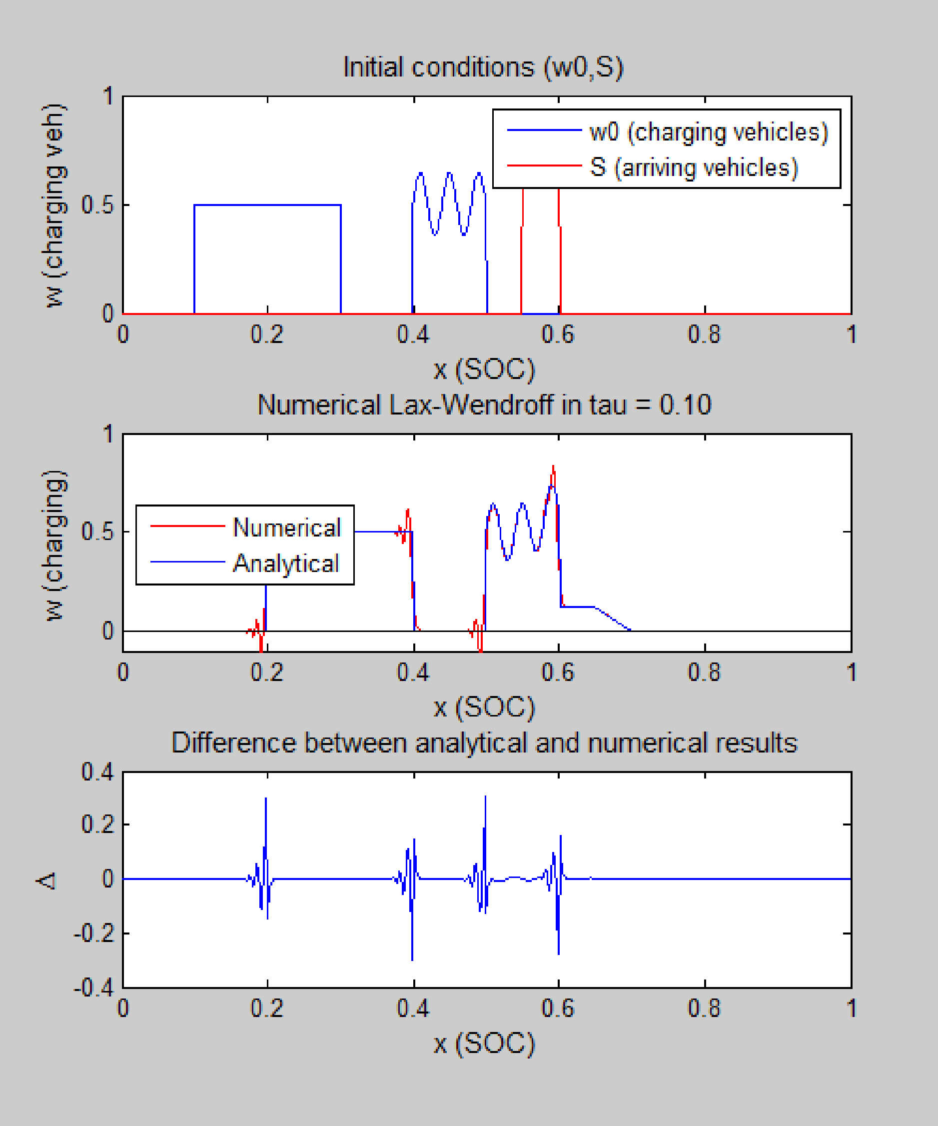
- Initial-boundary value problem.
  For problem to be well posed, initial and boundary conditions are needed.

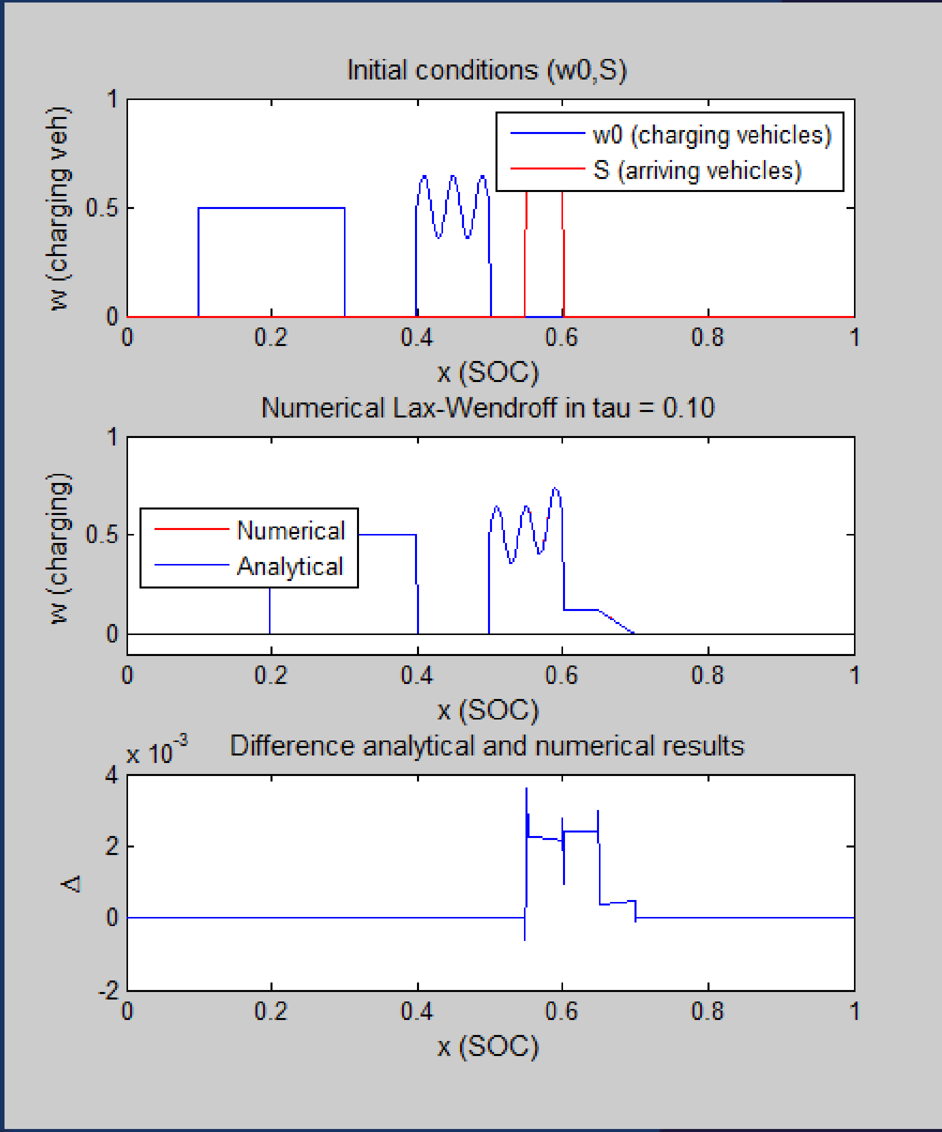
- Lax Wendroff second-order accuracy in both space and time

- Courant–Friedrichs–Lewy necessary condition for stability:

$$\left|q\frac{\Delta x}{\Delta t}\right| \leq 1$$

- In our case we choose $\left|q\frac{\Delta x}{\Delta t}\right| = 1$ because of phase problems





$$q\frac{\Delta x}{\Delta t} < 1$$

$$q\frac{\Delta x}{\Delta t} = 1$$

Berkeley
UNIVERSITY OF CALIFORNIA

Model presentation | Solving the optimization problem

$$\min_{\sigma_{i\to d},\sigma_{i\to c},Dep} C = \int_{t=0}^{T_{max}} C_{elec}(t) \int_0^{x_{max}} w(x,t)q\,dx\,dt$$

subject to

$$w_t(x,t) = -q_c w_x(x,t) + \sigma_{i\to c}(x,t)$$
$$\gamma_t(x,t) = q_d \gamma_x(x,t) + \sigma_{i\to d}(x,t)$$
$$v_t(x,t) = -[\sigma_{i\to c}(x,t) + \sigma_{i\to d}(x,t)] + Arr(x,t) - Dep(x,t)$$
$$w(x,t) = 0 \ \forall x \geq x_{Cmax}$$
$$\gamma(x,t) = 0 \ \forall x \leq x_{Dmin}$$
$$v(x,t) = 0 \ \forall x \geq x_{Dmin}$$
$$w(x,0) = w_0(x),\ \gamma(x,0) = \gamma_0(x),\ v(x,0) = v_0(x)$$

$$P_{supply}(t) = \int_0^{x_{max}} \gamma(x,t)q\,dx = P_{regul}^{des}(t)$$
$$\int_{X_{dep}}^{X^{max}} Dep(x,t)dx = Dem(t)\ \forall t$$
$$\int_{X_{dep}}^{X^{max}} w(x,T_{max}) + \gamma(x,T_{max}) + v(x,T_{max}) \geq N_{min}$$

$$\min_{\sigma_d,\sigma_c,Dep} \Delta t \Delta x q \sum\sum p_{elec}^n w_j^n$$

subject to

$$w^{n+1} = M_c w^n + \sigma_c^{n+1}$$
$$\gamma^{n+1} = M_d \gamma^n + \sigma_d^{n+1}$$
$$v^{n+1} = v^n - (\sigma_d^{n+1} + \sigma_c^{n+1}) + (\frac{Arr^{n+1}}{\Delta x} - \frac{Dep^{n+1}}{\Delta x})$$
$$w_j^n = 0 \forall j > Jc$$
$$\gamma_j^n = 0 \forall j < Jd$$
$$v_j^n = 0 \forall j < Jd$$
$$w^0 = w_{init}, \gamma^0 = \gamma_{init}, v^0 = v_{init}$$
$$w, v, \gamma, Dep \geq 0$$

$$q\Delta x \sum_j \gamma_j^n \geq P^n$$
$$\sum_{j=J_{dep}}^{J_{max}} Dep_j^n = Dem^n$$
$$dx \sum_{j=J_{dep}}^{J_{max}} w_j^N + v_j^N + \gamma_j^N \geq N_{final}^{min}$$

LP formulation

Discretization in SOC space:
K steps
Discretization in time:
N steps

Variable size : 4KN

$$\min_{w,v,\gamma,Dep} \Delta t \Delta x q \sum \sum p_{elec}^{n} w_{j}^{n}$$

subject to

$$w^{n+1} + \gamma^{n+1} + v^{n+1} + \frac{Dep^{n+1}}{\Delta x} = M_c w^n + M_d \gamma^n + \frac{Arr^{n+1}}{\Delta x}$$

$$w_j^n = 0 \forall j > Jc$$

$$\gamma_j^n = 0 \forall j < Jd$$

$$v_j^n = 0 \forall j < Jd$$

$$w^0 = w_{init}, \gamma^0 = \gamma_{init}, v^0 = v_{init}$$

$$w, v, \gamma, Dep \geq 0$$

$$q \Delta x \sum_j \gamma_j^n \geq P^n$$

$$\sum_{j=J_{dep}}^{J_{max}} Dep_j^n = Dem^n$$

$$\Delta x \sum_{j=J_{dep}}^{J_{max}} w_j^N + v_j^N + \gamma_j^N \geq N_{final}^{min}$$

Berkeley UNIVERSITY OF CALIFORNIA

Value of parameters

- Initial fleet of 250 vehicles

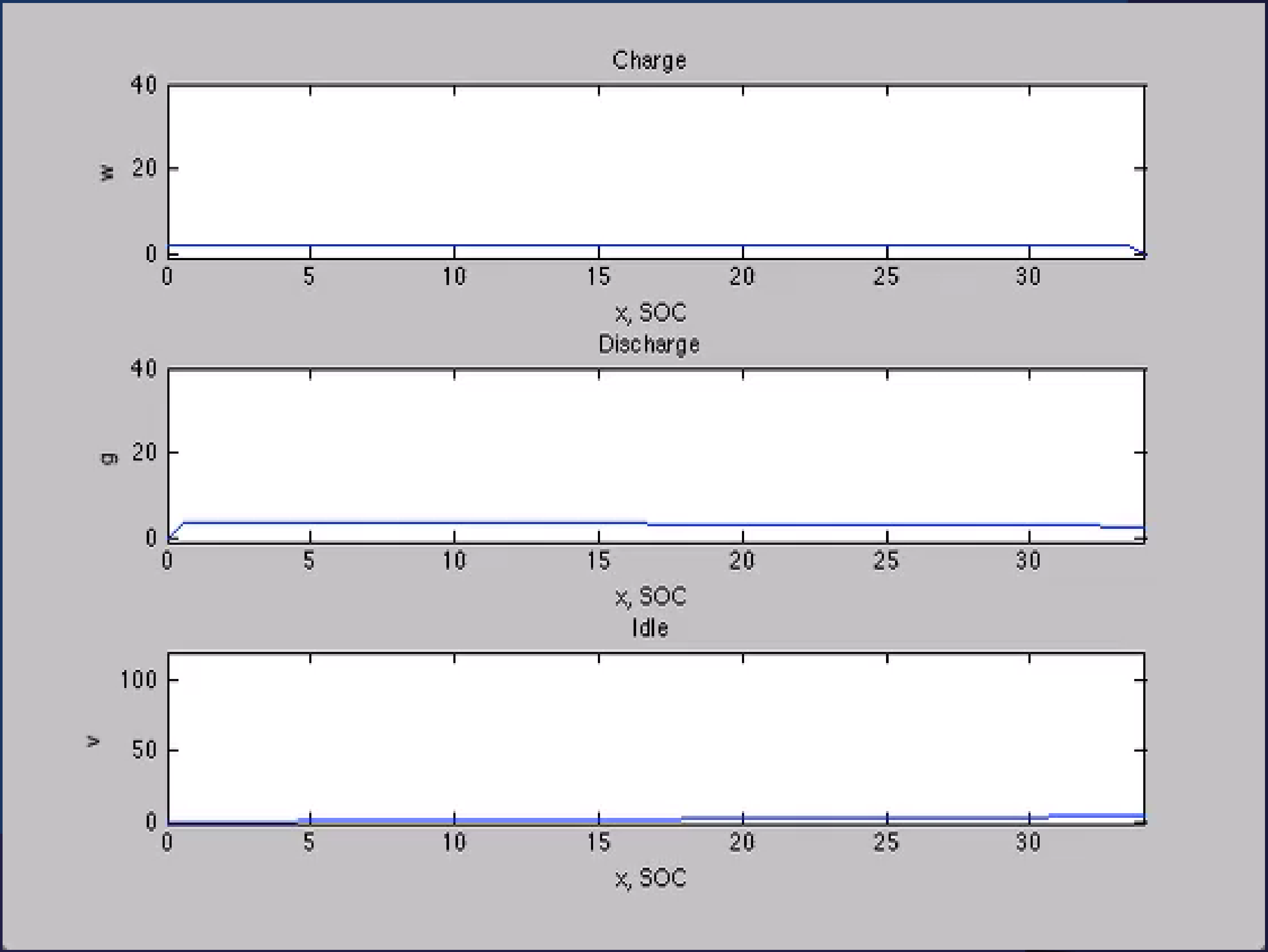
- Maximum capacity of battery: 34kWh
- Instantaneous charging power 1kW
- Tmax: 24h
- dt: 30min
- dx: 0.5 kWh

Result| Results

Charge

Discharge

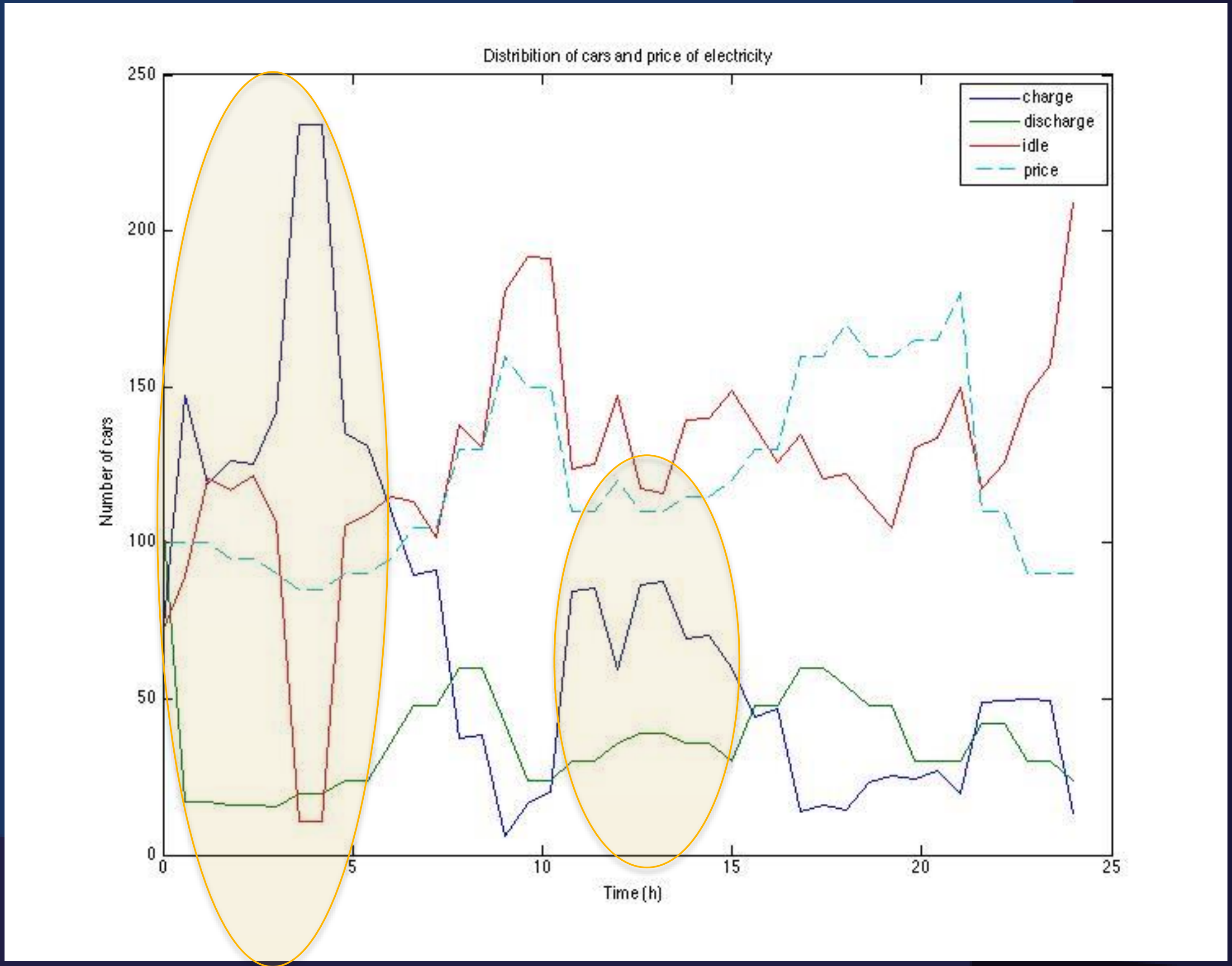
Idle



Berkeley
UNIVERSITY OF CALIFORNIA

Result

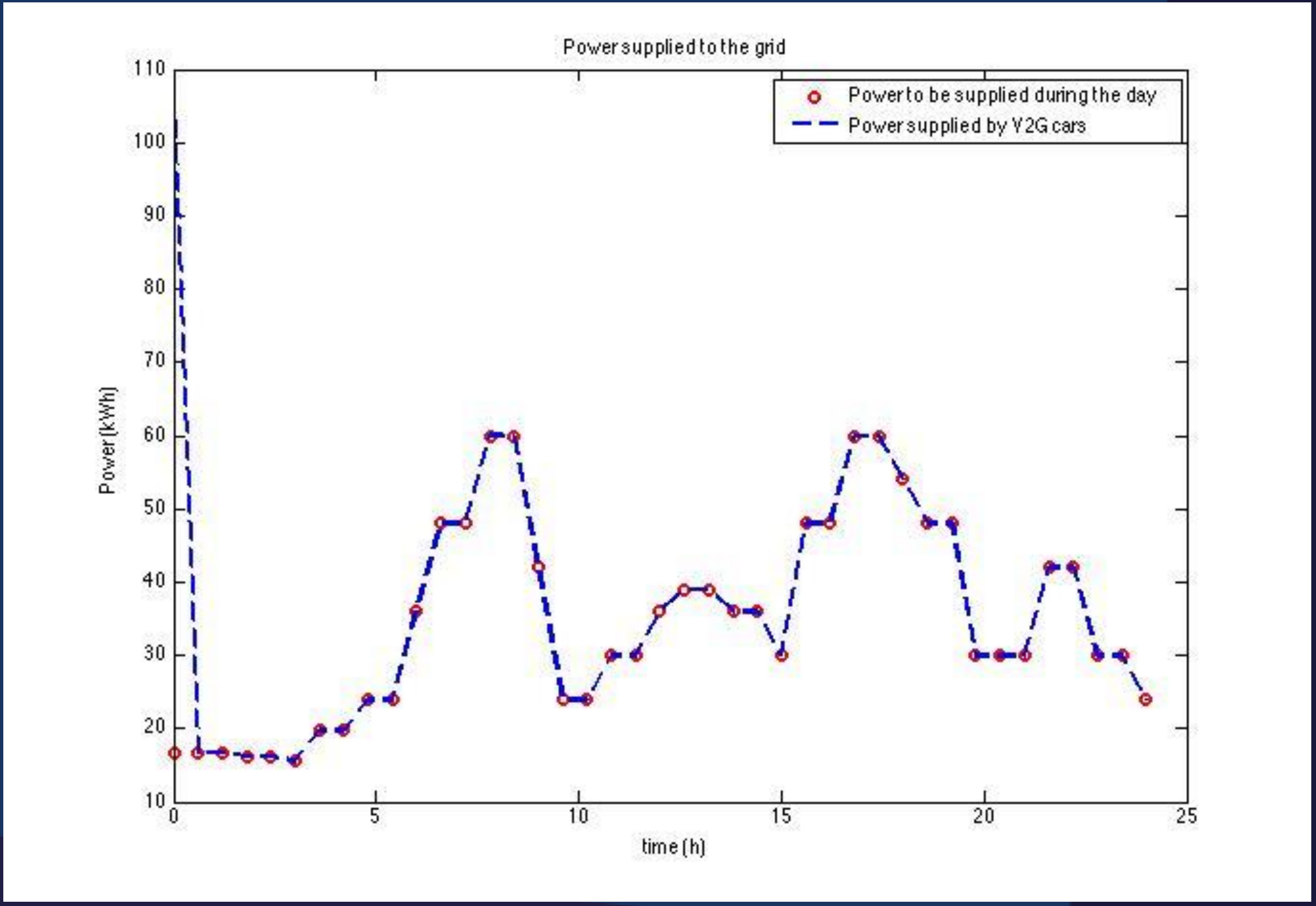
Charge
Discharge
Idle



Charges at low prices of electricity

Result

V2G cars follow grid electricity demand



FURTHER WORK

Battery degradation and smoothing

$$\min_{\sigma_{i\to d},\sigma_{i\to c},Dep} C = \int_{t=0}^{T_{max}} C_{elec}(t) \int_{0}^{x_{max}} w(x,t) q \, dx dt + \lambda \int_{t=0}^{T_{max}} \int_{0}^{x_{max}} |\sigma_{i\to d}| + |\sigma_{i\to c}| \, dx dt$$

$$\min_{X} C = c^T X + \lambda ||AX||_1$$

Penalization L1:

Penalizes large controls

Adapt value of λ to have a ‘smoother policy’

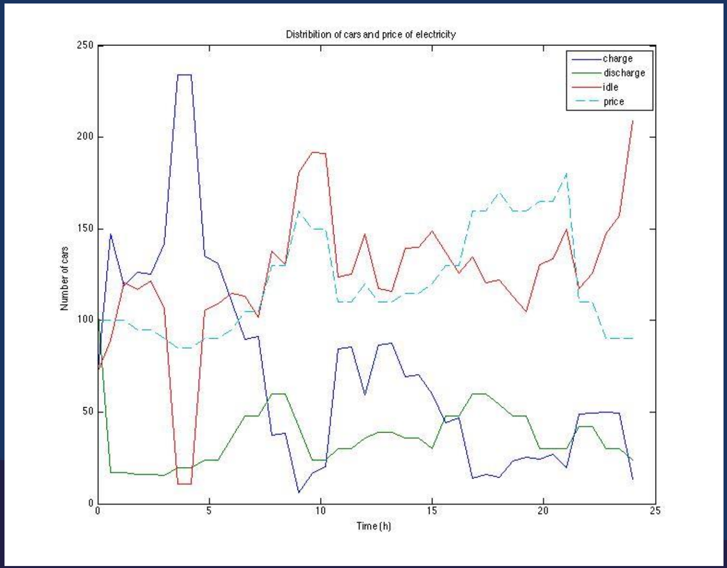
Cost function is still convex

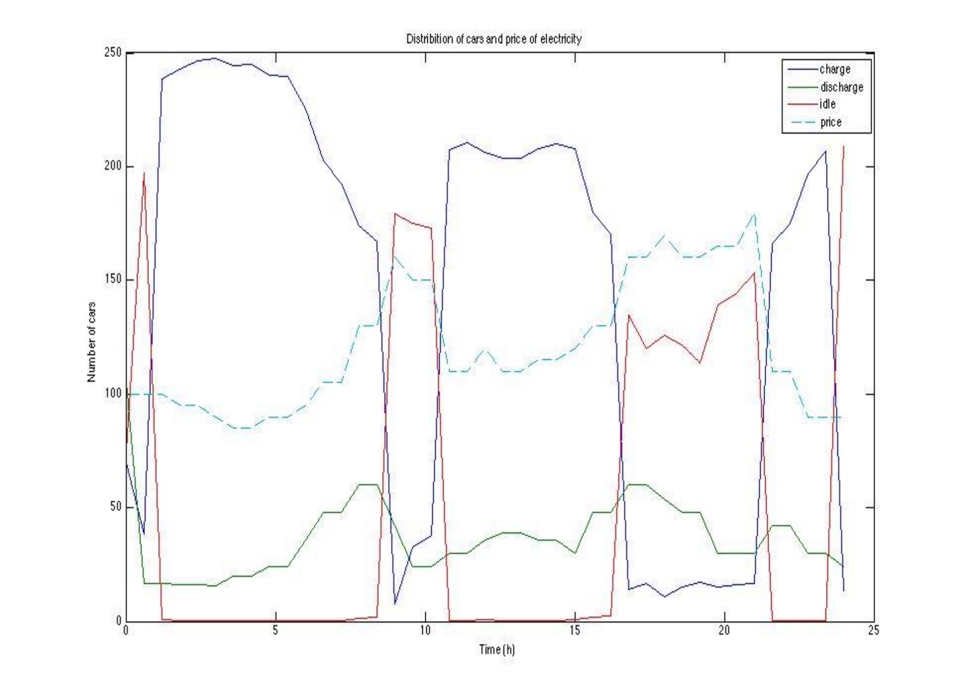
FURTHER WORK

Battery degradation and smoothing

$$\min_{\sigma_{i\to d},\sigma_{i\to c},Dep} C = \int_{t=0}^{T_{max}} C_{elec}(t) \int_{0}^{x_{max}} w(x,t) q\, dx dt + \lambda \int_{t=0}^{T_{max}} \int_{0}^{x_{max}} |\sigma_{i\to d}| + |\sigma_{i\to c}| dx dt$$

$$\min_{X} C = c^T X + \lambda ||AX||_1$$





Conclusion:

- Continuous representation of the system
- Optimizes G2V and V2G over both time and SOC space
- Very scalable method: can be applied for any size of fleets

Work not presented:

- Numerical schemes and stabilization
- Validation : Monte-Carlo simulator
- Smoothing of control policies

Further work:

- Validation on different simulators
- Include stochasticity

Thank you

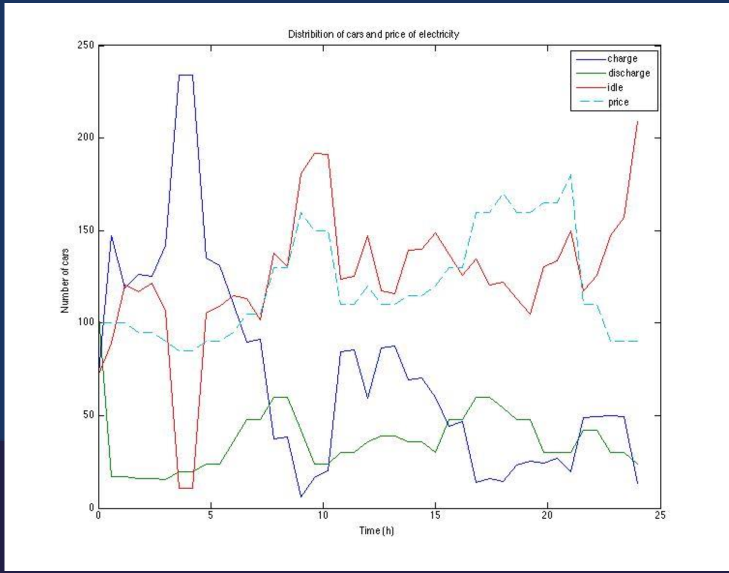
Berkeley
UNIVERSITY OF CALIFORNIA

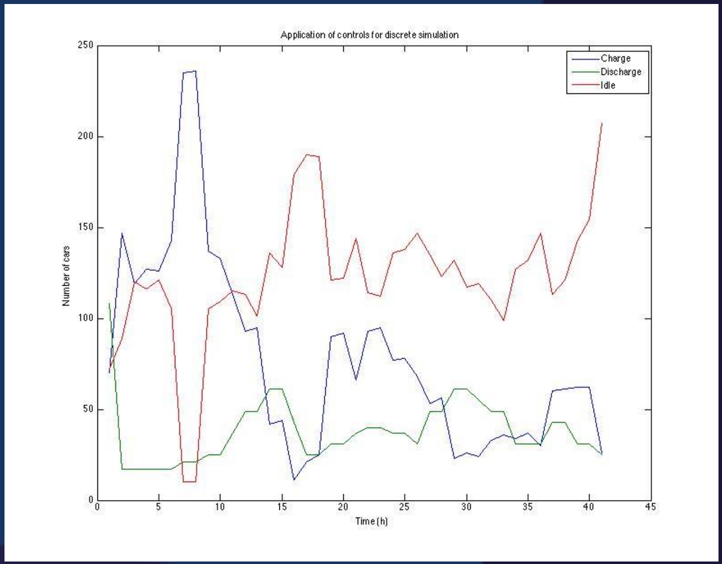
Questions?

www.Cars.com

Result| Validation

Continuous model → Value of control parameters → Discrete, rounded controls → **Monte Carlo simulator:** Cars simulated one by one, application of control parameters to regulate flows

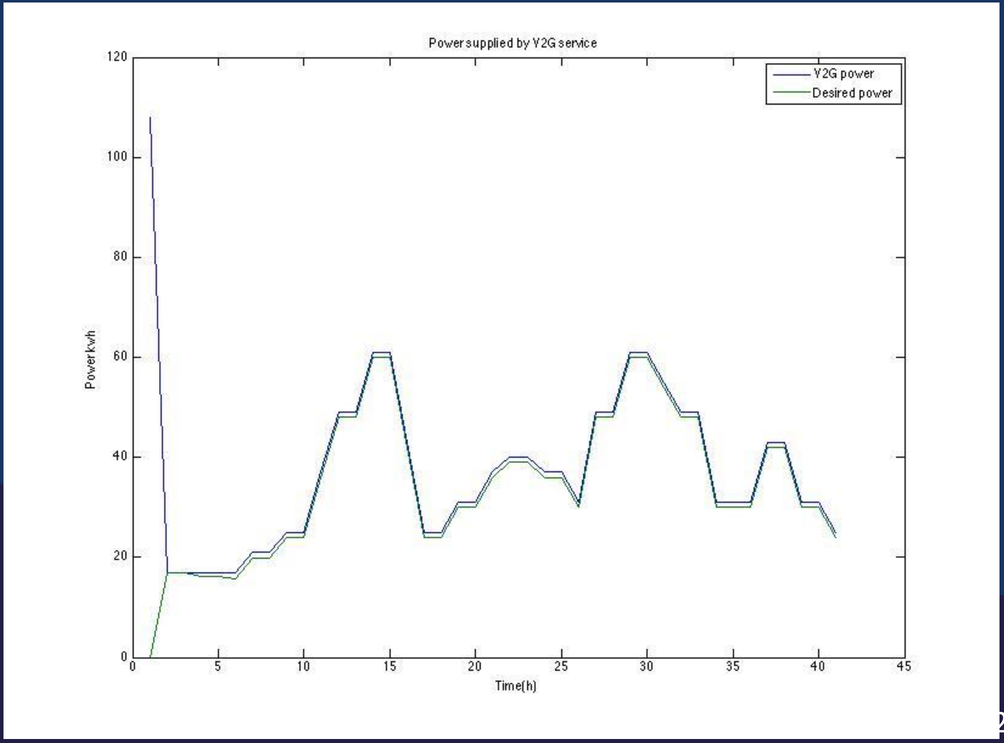




Berkeley
UNIVERSITY OF CALIFORNIA

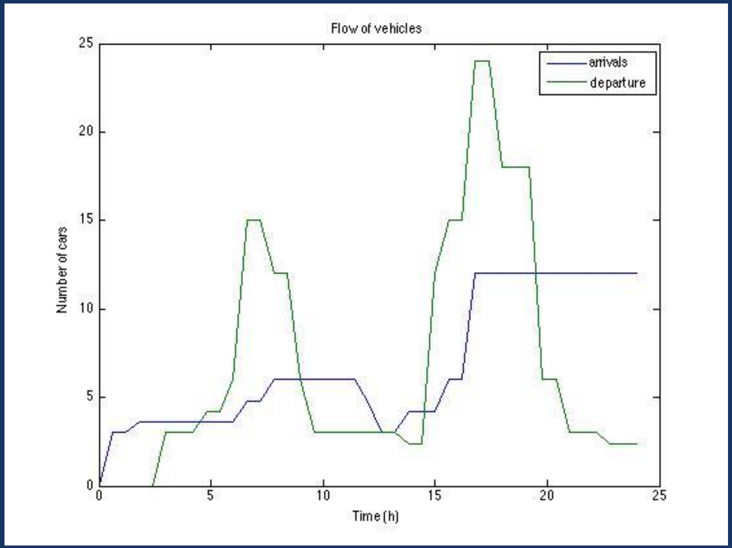
Result| Validation

Continuous model → Value of control parameters → Discrete, rounded controls → **Monte Carlo simulator:** Cars simulated one by one, application of control parameters to regulate flows

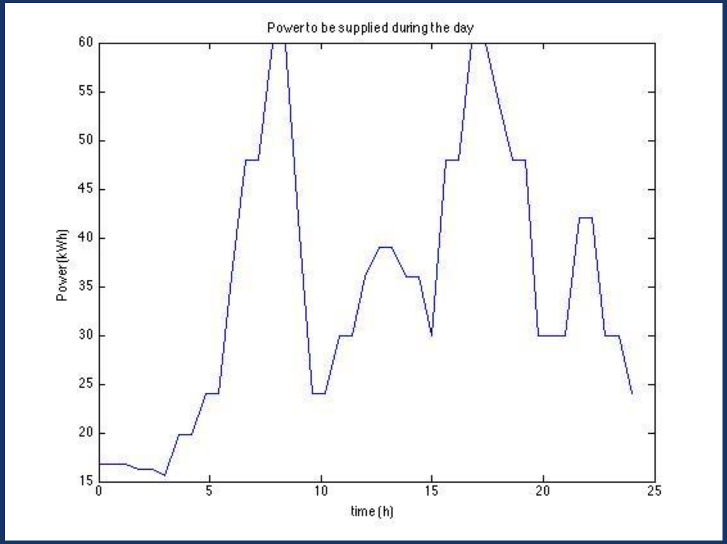


Result| Parameters

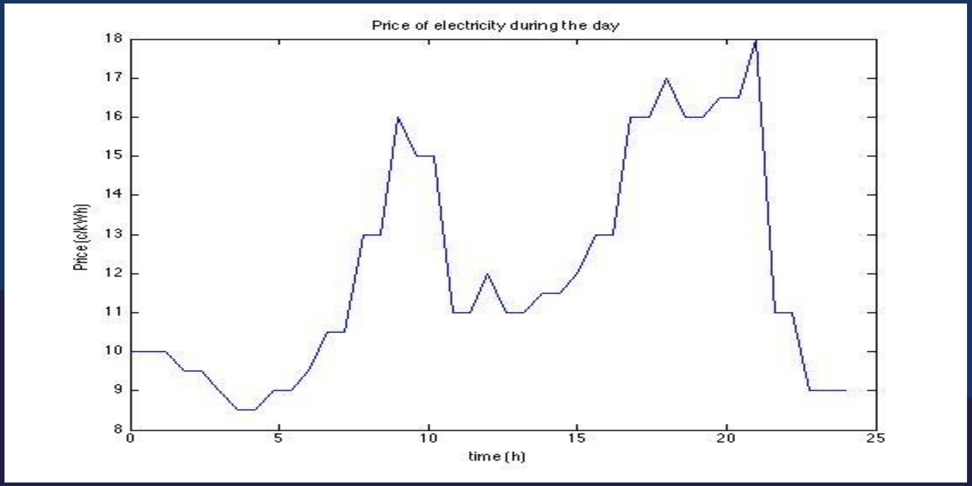
Flow: total arrivals and departure



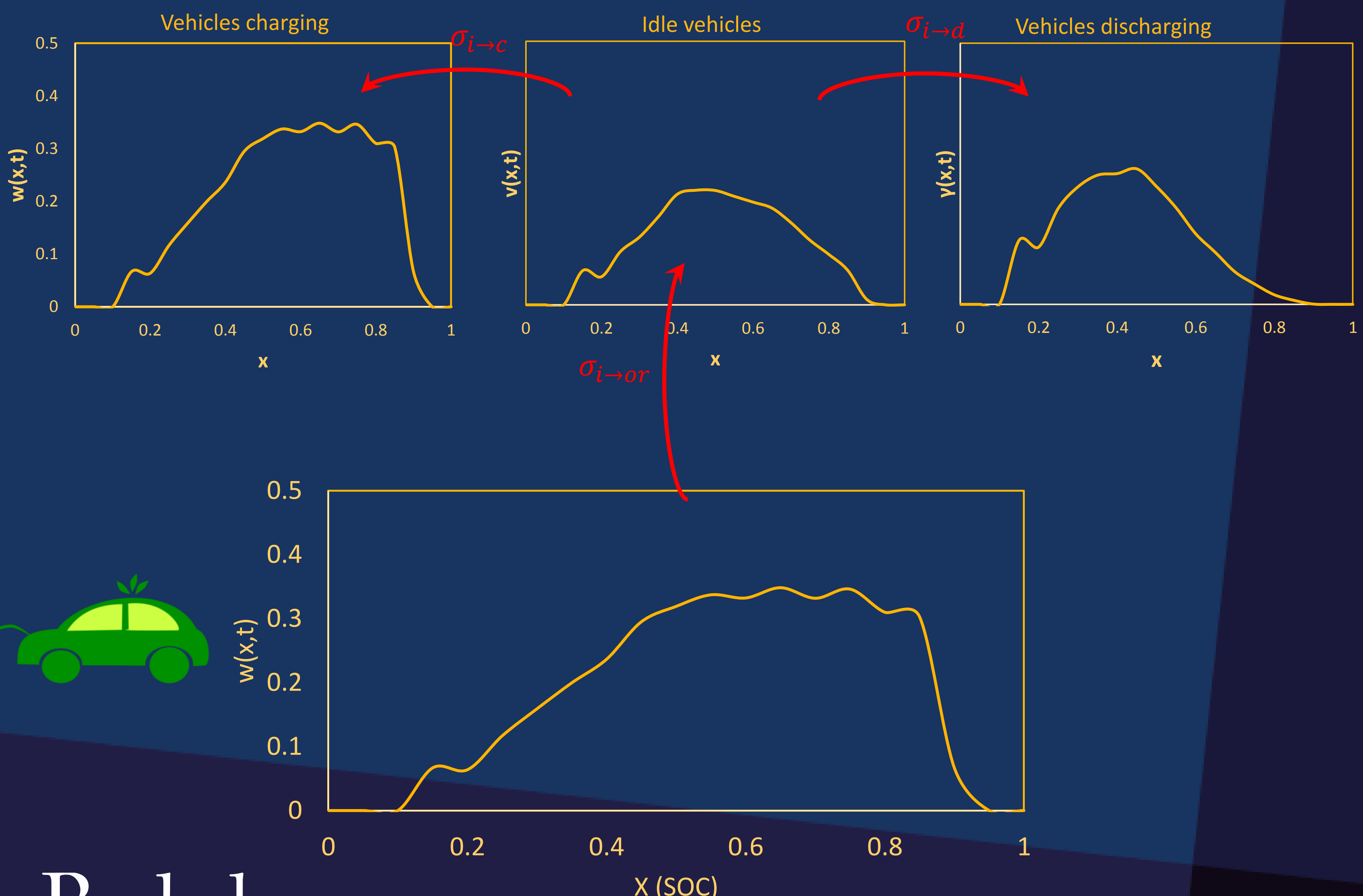
Power to be supplied



Price of electricity



Berkeley
UNIVERSITY OF CALIFORNIA



Vehicles charging
Idle vehicles
Vehicles discharging
$\sigma_{i\to c}$
$\sigma_{i\to d}$
$\sigma_{i\to or}$
w(x,t)
v(x,t)
y(x,t)
x
x
x
0 0.2 0.4 0.6 0.8 1
0 0.1 0.2 0.3 0.4 0.5
w(x,t)
X (SOC)