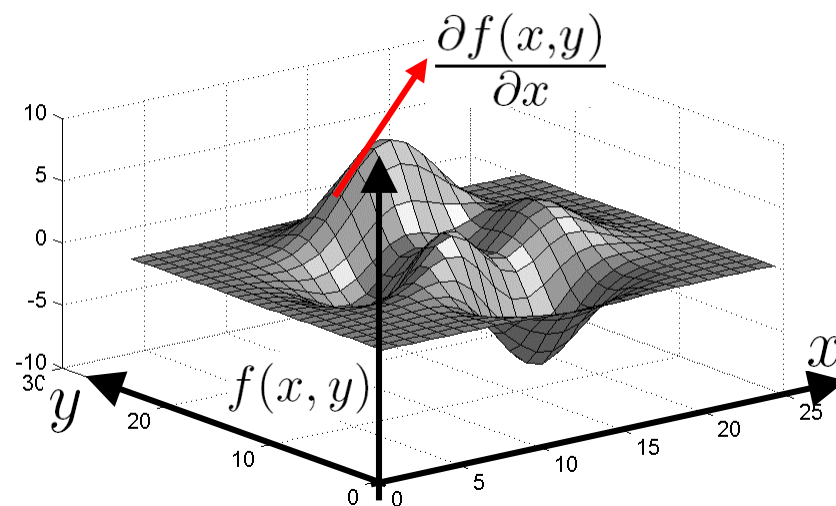


## Lecture 3½ : gradient refresher

- Definition of the gradient in 2D
- Definition of the gradient in nD
- Graphical interpretation of the gradient
- Interpretation for linear programs
- Application for integer programming

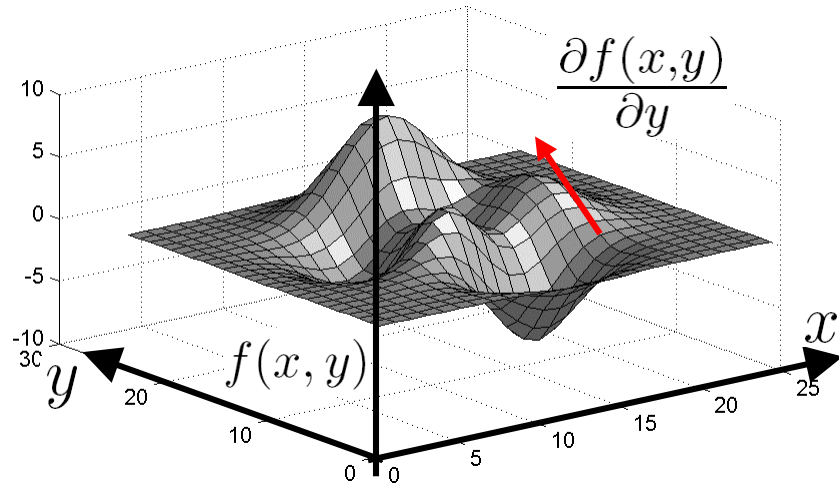
## Illustration of the gradient in 2D

Illustration of the gradient in 2D



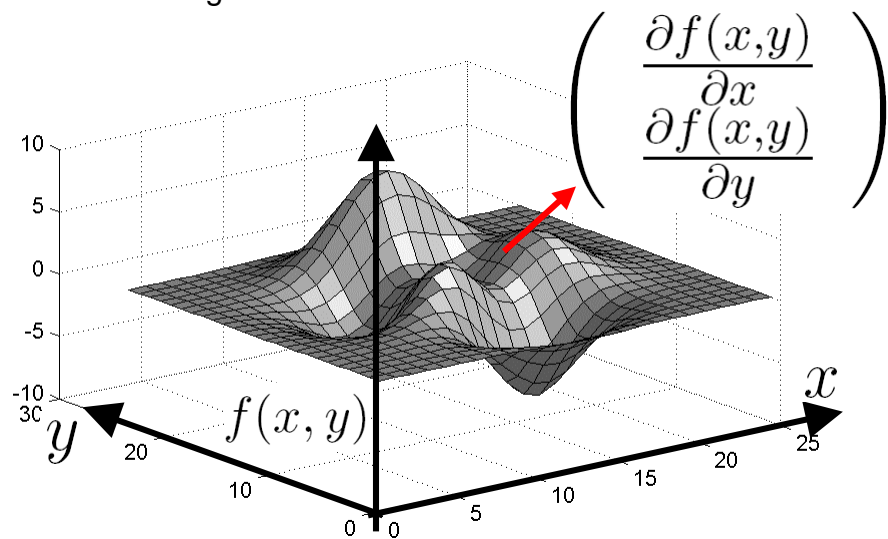
## Illustration of the gradient in 2D

Illustration of the gradient in 2D



## Illustration of the gradient in 2D

Illustration of the gradient in 2D



## Illustration of the gradient in 2D

---

Definition of the gradient in 2D

$$\nabla f(x, y) = \begin{pmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{pmatrix}$$

This is just a generalization of the derivative in two dimensions.  
This can be generalized to any dimension.

## Multiple dimensions

---

Everything that you have seen with derivatives can be generalized with the gradient.

For the descent method,  $f'(x)$  can be replaced by

$$\nabla f(x, y) = \begin{pmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{pmatrix}$$

In two dimensions, and by

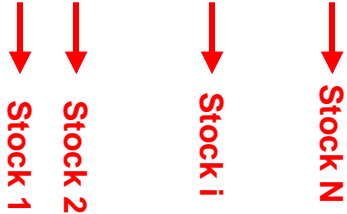
$$\nabla f(x_1, x_2, \dots, x_i, \dots, x_N) = \begin{pmatrix} \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_i}, \dots, \frac{\partial f}{\partial x_N} \end{pmatrix}$$

in N dimensions.

## Example of 2D gradient: MATLAB demo

The cost to buy a portfolio is:

$$f(x_1, x_2, \dots, x_i, \dots, x_N) = x_1^2 \cdot (x_2 - 4)^3 + \sum_{i=3}^N x_i^2$$

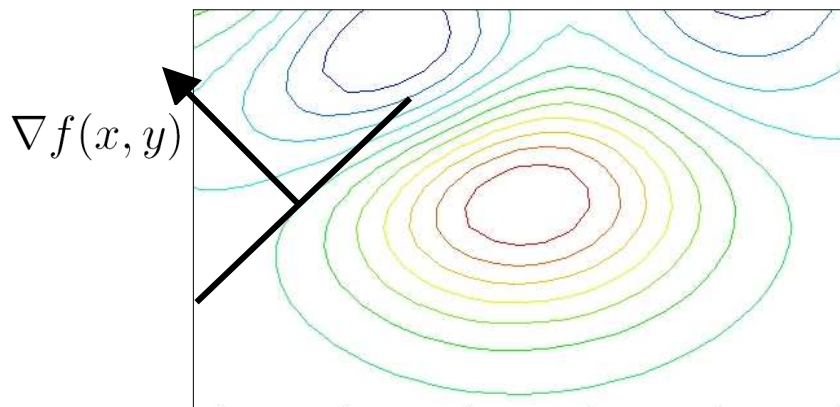


If you want to minimize the price to buy your portfolio, you need to compute the gradient of its price:

$$\nabla f(x_1, x_2, \dots, x_i, \dots, x_N) = \left( \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_i}, \dots, \frac{\partial f}{\partial x_N} \right)$$

## Graphical interpretation of the gradient

The gradient of a scalar field (function of multiple variables) is perpendicular to the isolines of this function.



## Interpretation: for linear programs

**Take your favorite linear program:**

$$\begin{array}{ll} \text{min:} & c_1x_1 + c_2x_2 + \cdots + c_Nc_N \\ \text{s.t.:} & a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,j}x_j \cdots + a_{1,N}x_N \leq b_1 \\ & a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,j}x_j \cdots + a_{2,N}x_N \leq b_2 \\ & \vdots \\ & a_{M,1}x_1 + a_{M,2}x_2 + \cdots + a_{M,j}x_j \cdots + a_{M,N}x_N \leq b_M \end{array}$$

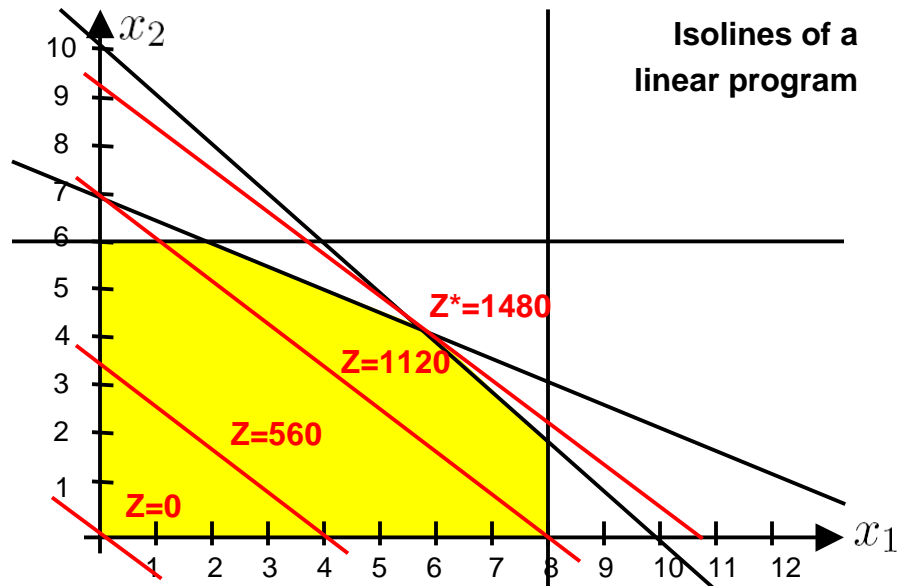
**Cost function of the linear program reads:**

$$J = c_1x_1 + c_2x_2 + \cdots + c_Nc_N \quad |$$

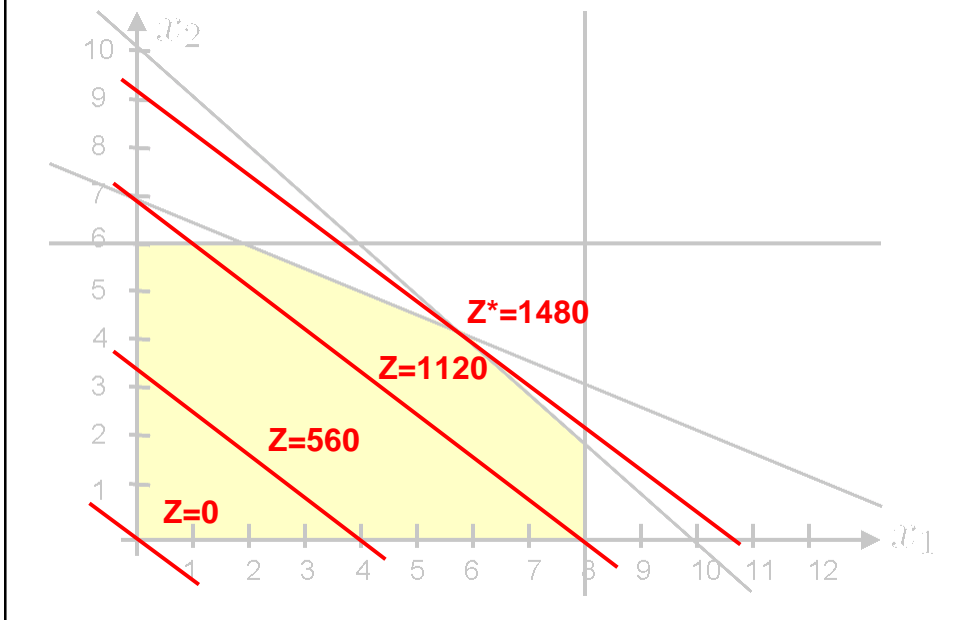
**Or in compact form:**

$$J = \mathbf{c} \cdot \mathbf{x} \quad |$$

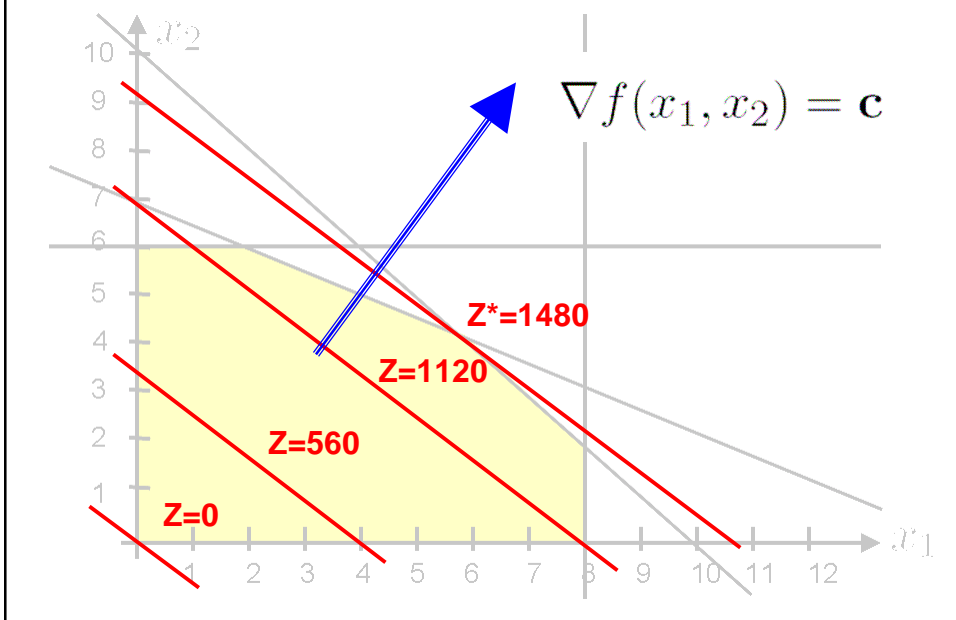
## Interpretation for linear programs



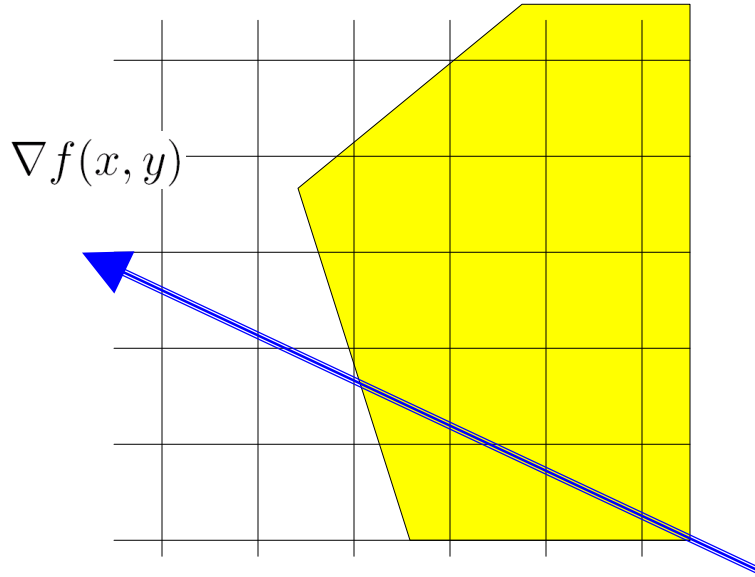
### Isolines for the cost



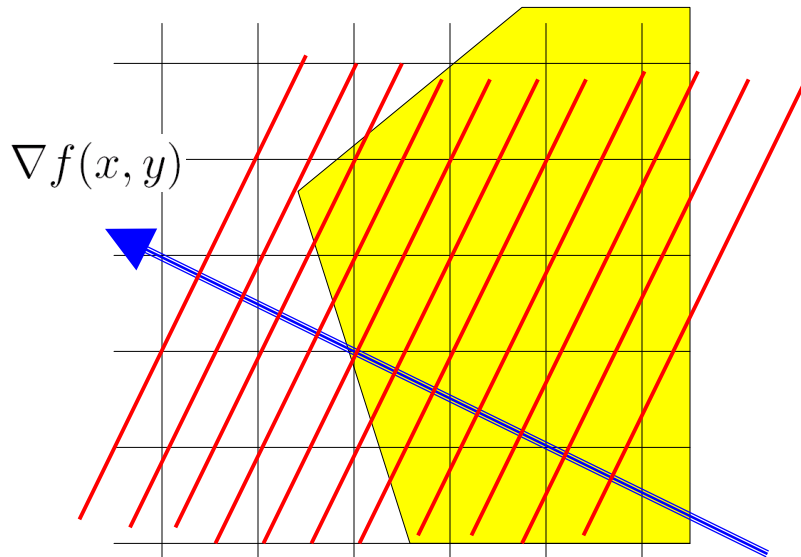
### Isolines for the cost



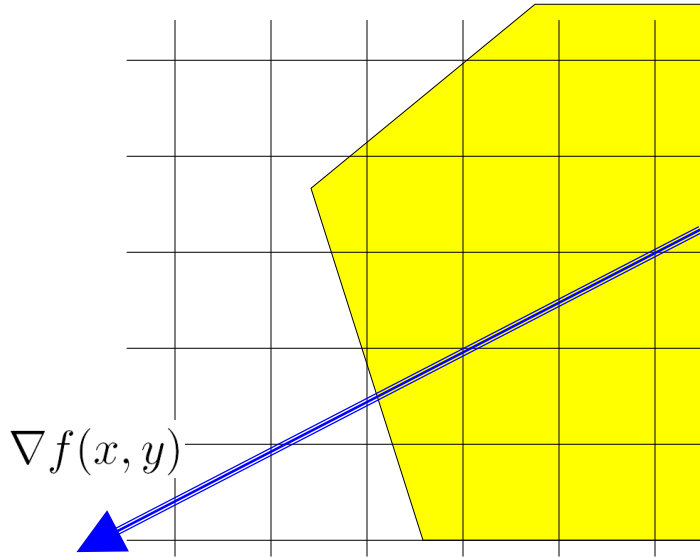
Application for integer programming



Application for integer programming



Application for integer programming



Application for integer programming

