

Measuring regret in routing: assessing the impact of increased app usage

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Abstract—This article is focused on measuring the impact of navigational apps on road traffic patterns. We first define the *marginal regret*, which characterizes the difference between the travel time experienced on the most optimal path and the path of interest between the same origin destination pair. We then introduce a new metric, the *average marginal regret*, which is the average of *marginal regret*, taken over all possible OD pairs in the network. We evaluate the *average marginal regret* in simulations with varying proportions of app and non-app users (information vs. no information) using the microsimulation software *Aimsun*. We conduct experiments on a benchmark network as well as a calibrated corridor model of the I-210 in Los Angeles for which OD demand data is gathered from several sensing sources as well as actual signal timing plans. In both cases (i.e. the benchmark and I-210) experiments demonstrate that the use of apps leads to a system-wide convergence towards Nash equilibrium.

I. INTRODUCTION

The increased use of GPS-enabled navigational applications (i.e. routing apps) and ensuing political and practical concerns have become popular topics in media over the last several years [1], [2], [3], [4], [5]. By extending the regret notion [6] to non-repeated game, this article introduces a new metric, the *average marginal regret*, which quantifies the impact of routing app usage on the traffic network. It defines the *marginal regret* which characterizes the difference between the travel time experienced on the most optimal path and the path of interest between the same origin and destination (OD). Then, the *average marginal regret* is defined as the average of *marginal regret*, taken over all possible OD pairs in the network. One possible use of the *average marginal regret* is the quantification of the gap between the actual behavior of drivers and a purely selfish behavior (i.e. a Nash equilibrium). We first present the context and background of this problem and discuss relevant work in addition to identifying the specific contributions of this article. We then present our problem framework and define the *average marginal regret*. Following this, we show simulation results for both a benchmark network and a large segment of the I-210 corridor in LA.

A. Context

1) *Traffic congestion and routing apps*: Traffic congestion is an increasingly present condition of urban life in the U.S. [7]. This congestion in addition to the increased use of routing apps have resulted in new traffic patterns which we refer to as *cut-through* traffic. *Cut-through* traffic caused by routing apps has garnered the attention of city residents and officials due

to congestion and safety concerns [8]. Negative externalities imposed by these routing apps include: increased travel time for residents, safety concerns for pedestrians in affected areas, public policy challenges, increased GHG emissions, and increased rate of infrastructure decay [9], [10].

2) *Use of routing apps*: The advent of the mobile internet and corresponding explosion in cell phone use have led to the popularity of routing apps [11]. These routing apps (e.g. INRIX, Here, TOMTom, Google, Waze, Apple, etc.) provide information about the state of the transportation network at a global scale. In the U.S. at least 60M motorists use Google Maps/Waze while 40M motorists use Apple Maps [12]. Furthermore, *Mobility as a Service* (MaaS) companies like Uber and Lyft use single routing apps for nearly all of their drivers, of which there are 45,000 registered in SF alone [13].

3) *Selfish routing*: Routing apps typically provide vehicles with shortest paths based on the current state of the network. We assume that people using apps are routed on the fastest possible path based on the current state of the network. This is desirable for individuals looking to minimize their own travel time. Unfortunately, shortest travel time routing (i.e. “selfish” routing) does not lead to socially optimal travel patterns [14], [15]. At best, traffic conditions under selfish routing can achieve a *Nash equilibrium*, sometimes also referred to as Wardrop’s equilibrium in traffic theory [14], [16].

B. Background

This article uses a game theoretic approach to model traffic using routing games [17]. In numerous traffic network models, drivers are assumed to possess perfect information over the state of the network. To model the impact of app usage on driver behavior, this article separates drivers into two populations: those who use routing apps (app users) and those who do not (non-app users) [10], [18]. Extending the concept of regret from decision and game theory [6], [19], this article introduces a new concept to evaluate the impact of apps on traffic patterns: the *average marginal regret*. Through dynamic simulations performed with *Aimsun* [20], this article presents the evolution of *average marginal regret* in the network as app usage increases.

C. Contributions of this article

By assessing the impact of increased app usage on traffic routing using *average marginal regret*, this article makes the

following contributions:

- 1) Extending regret to non-repeated and non-atomic routing games in the context of traffic assignment.
- 2) Quantification of the impact of routing apps on the transportation network with a new metric: the *average marginal regret*.
- 3) Modeling dynamic behaviors of app users and non-app users in *Aimsun*.
- 4) Microsimulations with app and non-app users that demonstrate that *average marginal regret* decreases while app usage increases on both a benchmark network and the I-210 calibrated corridor model.

II. PROBLEM FRAMEWORK AND AVERAGE MARGINAL REGRET

A. Mathematical formulation and network notation

We consider a transportation network represented by a directed graph $G = (\mathcal{V}, \mathcal{E})$, where $v \in \mathcal{V}$ are the vertices in the network and $e \in \mathcal{E}$ are the edges. We call the set of paths without cycles between two nodes $o, d \in \mathcal{V}$: \mathcal{P}^{od} . For all possible paths,

$$\mathbf{p} \in \mathcal{P} := \bigcup_{o, d \in \mathcal{V}} \mathcal{P}^{od}$$

We define the flow using path \mathbf{p} as $h_{\mathbf{p}}$ (i.e. path flow of path \mathbf{p}). Using this path flow, $h_{\mathbf{p}}$, we also define the path flow vector, \mathbf{h} , as

$$\mathbf{h} := (h_{\mathbf{p}})_{\mathbf{p} \in \mathcal{P}}$$

For each path $\mathbf{p} \in \mathcal{P}$ we define the cost function $c_{\mathbf{p}}(\mathbf{h})$ of the path \mathbf{p} which gives the path cost given a flow allocation \mathbf{h} as:

$$c_{\mathbf{p}} : \mathbf{h} \rightarrow c_{\mathbf{p}}(\mathbf{h})$$

We define the total cost of the network $C(\mathbf{h})$ as:

$$C(\mathbf{h}) := \sum_{\mathbf{p} \in \mathcal{P}} h_{\mathbf{p}} \cdot c_{\mathbf{p}}(\mathbf{h}).$$

B. Traffic assignment problem

The traffic assignment problem consists of assigning vehicular flow to specific routes in a network given OD demand information [15]. We call the total number of vehicles traveling from o to d per unit of time $r_{od} \geq 0$. From this demand $r_{od} \geq 0$ we define the demand matrix: $\mathbf{r} := (r_{od})$ with $o, d \in \mathcal{V}$. Given a demand $\mathbf{r} \in \mathbb{R}_{\geq 0}^{|\mathcal{V}| \times |\mathcal{V}|}$ we define the set of feasible path flow allocations, $\mathcal{H}_{\mathbf{r}}$:

$$\mathcal{H}_{\mathbf{r}} := \{\mathbf{h} \in \mathbb{R}_{\geq 0}^{|\mathcal{P}|} : \forall r_{od} \in \mathbf{r}, \sum_{\mathbf{p} \in \mathcal{P}^{od}} h_{\mathbf{p}} = r_{od}\}.$$

We say that a path \mathbf{p} is used if $h_{\mathbf{p}} > 0$.

Definition 1 (User equilibrium [15]): For the traffic demand \mathbf{r} , a flow allocation $\mathbf{h} \in \mathcal{H}_{\mathbf{r}}$ is called a user equilibrium if and only if:

$$h_{\mathbf{p}} \cdot (c_{\mathbf{p}}(\mathbf{h}) - \min_{\mathbf{p} \in \mathcal{P}^{od}} c_{\mathbf{p}}(\mathbf{h})) = 0 \\ \forall \mathbf{p} \in \mathcal{P}^{od} \quad \forall o, d \in \mathcal{V}$$

The user equilibrium definition, equivalent to Wardrop's first principle, states that the cost on all used paths between

OD pairs are equal and less than the cost which would be experienced on any unused path between the same origin and destination [16]. User equilibrium can be seen as an extension of Nash equilibrium in a game with an uncountable number of players (i.e. the non atomic routing game [17]).

C. Regret in the context of routing

We now define *marginal regret* to quantify the difference between the actual costs in the network and the minimum costs achieved between OD pairs. We define the minimum cost between $o, d \in \mathcal{V}$ as:

$$\pi_{o,d}(\mathbf{h}) := \min_{\mathbf{p} \in \mathcal{P}^{od}} c_{\mathbf{p}}(\mathbf{h}).$$

Definition 2 (Marginal regret): We define the *marginal regret*, $\mathcal{R}(\mathbf{h}, \mathbf{p})$, on path \mathbf{p} as

$$\mathcal{R}(\mathbf{h}, \mathbf{p}) := c_{\mathbf{p}}(\mathbf{h}) - \pi_{o,d}(\mathbf{h})$$

which is the cost a vehicle on path \mathbf{p} could have saved by using the optimal path (i.e. by rerouting).

We call this metric *marginal regret* because it characterizes, for a given OD pair, the difference between the travel time experienced on the most optimal path and the path of interest. It is important to note that the *marginal regret* is defined on a specific path \mathbf{p} and for a specific flow allocation \mathbf{h} .

This term is used in both decision and game theory to describe a notion similar to that which we express [21], [22]. Note that *regret* has a meaning in repeated game which is different from how this article defines it. In the context of repeated games, what we call *marginal regret* is called instantaneous regret [23].

Definition 3 (Average marginal regret): We define the *average marginal regret*, $\mathcal{R}(\mathbf{h})$, as the arithmetic average of *marginal regret* weighted by the path flow, taken over all possible paths in the network:

$$\mathcal{R}(\mathbf{h}) := \frac{1}{\|\mathbf{r}\|_1} \cdot \sum_{\mathbf{p} \in \mathcal{P}} h_{\mathbf{p}} \cdot \mathcal{R}(\mathbf{h}, \mathbf{p}) \\ \|\mathbf{r}\|_1 = \sum_{o, d \in \mathcal{V}} r_{od}.$$

Definition 4 (Relative average marginal regret): We define the *relative average marginal regret* as

$$\bar{\mathcal{R}}(\mathbf{h}) := \frac{\sum_{\mathbf{p} \in \mathcal{P}} h_{\mathbf{p}} \cdot \mathcal{R}(\mathbf{h}, \mathbf{p})}{\sum_{\mathbf{p} \in \mathcal{P}} h_{\mathbf{p}} \cdot c_{\mathbf{p}}(\mathbf{h})}$$

which is the *average marginal regret* of an OD pair normalized by the average travel time of the OD pair.

The *relative average marginal regret* is unitless and represents the percent delay. The *relative average marginal regret* is useful for comparing regret between different OD pairs, since these ODs might contain travel times of vastly different time scales.

D. Properties of the average marginal regret

We note by Definition 3 that when the network is in a state of user equilibrium the *average marginal regret*, $\mathcal{R}(\mathbf{h})$, is zero.

Proof 1: Given feasible path flows $\mathbf{h} \in \mathcal{H}_r$ positive *marginal regret* $\mathcal{R}(\mathbf{h}, \mathbf{p}) \geq 0$ for all paths $\mathbf{p} \in \mathcal{P}$ we see that if $\mathcal{R}(\mathbf{h}) = 0$, then we have:

$$h_{\mathbf{p}} \cdot (c_{\mathbf{p}}(\mathbf{h}) - \min_{\mathbf{p} \in \mathcal{P}^{od}} c_{\mathbf{p}}(\mathbf{h})) = 0 \quad (1)$$

which defines a user equilibrium as per Definition 1.

Note that the *average marginal regret* can be interpreted from a probabilistic point of view. If we randomly choose a vehicle (i.e. an infinitesimal fraction of path flow) the expected cost it can save by changing its path choice is the average of the marginal regret of all vehicles.

In a state of social optimality, the *average marginal regret*, $\mathcal{R}(\mathbf{h})$, might not be zero. By definition, in a socially optimal state the total cost of the network, $C(\mathbf{h})$, is minimum. However, given a specific OD pair, some vehicles can experience a travel time greater than that on the best path between this OD pair which results in $\mathcal{R}(\mathbf{h}) \geq 0$. This non-zero regret is due to the difference between a social optimum and a Nash equilibrium [24].

Throughout the remainder of this article, *average marginal regret* will be used to evaluate the impact of routing apps on traffic. We assume that people aim to minimize their own travel time, therefore, in the remainder of this article, the cost of a path consists the travel time of this path.

III. EXPERIMENTAL ASSESSMENT OF THE IMPACT OF APPS ON TRAFFIC

A. Dynamic simulation with Aimsun

To evaluate the impact of routing apps on traffic we perform microsimulations using *Aimsun*. *Aimsun* uses a car following model to describe the movement of individual vehicles through a network. We explicitly model the effect of information on routing behavior by considering app users and non-app users.

We differentiate app users from non-app users in the scenarios by prescribing different routing behavior between the two groups. App users choose the lowest cost paths (i.e. lowest travel time) with high probability and are allowed to change paths throughout the simulation as path costs change due to congestion. We assume that apps give the fastest route and app users follow the recommendation of the apps. Non-app users follow prescribed paths. Their prescribed paths are determined by solving the static user equilibrium problem for the same demand. We assume that non-app users mainly follow road signs. So, they follow a prescribed path. We also assume that these paths induced by the road signs are designed to be the path obtained by solving the static user equilibrium. Therefore, non-app users are required to follow the paths that were found by solving the static user equilibrium problem. Non-app users are unable to change routes during the simulation since they are following predetermined routes.

Since path costs (i.e./ path travel times) are an essential component of app user behavior, these costs have to be updated frequently in order to guarantee that vehicles are routed based on up-to-date travel time information. A high cost cycle (e.g. 20 minutes in a 60 minute simulation) will lead to undesired effects. For instance, assume that a path has low travel time (i.e. a low cost) due to low traffic flow. App users will start routing themselves onto the path, which will lead to the congestion of

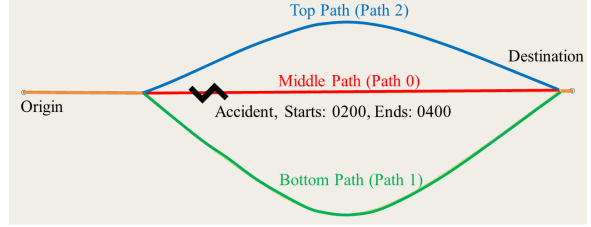


Fig. 1. Illustration of the benchmark network with the location of the accident shown in black.

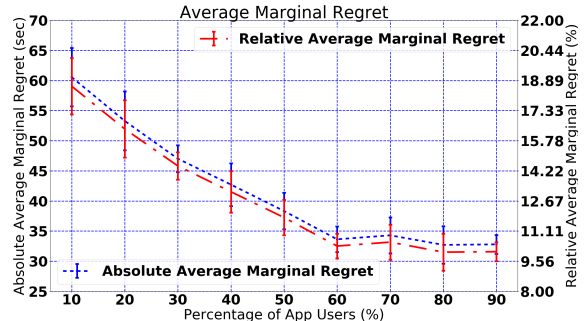


Fig. 2. The benchmark network scenario without an accident shows the decreases in *average marginal regret* as the percentage of app users in the network increases. The *average marginal regret* indicates that the traffic state converges to a user equilibrium as more drivers use routing apps.

the path. However, the cost of the path is not updated (since the cost cycle is high) so app users will continue to route onto the path, further worsening the congestion. To prevent such effects, we use a one minute cost cycle time.

B. Scenario setup

We conduct simulations on two networks, the first of which is a benchmark shown in Fig. 1. We demonstrate the use of *average marginal regret* on the benchmark network in order to show how it can be calculated and interpreted. In the benchmark network we consider the general impact of increased app usage on the state of the network in addition to the impact of a capacity decrease due to an accident.

The second network that we consider is the I-210 corridor in LA. The *Aimsun* model of the I-210 is part of an ongoing project to build a calibrated corridor model [25]. Data from the California DOT freeway loop sensors and city traffic studies are used to establish realistic OD demand. Traffic control plans from the California DOT, Arcadia, and Pasadena are incorporated into the model. The Connected Corridors project is a fundamental component of creating response plans for incident response and congestion mitigation in the I-210 corridor. As a result, the *Aimsun* model of the I-210 realistically simulates the evolution of traffic over the network.

For the both the benchmark network and the I-210 corridor simulation, we consider only impact of increased app usage and use the *average marginal regret* to quantify the evolution of the traffic state. In both networks, we fix the demand between OD pairs and perturb the percentage of app users between a single

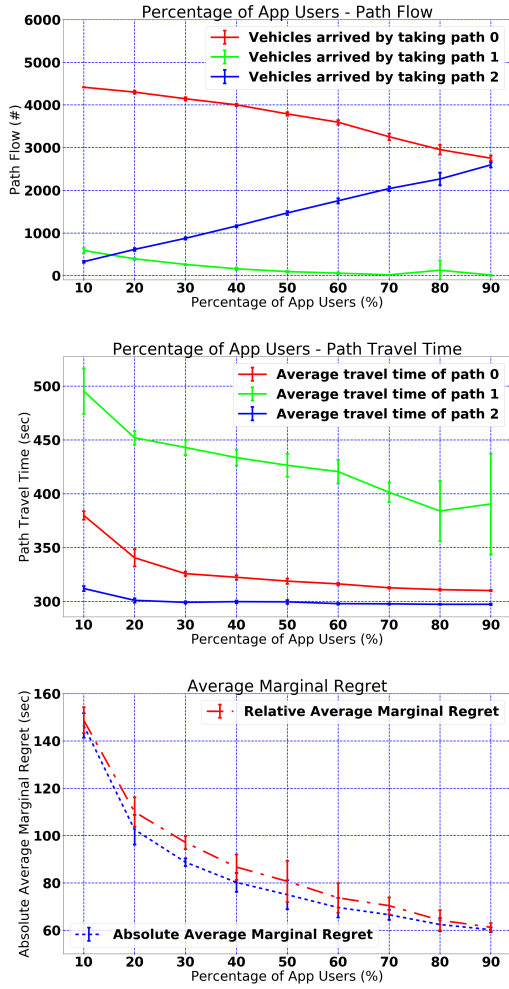


Fig. 3. The benchmark network scenario with an accident shows the decreases in *average marginal regret* as the percentage of app users in the network increases. **Top:** Path flow. **Middle:** Path travel time. **Bottom:** Absolute and relative *average marginal regret*. The *average marginal regret* decreases with the increase of app usage: traffic state converges to a user equilibrium as more drivers use routing apps. Note how the *average marginal regret* of the accident scenario is higher than that of the scenario without accident under the same percentage of app usage.

OD pair, starting with 10% app users and increase to 90%, using 10% increments. Negative externalities of app usage have previously been shown using static traffic assignment models and field data from the I-210 corridor [18].

C. Benchmark Network: No Accident

The benchmark network is shown in Fig. 1. In this case no accident occurs. The network consists of a single OD pair connected by three paths: the top path (blue), the middle path (red), and the bottom path (green). The common links (i.e., road sectors) shared by the three paths have three lanes, while the links owned exclusively by the paths have one lane each, with no traffic controls at intersections between the links. With every lane having the identical capacity of 2000 *veh/hr*,

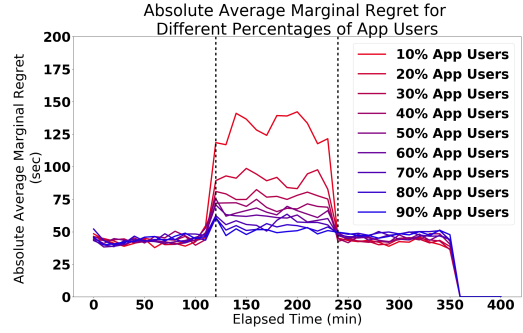


Fig. 4. Evolution of *average marginal regret* in the benchmark network with an accident. The accident start and end time are marked by black dashes. The accident leads to an increase in *average marginal regret* as vehicles on the middle path (red) suffer a higher delay than vehicles traveling on the top path (blue). **Best viewed in color.**

all links in this scenario have capacities of 2000 *veh/hr*. We perform a 6-hour simulation in which the demand of 2667 *veh/hr* exceeds capacity of the links, hence causing congestion.

We observe that the dynamic routing behavior of app users allows them to reroute to avoid this congestion. The bottom path (green) is never used because its travel time is so much longer than that of the other two paths. The results shown in Fig. 2 indicate that the *average marginal regret* decreases as the number of app users in the network increases, which demonstrates that the network is converging to a state of user equilibrium.

D. Benchmark Network: Accident

The scenario involving an accident on the benchmark network is shown in Fig. 1. It is nearly identical to the simple benchmark scenario, however the common links (shown in orange in Fig. 1) has six lanes, with the remaining links exclusive to the three paths having two lanes. The number of lanes for each link are increased in this scenario in order to ensure that the middle path (red) can still be used after the occurrence of the accident. We again perform a 6-hour simulation with demand of 2667 *veh/hr*. The accident occurs 2 hours after the simulation begins and lasts for 2 hours. There are 2 hours at the end of the simulation during which all roads are again operating at full capacity.

The major differences between the results obtained with the accident and the results obtained without the accident are the trends in path flow and path travel time. We can see in Fig. 3 that the path flow of the bottom path, illustrated in green, decreases as the percentage of app users increases. Since the bottom path (green) is considered an alternative path and hence should accommodate increased path flow as the percentage of app users increases, such phenomenon is counterintuitive. This can be explained by congestion at the common link, colored in orange in Fig. 1, which creates a barrier that blocks vehicles from rerouting themselves onto the top path (blue). Therefore, many app users are forced to take the bottom path (green) after a long wait at the intersection. As the percentage of app users increases, the congestion in the middle path is mollified, therefore decreasing the number of vehicles that are forced

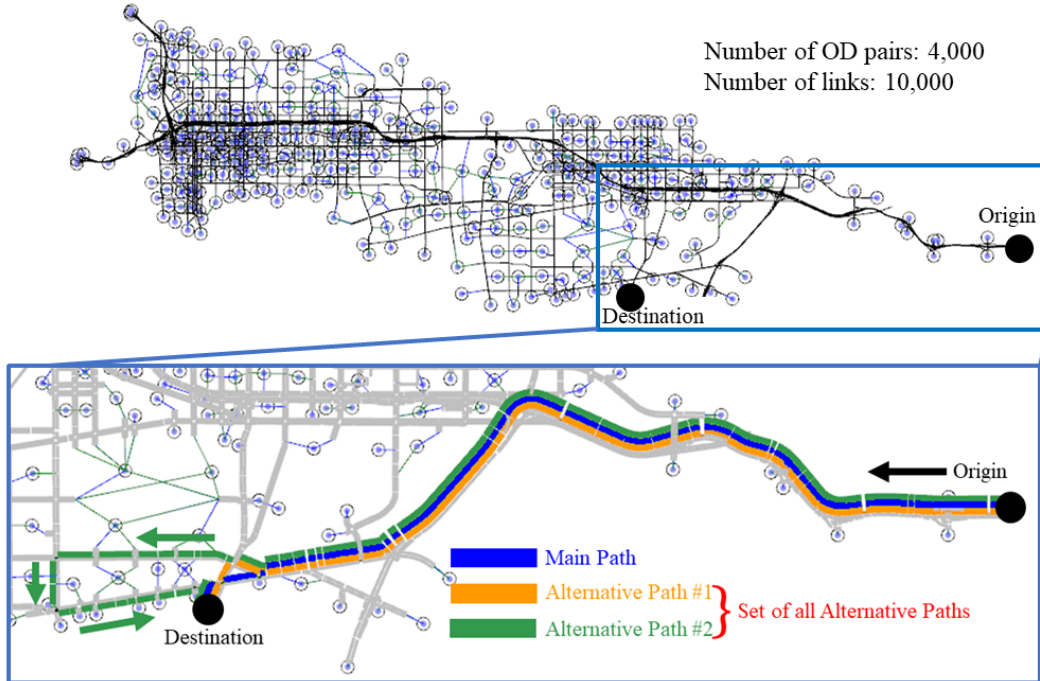


Fig. 5. **Top:** The I-210 corridor in LA on which we perform *Aimsun* simulations. **Bottom:** The OD pair for which we perturb the number of app users. **Best viewed in color.**

to take the bottom path. Between the two heavily used paths (red and blue) we see a convergence in path flow and path travel time as the number of app users in the network increases. This is reasonable because the system is approaching a state of user equilibrium as more vehicles have access to travel time information. We also show the *average marginal regret* over time in the case of an accident in Fig. 3 which demonstrates that the experienced *average marginal regret* is less severe when there are a large number of app users in the network.

E. I-210 Corridor: No Accident

We conduct simulations similar to those on the benchmark network on the I-210 corridor shown in Fig. 5. The I-210 corridor is composed of over 4,000 OD pairs and more than 10,000 links. There are four major highways in the I-210 corridor, namely the East/West-bound I-210, the North/South-bound I-605, the North/South-bound California 101, and the East/West-bound I-134. Background flow from a typical weekday (6:00 AM - 7:00 AM) is obtained from PeMS and city data collected for the Connected Corridors project [26]. During these peak hours over 75,000 vehicles enter the network hourly. As in the benchmark scenario, we fix the demand between OD pairs and then perturb the percentage of app users between a single OD as shown in Figure 5. We start with 10% app users and increase to 90% using 10% increments. We focus our analysis on a single OD pair because the complexity of the network is high and therefore results are difficult to interpret when all OD pairs are perturbed simultaneously.

We consider an OD pair that connects a freeway origin and a local destination as shown in Fig. 5. Since vehicles traveling between freeway and local ODs must use local (i.e.,

non freeway) links they are more incentivized to find routes with lower travel time. Furthermore, we choose to observe OD pairs with a high demand, over 250 *veh/hr*. We do this to ensure that perturbing the percentage of app users traveling between the OD pair creates a large enough impact on the network to demonstrate the use of the *average marginal regret*. In consideration of these two criteria, we observe the OD pair illustrated in Fig. 5.

We perturb the number of app users between this OD pair from 10% to 90% with results shown in Fig. 6. As described previously, non-app users follow paths determined by computing a static user equilibrium in the network given the same demand. Similar to the results in the benchmark network, we observe that the path flows and travel times of the main path (freeway) and alternative paths (shown in orange and green in Fig. 5) converge as the number of app users between the specific OD pair increases. The *average marginal regret* shown in Fig. 6 also decreases as the percentage of app users increases. However, there are some slight increases in *average marginal regret*, specifically at 50%, 80%, and 90% app users. This phenomenon may be caused by the cost (i.e., travel time) of a path which is calculated based on the current network state instead of the future or predicted state. As a result, large numbers of app users will reroute themselves onto alternative routes with low costs before the costs of these alternatives are updated, leading to a travel time longer than expected.

IV. CONCLUSION

This article assesses the impact of app use on traffic patterns. In particular, after extending regret to non repeated games, we define the *average marginal regret*. *Average marginal regret*

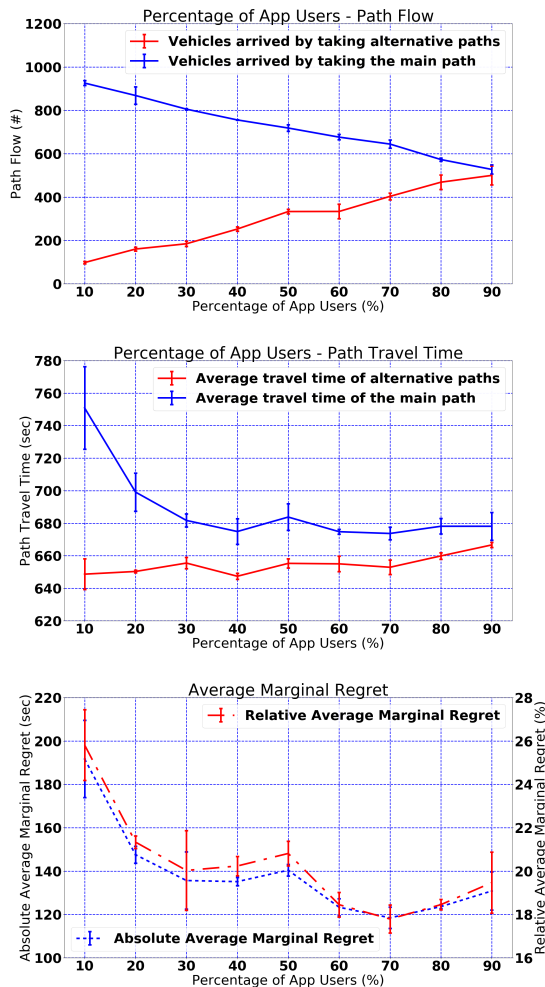


Fig. 6. I-210 network without accident. **Top:** Path flow on the main path (freeway shown in blue in Fig. 5) and all alternative paths (shown in orange and green in Fig. 5). **Middle:** Path travel time convergence between the main freeway path and all alternative paths. **Bottom:** Absolute and relative *average marginal regret* as percentage of app users in the network increases.

quantifies the time a driver could expect to save by having information about the state of the overall network. We use dynamic simulations conducted in *Aimsun* to show that regret decreases when app usage increase. On a benchmark network we demonstrate that 1) traffic tends to converge to a Nash equilibrium when app usage increases and 2) regret upon the occurrence of an accident is reduced when there are a large percentage of app users. We also show results for dynamic simulations run at scale on the I-210 corridor in LA which indicate a similar decrease in *average marginal regret* as the percentage of app users traveling between the considered OD pair increases. Experiments on both the benchmark network and the I-210 demonstrate that increased app usage leads to a system-wide convergence to Nash equilibrium.

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