Modeling of Human Response to Ground Motion using Discrete and Continuous Methods

Brenda Dix and Jack Reilly
CE 291F: Control and Optimization of Distributed Parameters Systems
Final Project
May 4, 2010

Motivation For Project

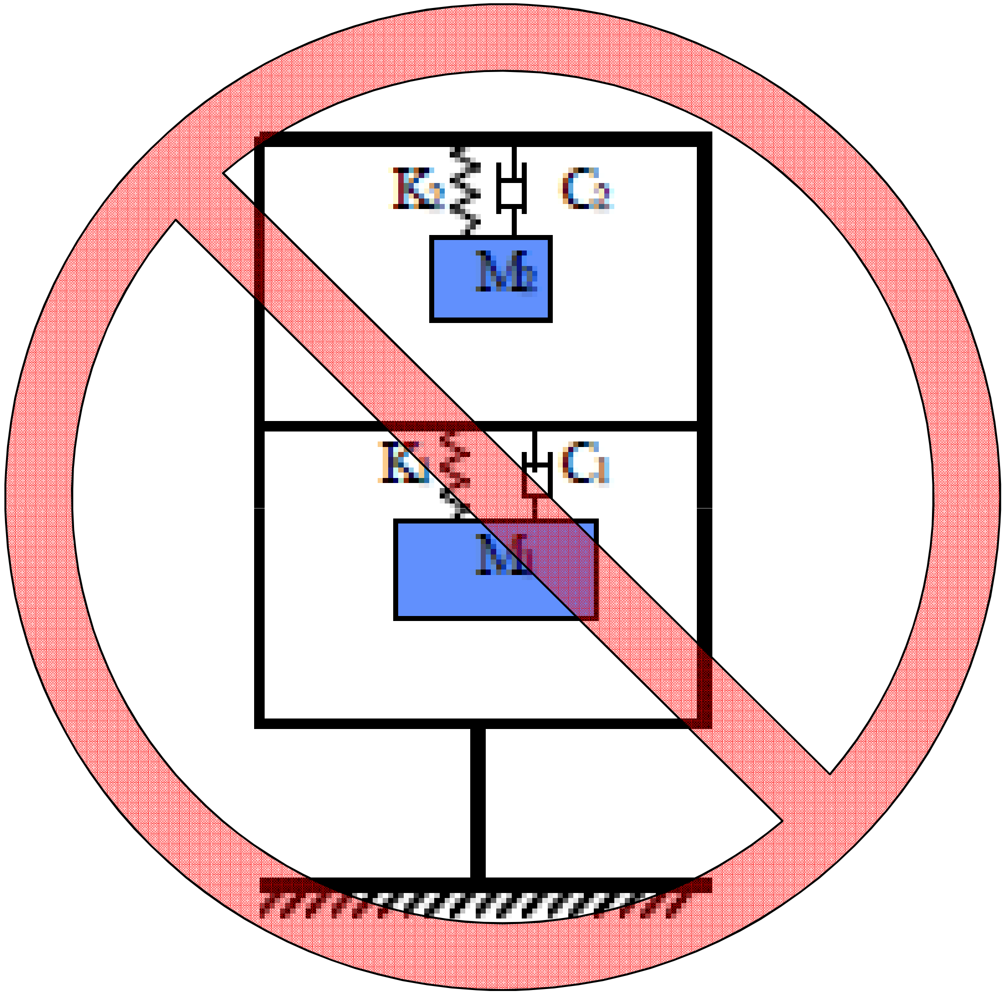
- Using cell phones as sensors for earthquakes

- Effects of human response to rigidly-attached accelerometer signal

- Extraction of ground acceleration from sensor

Discrete Models

ISO standards dictate a vibration simulator of 2DOF systems connected by a rigid frame

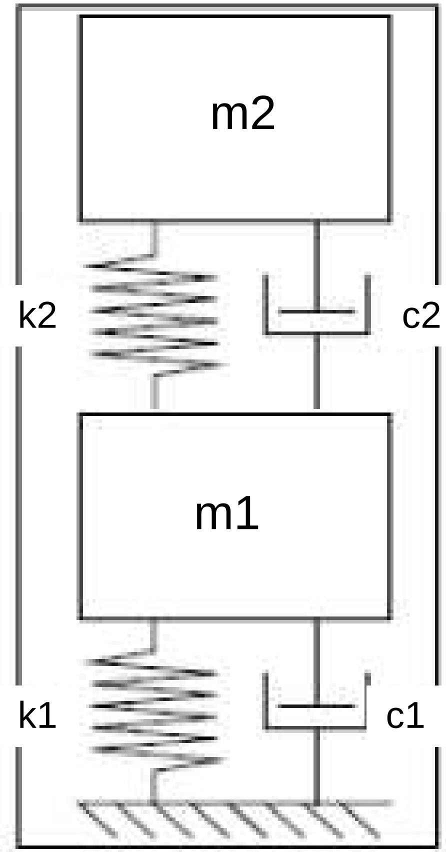


Problems with System:

- **Not really 2 DOF – just 2 SDOF systems connected by rigid frame**
- **Suggests that head and torso move independently of the lower body**
- **Doesn't account for building motion different from ground motion**

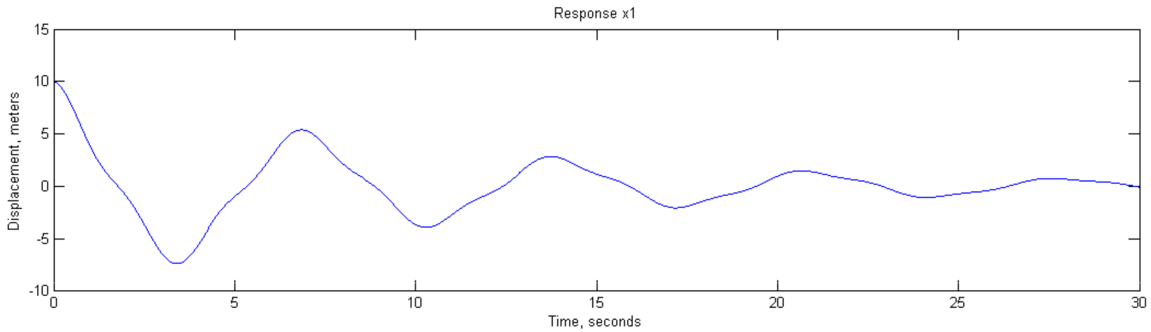
Better Discrete Models

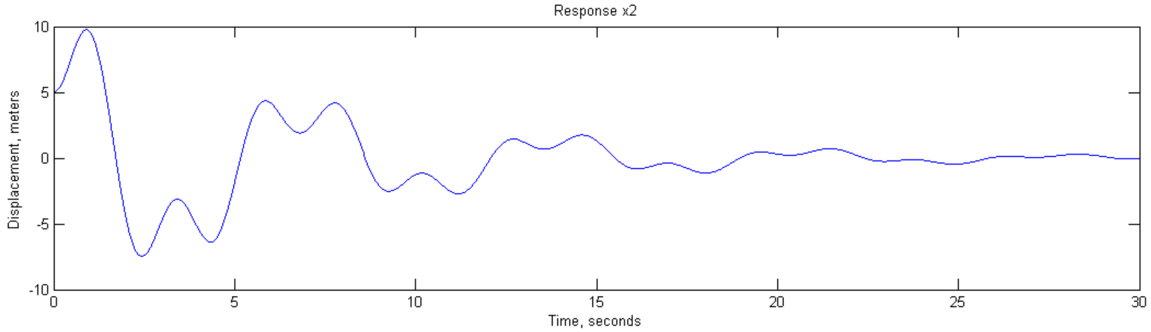
Damped 2DOF Model



$$m_1\ddot{x_1} + (c_1 + c_2)\dot{x_1} - c_2\dot{x_2} + (k_2 + k_1)x_1 - k_2 x_2 = 0$$

$$m_2\ddot{x_1} + c_2(\dot{x_2} - \dot{x_1}) + k_2(x_2 - x_1) = 0$$



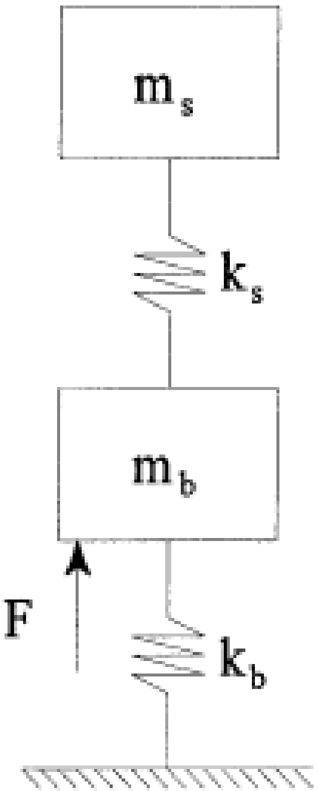


Problems:

- **Hard to estimate parameters**
- **Doesn't account for building motion separate from ground motion**
- **Still a discrete model**

Better Discrete Models Continued

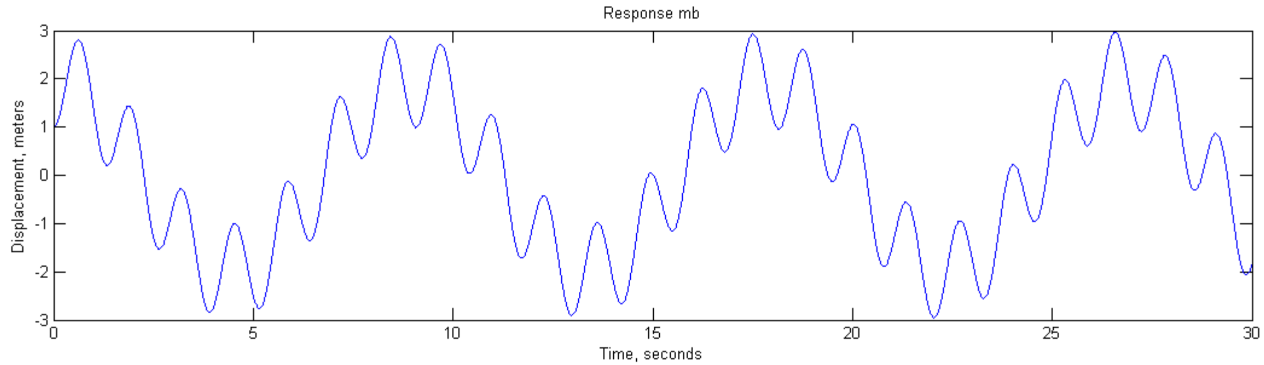
Subject-Beam Model

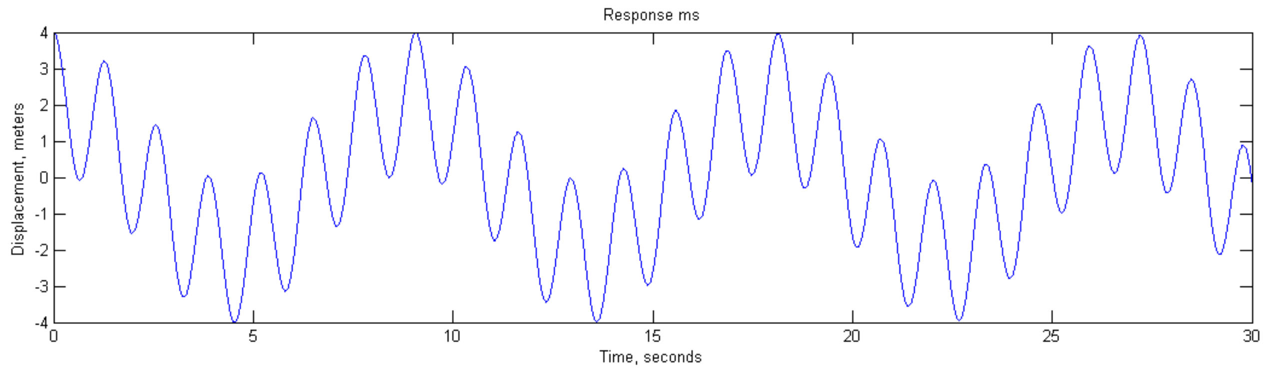


$$m_s \ddot{x}_s + k_s X_s k_s X_b = 0$$

$$-k_s X_s + m_b \ddot{X}_b + (k_s + k_b) X_b = F$$

$$\omega_s'^2 = \frac{\omega^2(\omega_b^2 - \omega^2)}{\omega_b^2 - (1+\alpha)\omega^2}$$



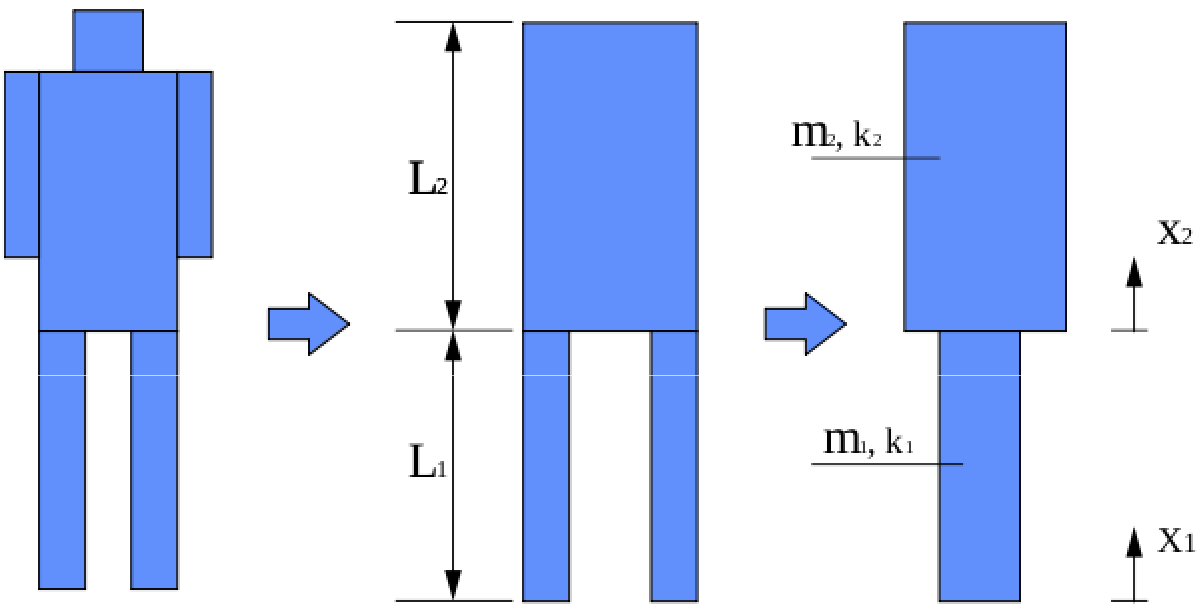


Problems:

- **SDOF – not very robust**
- **Does not account for damping**

Continuous Model

$$m\frac{\partial^2 U}{\partial t^2} - k\frac{\partial^2 U}{\partial x^2} = 0$$



- Two rods with independent properties (mass, length, stiffness)
- Assume $L_1 = L_2$, $m_2 = 2m_1$, and determine stiffnesses
- Force boundary conditions on the system of rods
- Axial Displacement, Forces =
  - @ 0, no force
  - @ x_1, no displacement
  - @ x_1+x_2,

Solution to PDE System

Separation of Variables

$$u(x,t) = T(t)\Phi(x)$$

Modal Shape

$$\Phi_i(x) = C_i \cos b_i x_i + D_i \sin b_i x_i \qquad i = 1,2$$

Application of BC's

$$\Phi_1(x_1) = D \sin b_1 x_1 \qquad 0 \le x_1 \le L_1$$

$$\Phi_2(x_2) = D\{\sin b_1 L_1 \cos b_2 x_2 + \sqrt{\frac{m_1 k_1}{m_2 k_2}} \cos b_1 L_1 \sin b_2 x_2\} \qquad 0 \le x_2 \le L_2$$

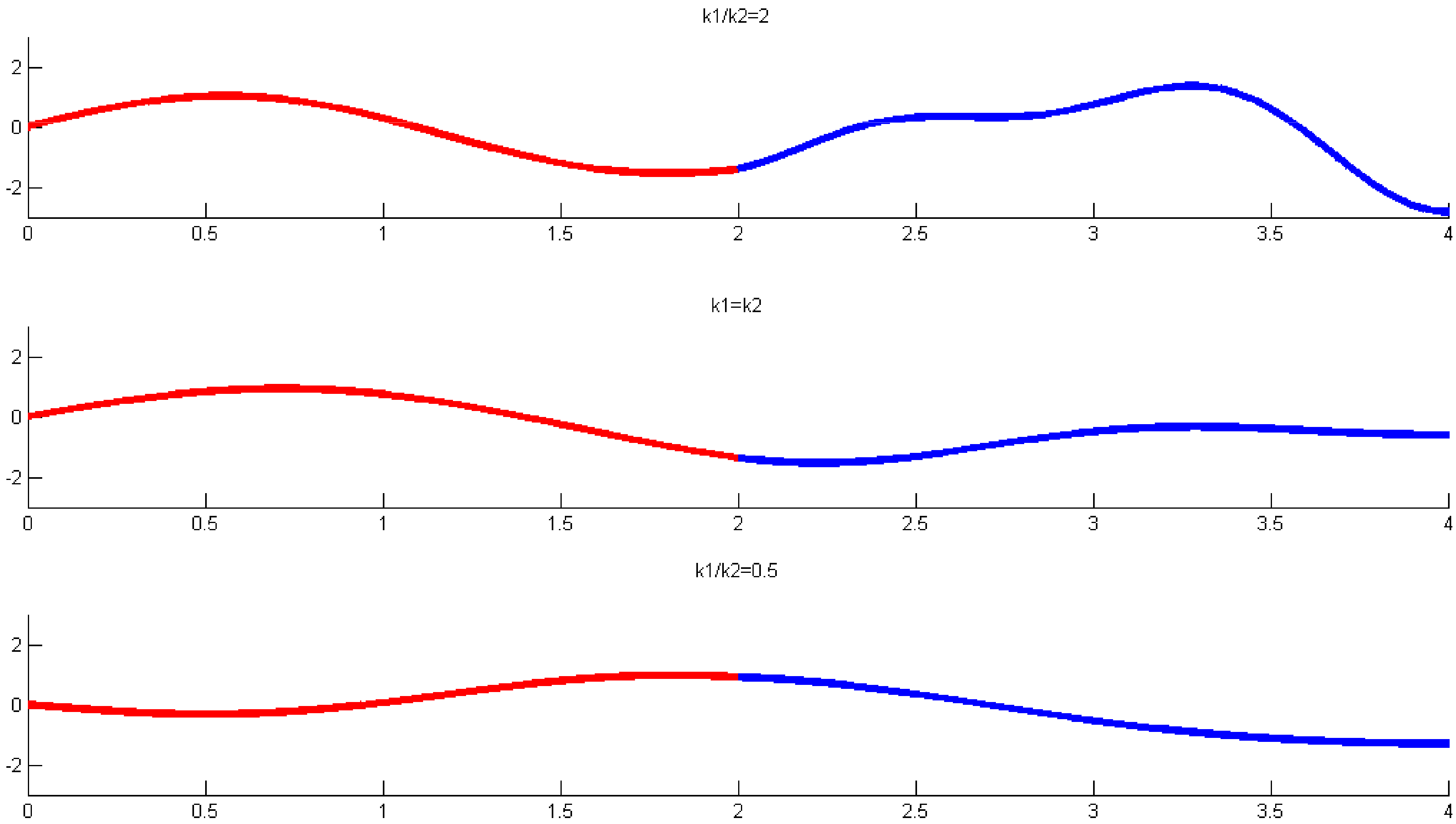
Full Model of PDE

- Stiffness Ratio Selection: for simplicity, a ratio of k1 / k2 = 2 is chosen
- Initial Conditions
  - No velocity
  - Assume sinusoidal initial shape

$$u(x_1, t) = \sum_{i=0}^{n} A_i sin(b_i x_1) cos(\omega_i t)$$

$$u(x_2, t) = \sum_{i=0}^{n} A_i sin(b_i L_1 (1 + \frac{2x_2}{L_2})) cos(\omega_i t)$$

3 Different Stiffness Ratios



Benefits of Continuous Model

- Reduces the number of parameters to be determined
  - In-between values embedded within the PDE, while discrete requires more nodes (more parameters)
- Can determine relative amplitudes of different modes as a function of location on body.
  - Know most likely position of sensor on human

$$u(x_1, t) = \sum_{i=0}^{n} A_i sin(b_i x_1) cos(\omega_i t)$$

$$u(x_2, t) = \sum_{i=0}^{n} A_i sin(b_i L_1 (1 + \frac{2x_2}{L_2})) cos(\omega_i t)$$

Further Study and Conclusions

FURTHER STUDY

- Additional sensors on the human participant to study local vibrational effects.
- Consider lateral movement and horizontal vibrations in order to understand 3-D picture of earthquake motions.
- Refine experiments to determine system parameters.
- Add damping to the model.

CONCLUSIONS

- Discrete models are commonly accepted but do not provide the resolution needed for the project.
- Our continuous model allows for the calculation of the modal response along the length of the body.
- Our continuous model does not meet our original goal of extraction of the earthquake ground motion signal.

References

ISO 5982:2001. "Mechanical vibration and shock -- Range of idealized values to characterize seated-body biodynamic response under vertical vibration". http://www.iso.org/iso/catalogue_detail.htm?csnumber=32917

Ji, Tianjian. "A Continuous Model for the Vertical Vibration of the Human Body in a Standing Position" *Proc. Of the UK Informal Group Meeting on Human Response to Vibration, Sept. 1995, Silsoe, UK.*

Kuchnicki, Stephen. December 2009. "Simple Vibration Problems with MATLAB (and Some Help from MAPLE)"

Randall, J. M. , Matthews, R. T. and Stiles, M. A.1997. "Resonant frequencies of standing humans" *Ergonomics,* 40: 9, 879 - 886

Thanks for listening!

Any questions?