Lecture 2: graphical solutions of LPs

- Graphical interpretation of constraints
- Feasible set
- Gradient of the cost function
- Unbounded feasible set
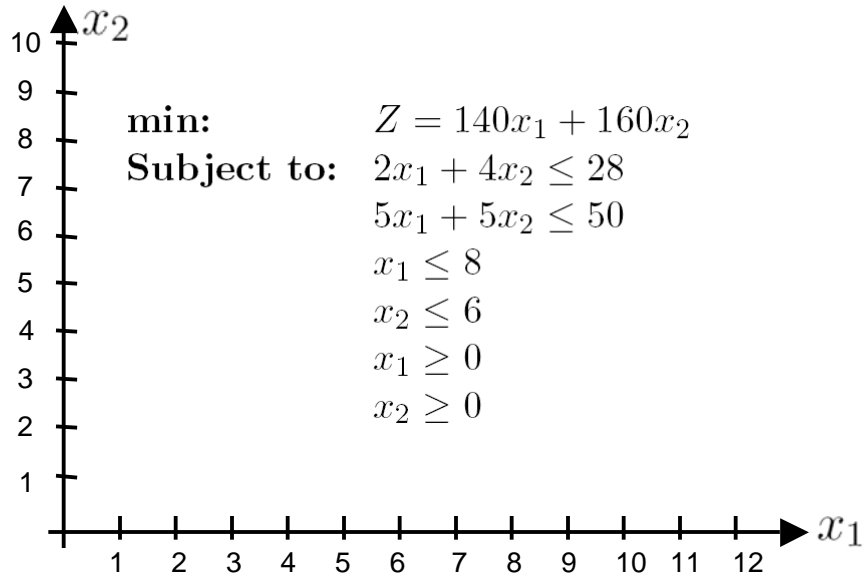- Unbounded cost function
- Infeasibility

Graphical solutions of linear programs
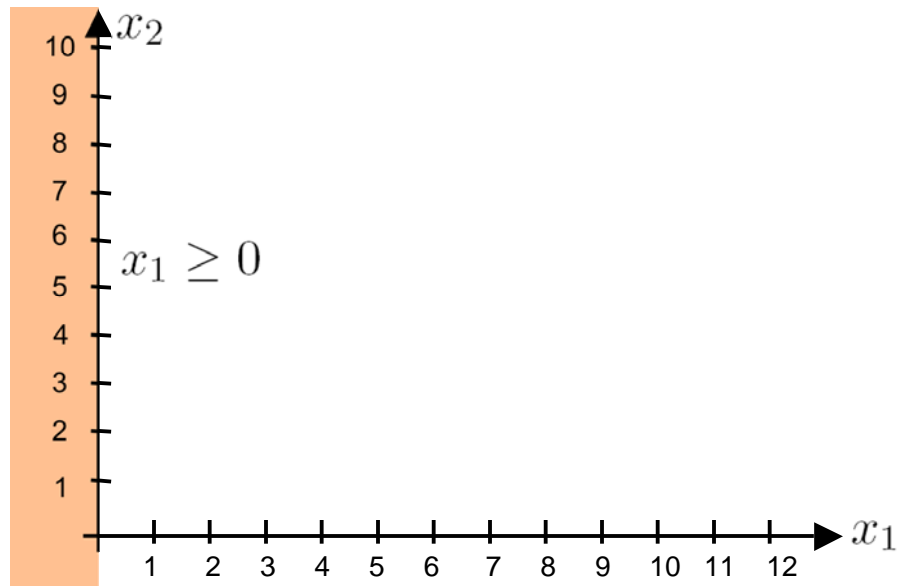
Example from the textbook [Chap 3., p. 43]

$$
\begin{aligned}
\textbf{min:} \qquad & Z = 140x_1 + 160x_2 \\
\textbf{Subject to:} \quad & 2x_1 + 4x_2 \leq 28 \\
& 5x_1 + 5x_2 \leq 50 \\
& x_1 \leq 8 \\
& x_2 \leq 6 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

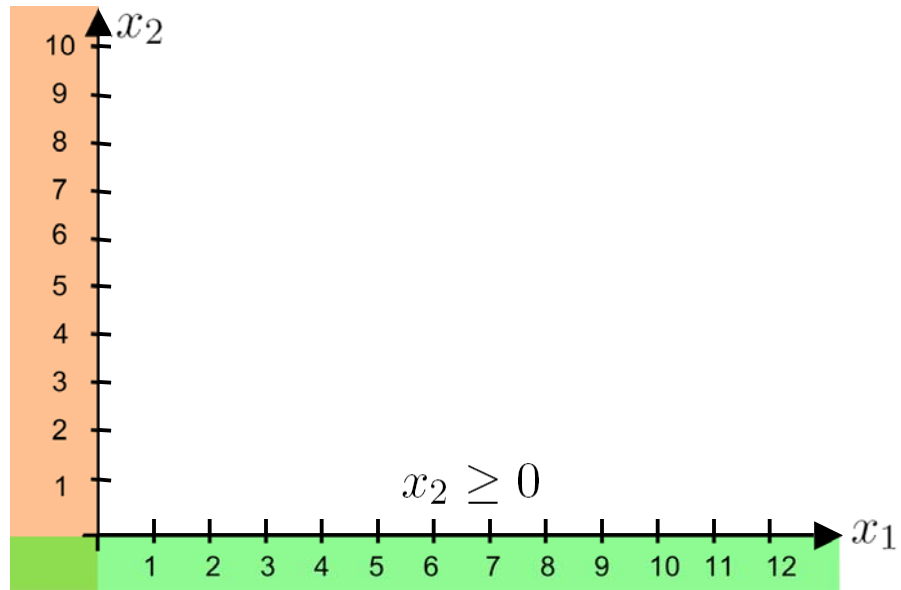**[Civil and Environmental Engineering, Revelle, Whitlatch and Wright]**
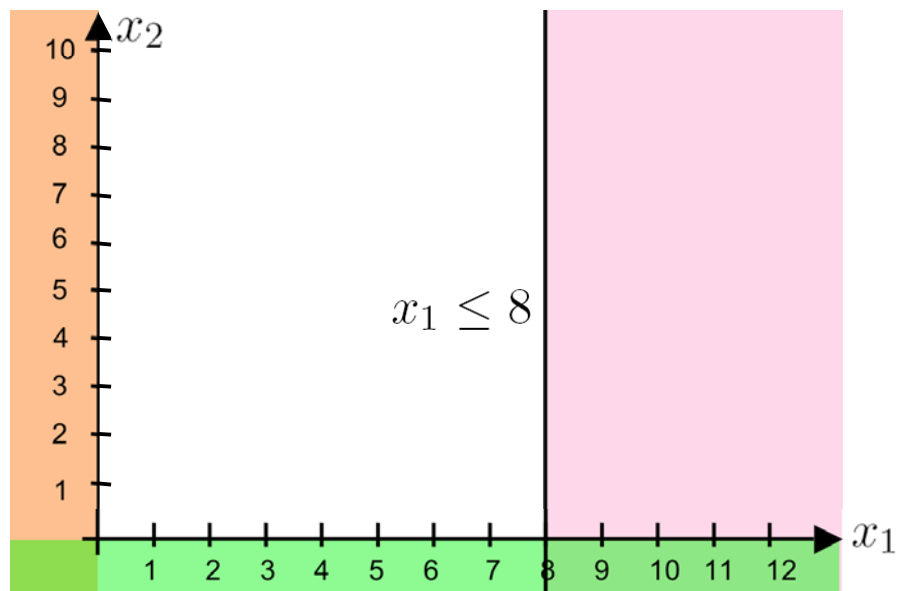
1

Construction of the feasible set

x_2

$$\text{min:} \qquad Z = 140x_1 + 160x_2$$
$$\textbf{Subject to:} \quad 2x_1 + 4x_2 \leq 28$$
$$5x_1 + 5x_2 \leq 50$$
$$x_1 \leq 8$$
$$x_2 \leq 6$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Construction of the feasible set

x_2

$x_1 \geq 0$

x_1

Construction of the feasible set

$$x_2 \geq 0$$

Construction of the feasible set

$$x_1 \leq 8$$

Construction of the feasible set

$x_2 \leq 6$

Construction of the feasible set

$5x_1 + 5x_2 \leq 50$

Construction of the feasible set

$2x_1 + 4x_2 \leq 28$

Feasible set: result

x_2

**the optimum
must be found
inside this set**

x_1

Optimal solution: isolines

$$Z = 140x_1 + 160x_2$$

Z=0

Optimal solution: isolines

$$Z = 140x_1 + 160x_2$$

Z=560

Optimal solution: isolines

$$Z = 140x_1 + 160x_2$$

Z=1120

Optimal solution: isolines

$$Z = 140x_1 + 160x_2$$

Z*=1480

7

Gradient of the cost function

$$Z = 140x_1 + 160x_2$$

Z*=1480
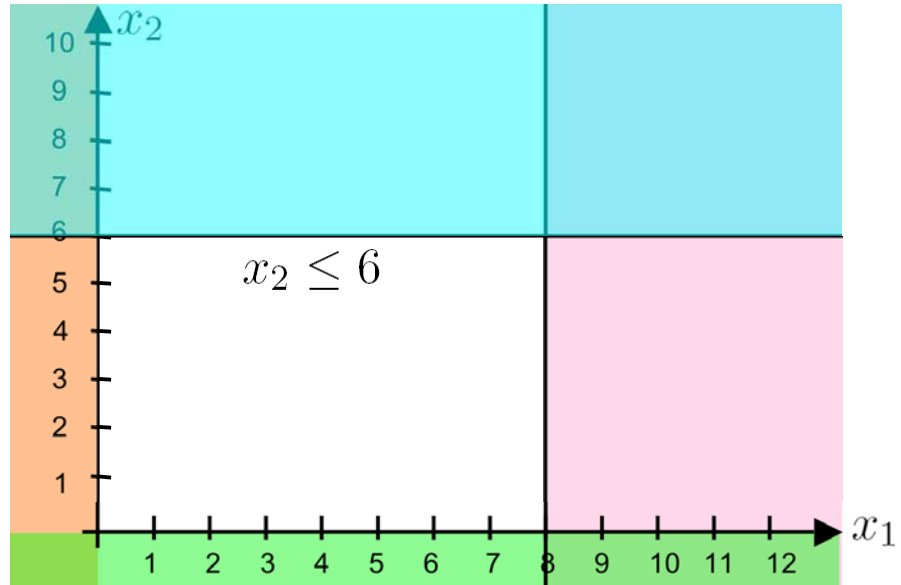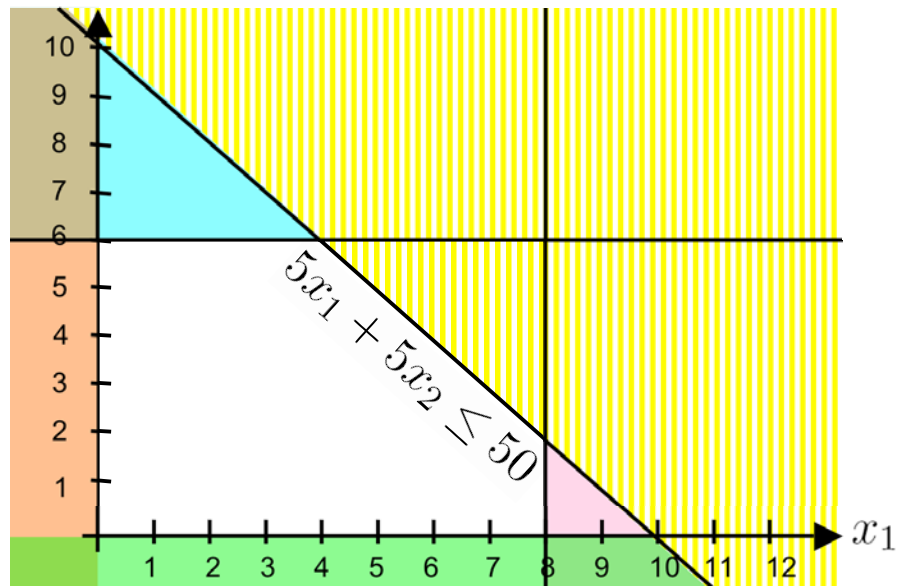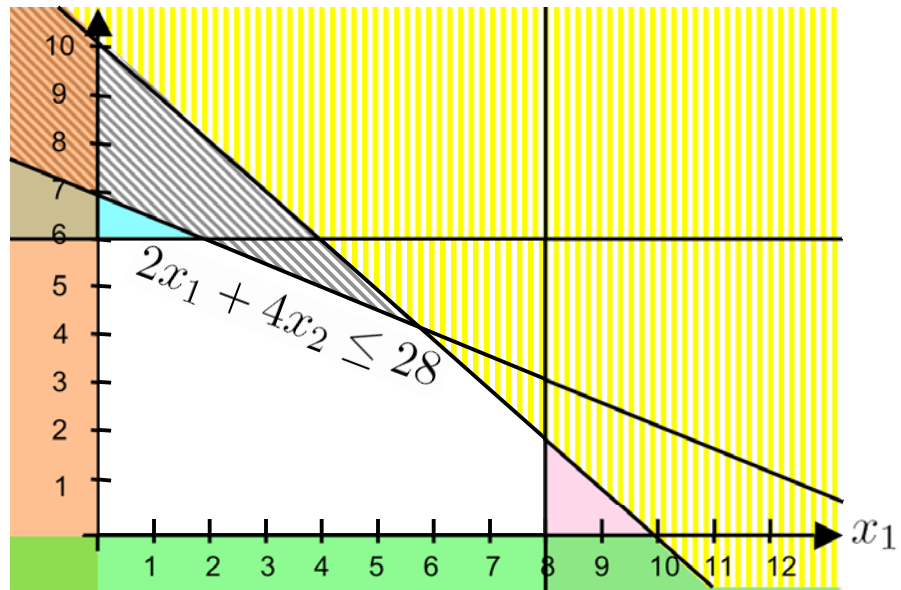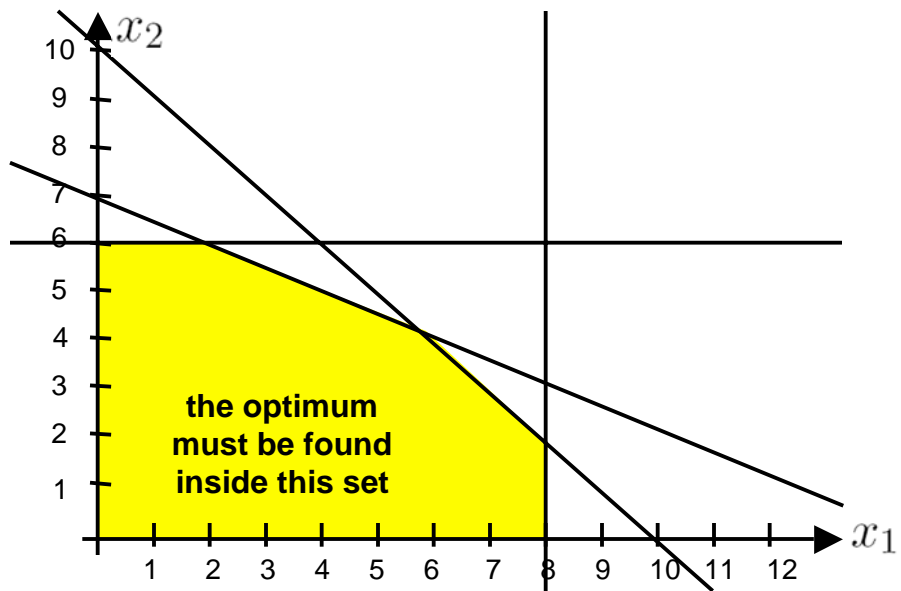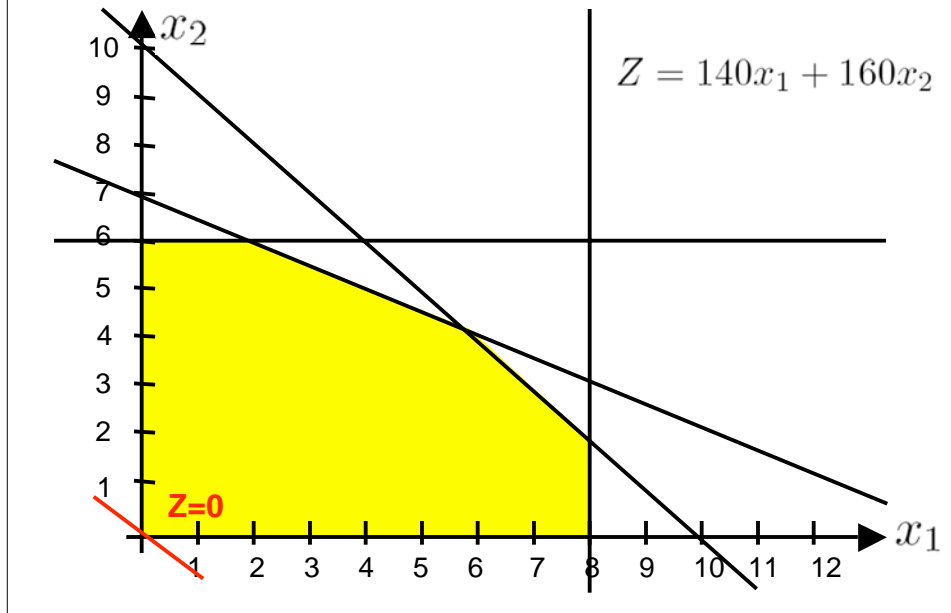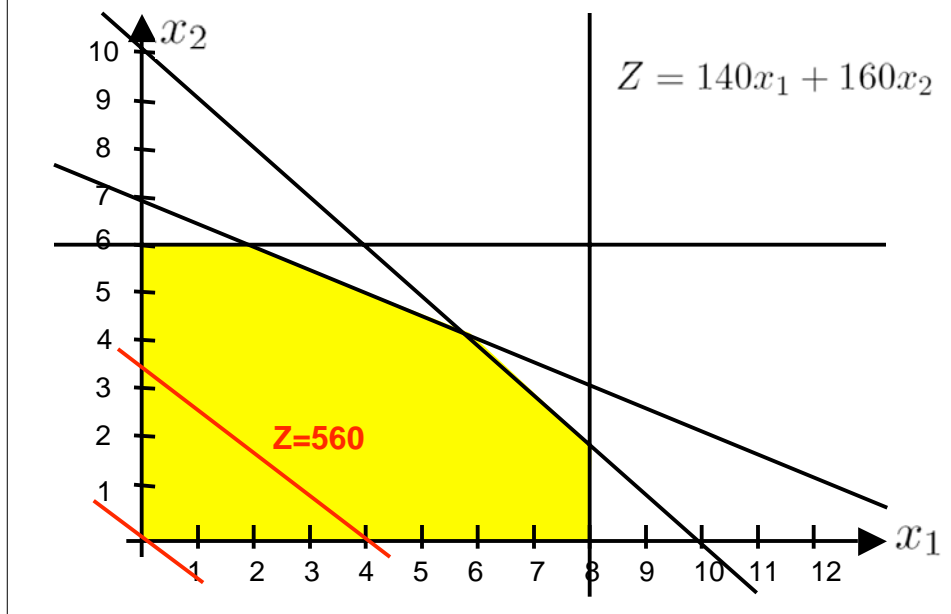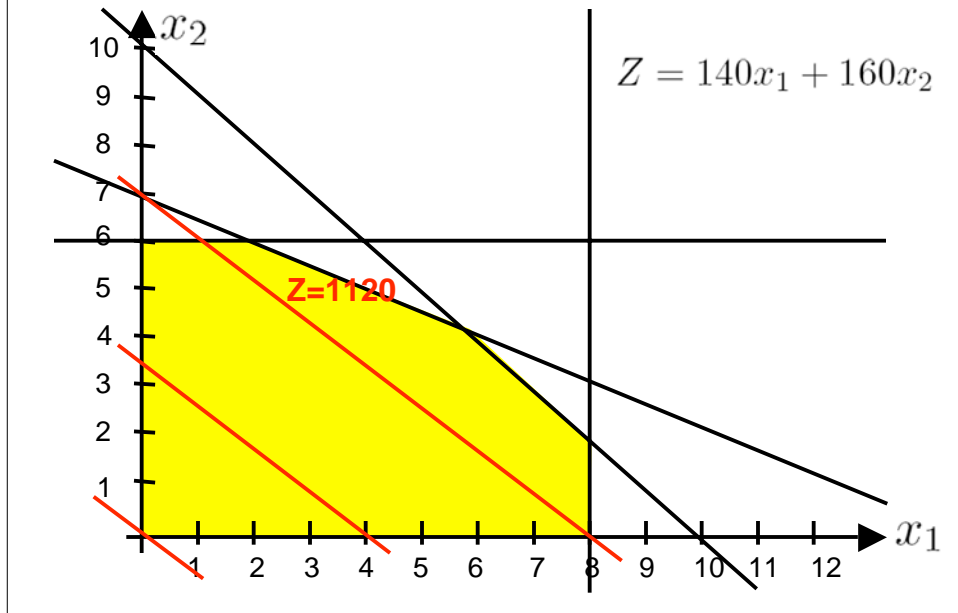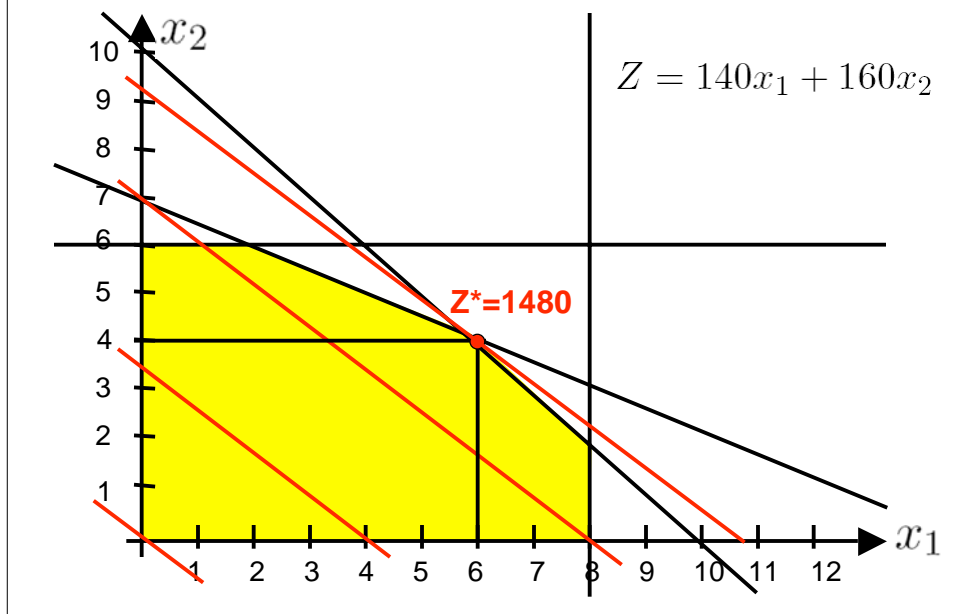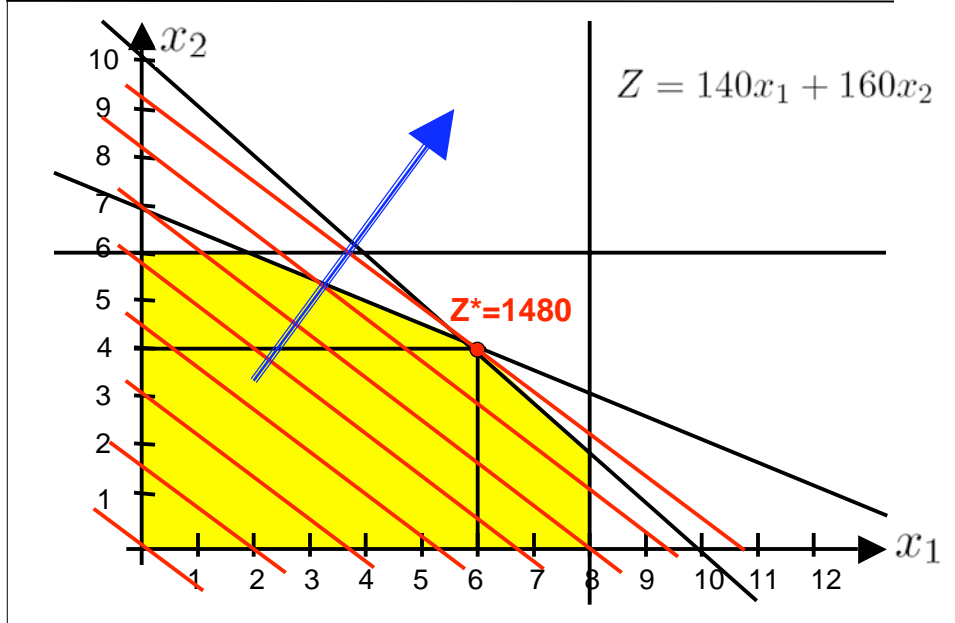
Uniqueness (or not) of the optimum
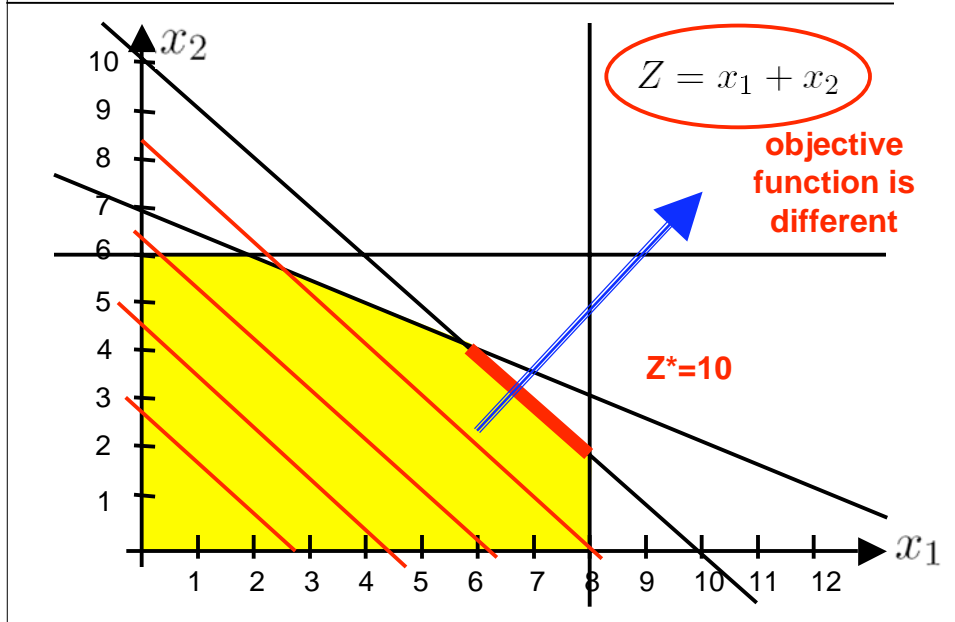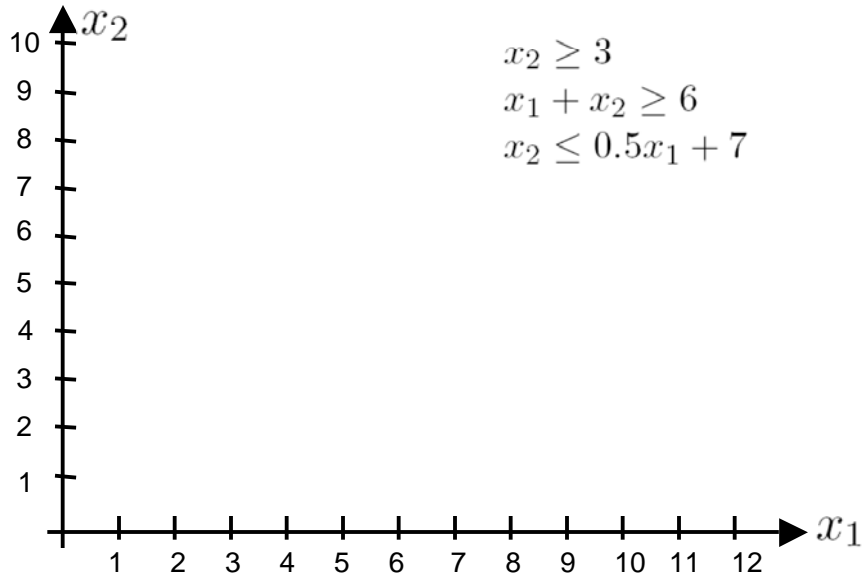
$$Z = x_1 + x_2$$

objective function is different

Z*=10

Features of the feasible set

$$x_2 \geq 3$$
$$x_1 + x_2 \geq 6$$
$$x_2 \leq 0.5x_1 + 7$$

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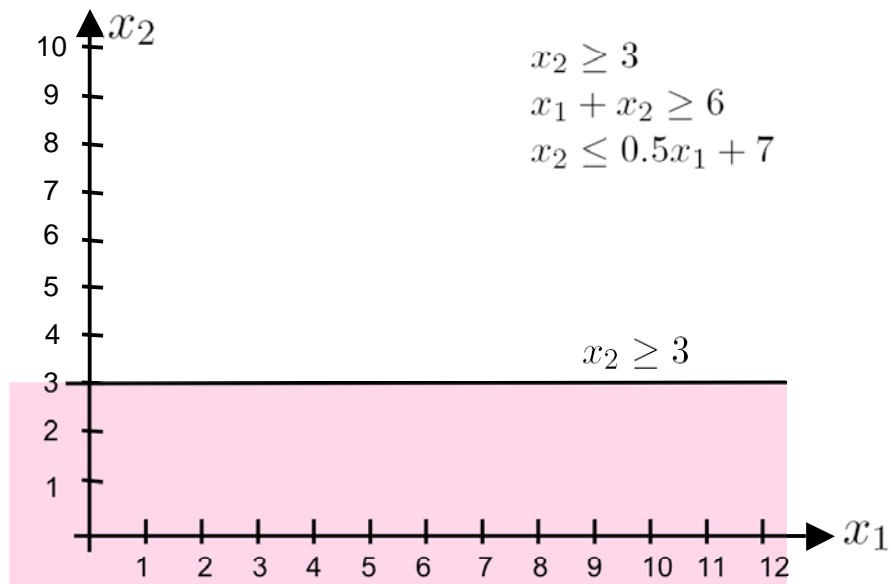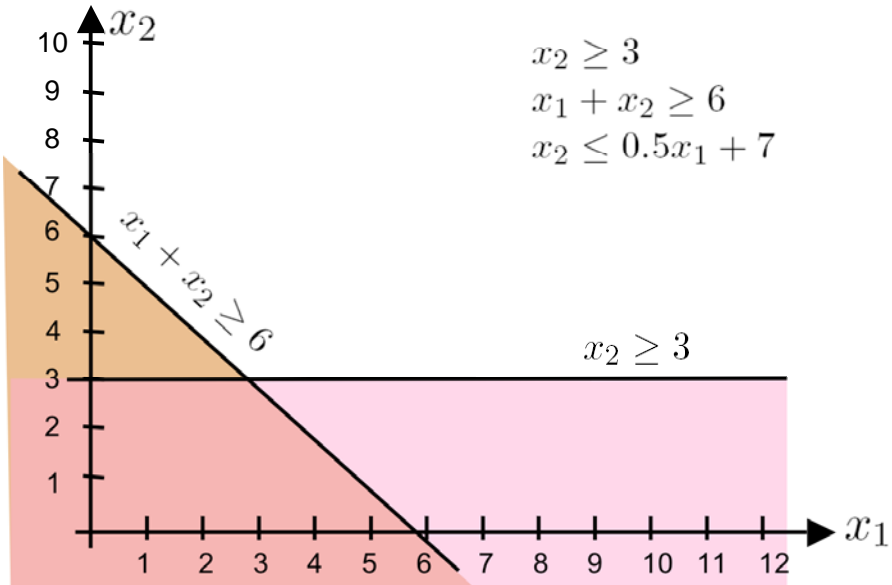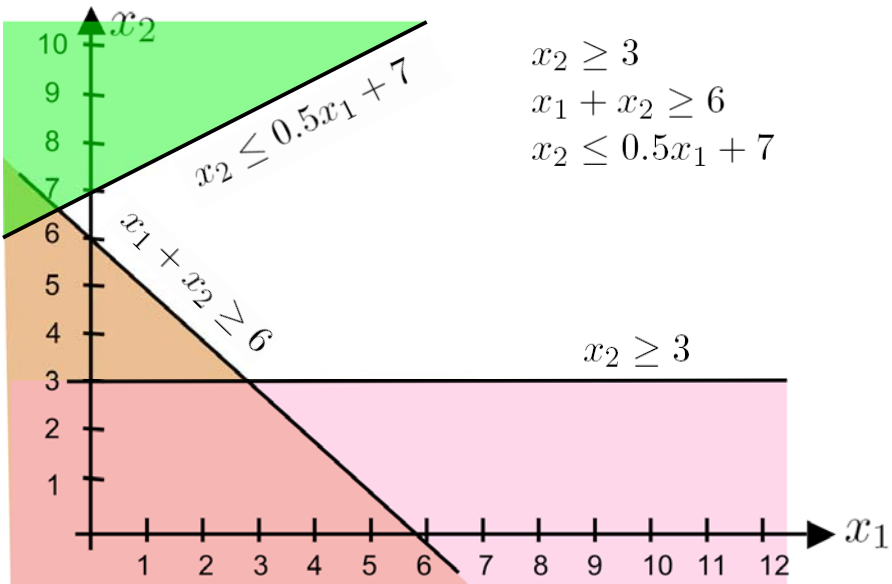
Feasible set is unbounded
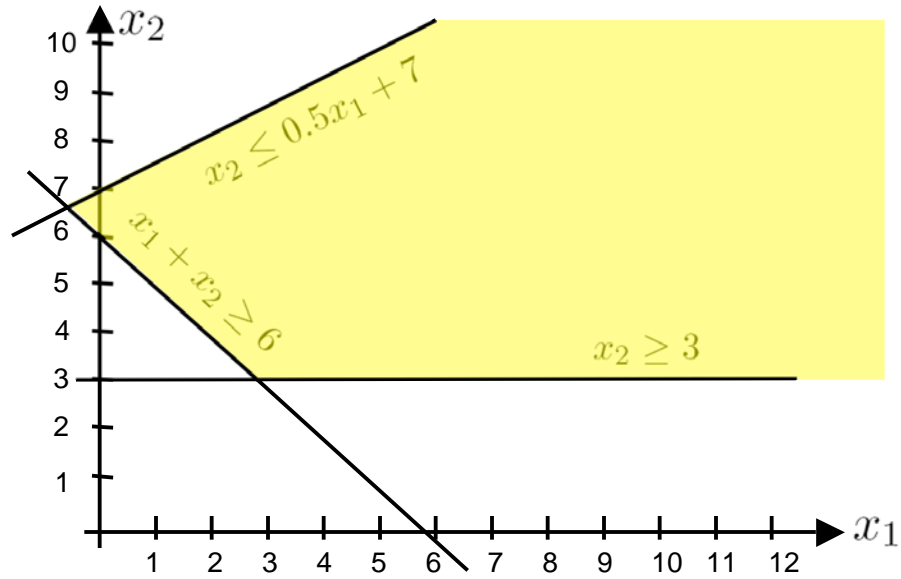
Objective function might be unbounded too

$$Z = 3.24x_1$$

Objective function might be unbounded too

$x_2 \leq 0.5x_1 + 7$

$x_1 + x_2 \geq 6$

$x_2 \geq 3$

$Z = 3.24x_1$
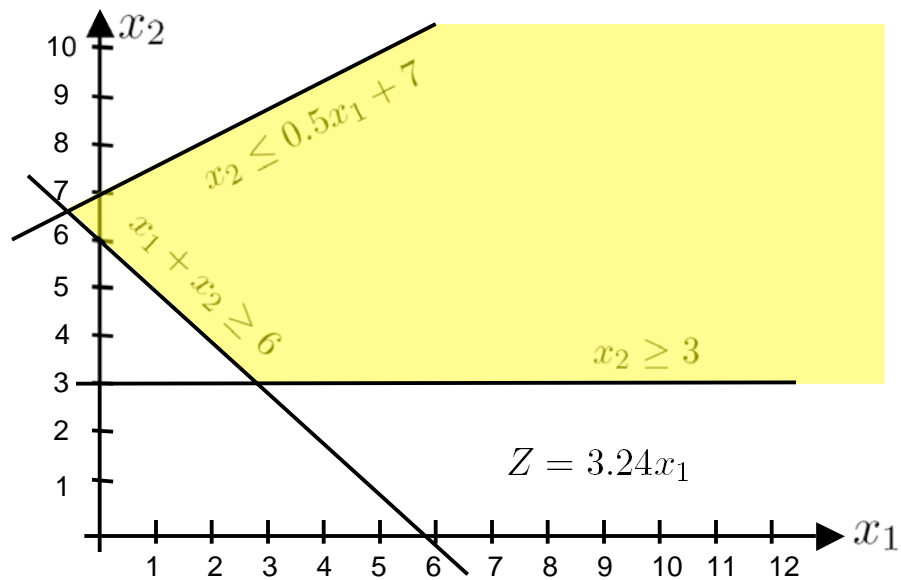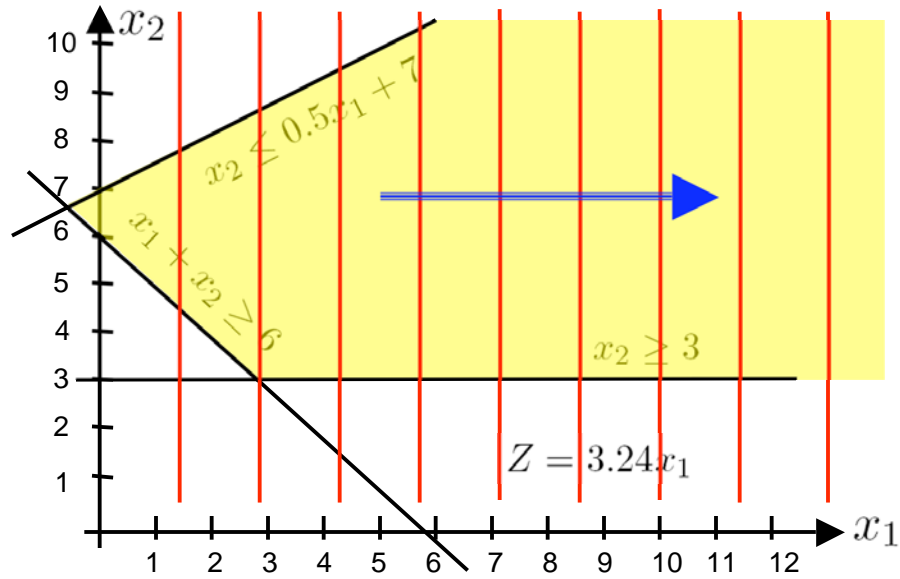
Objective might be bounded

$x_2 \leq 0.5x_1 + 7$

$x_1 + x_2 \geq 6$

$x_2 \geq 3$

Z*=12

$Z = x_1 + 3x_2$
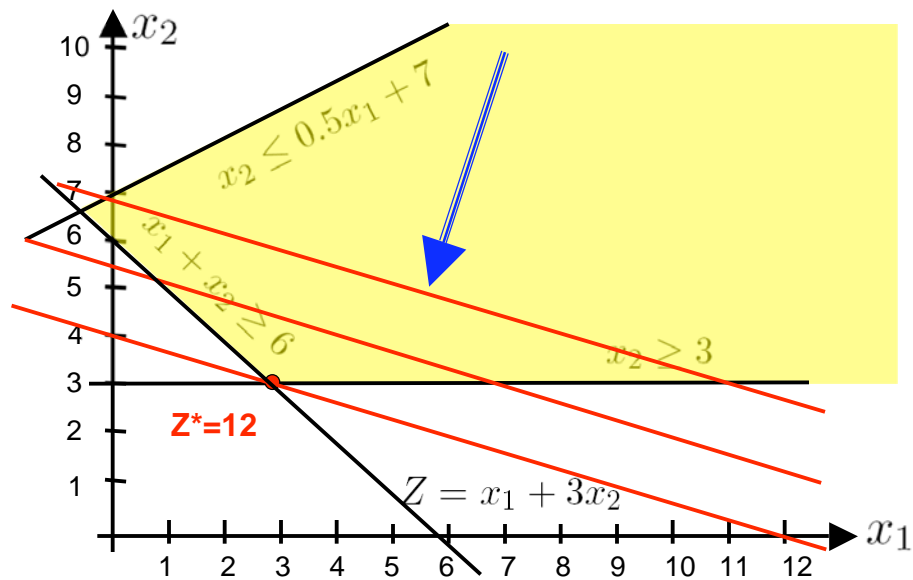
Objective might not be unique
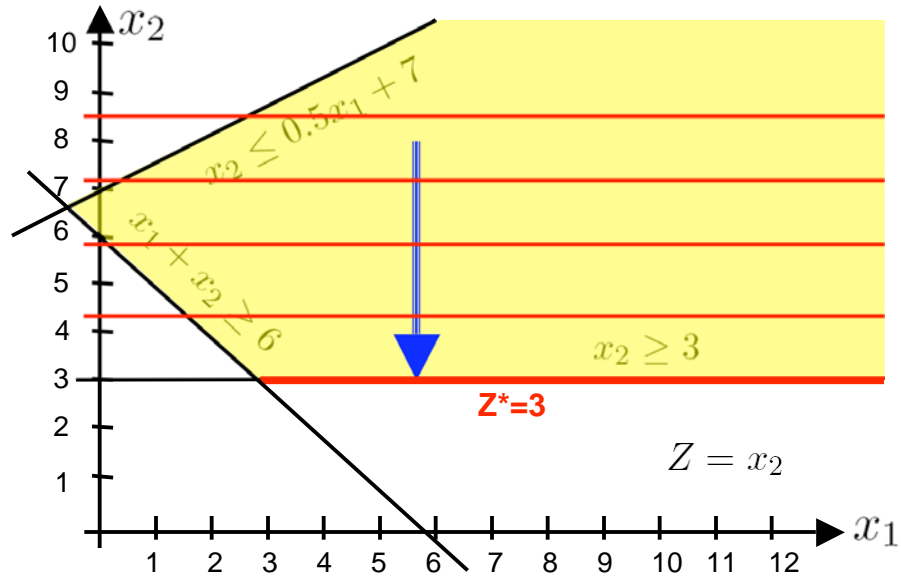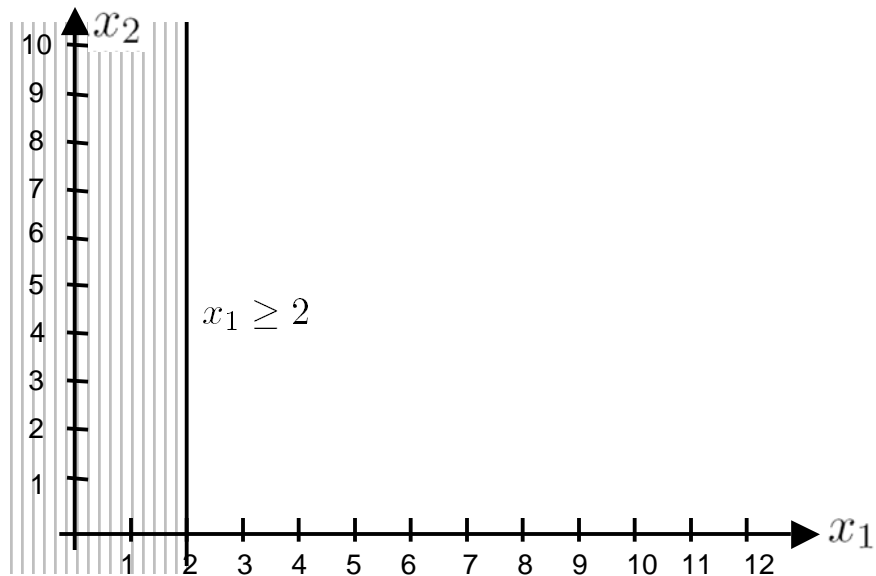
$x_2 \leq 0.5x_1 + 7$

$x_1 + x_2 \geq 6$

$x_2 \geq 3$

Z*=3

$Z = x_2$

Feasible set might be empty

$x_1 \geq 2$

Feasible set might be empty

$x_2 \leq 4$

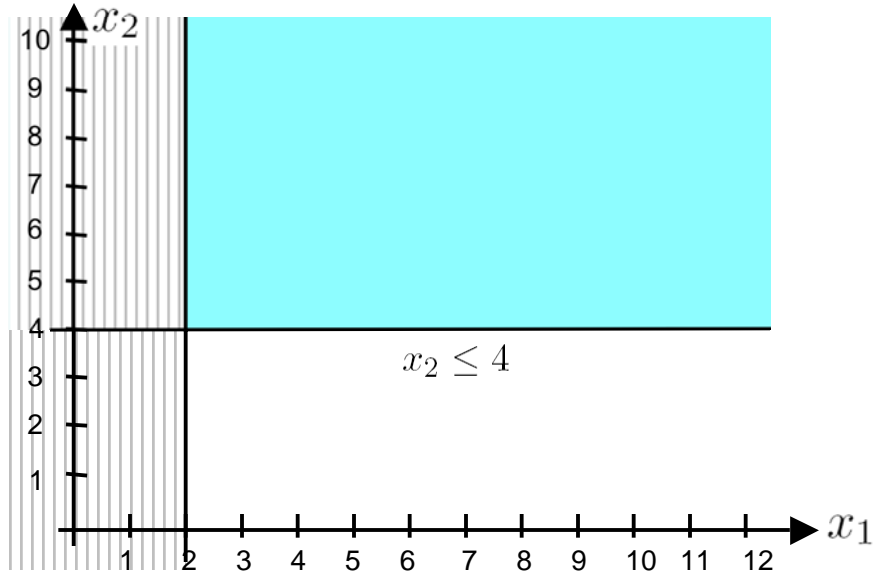## Feasible set might be empty
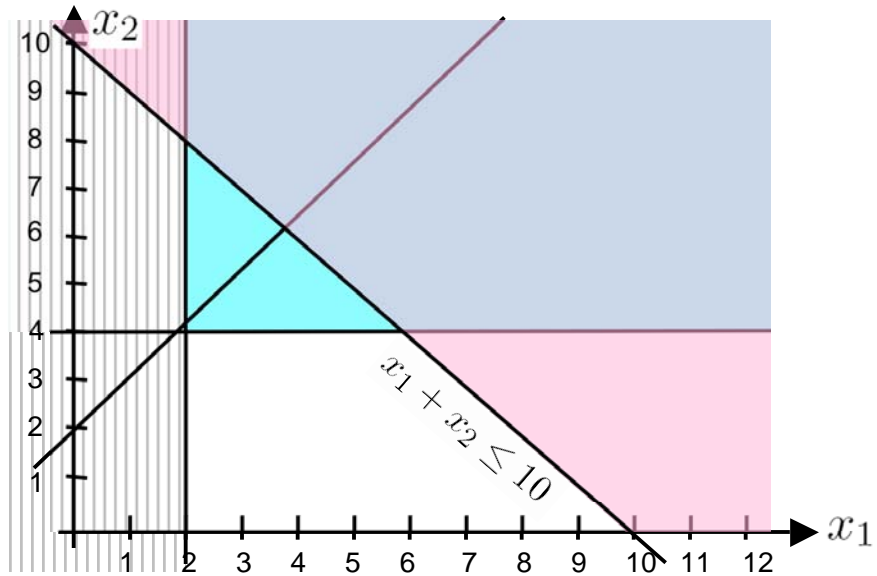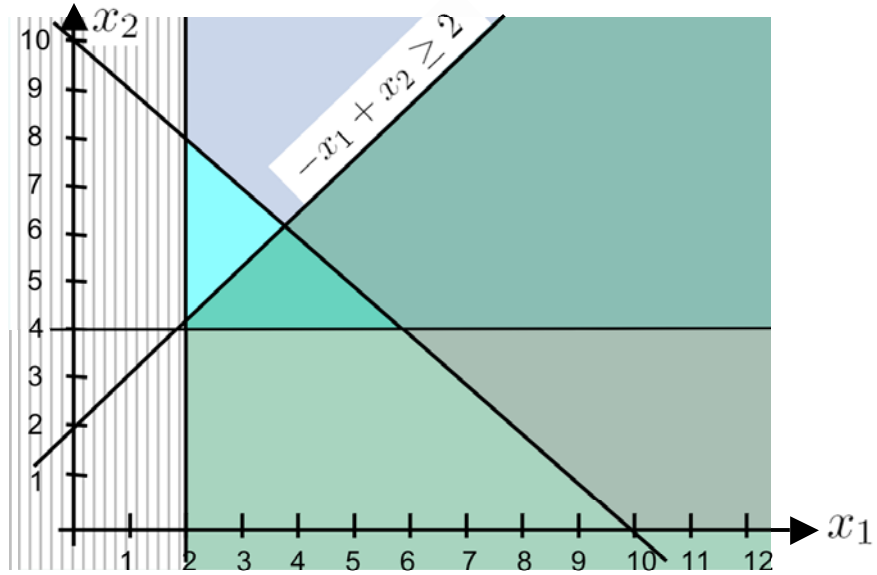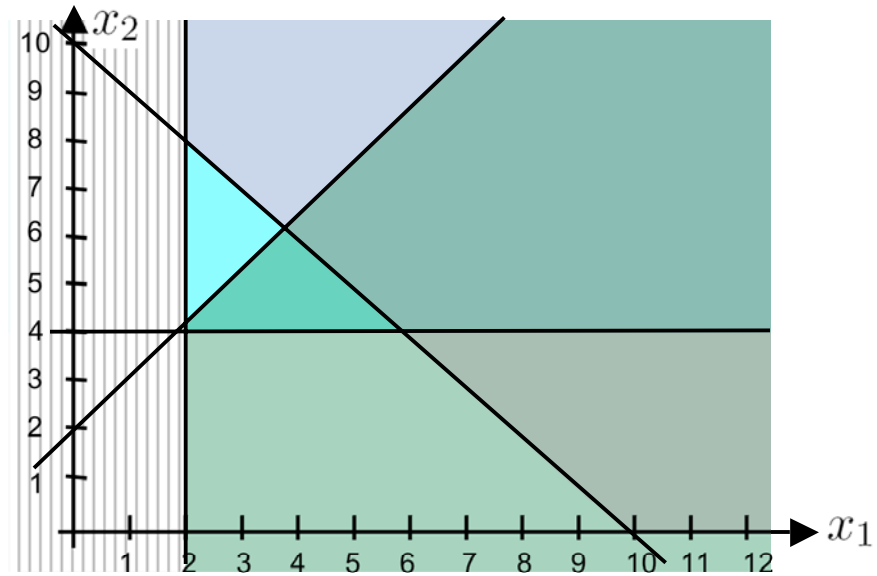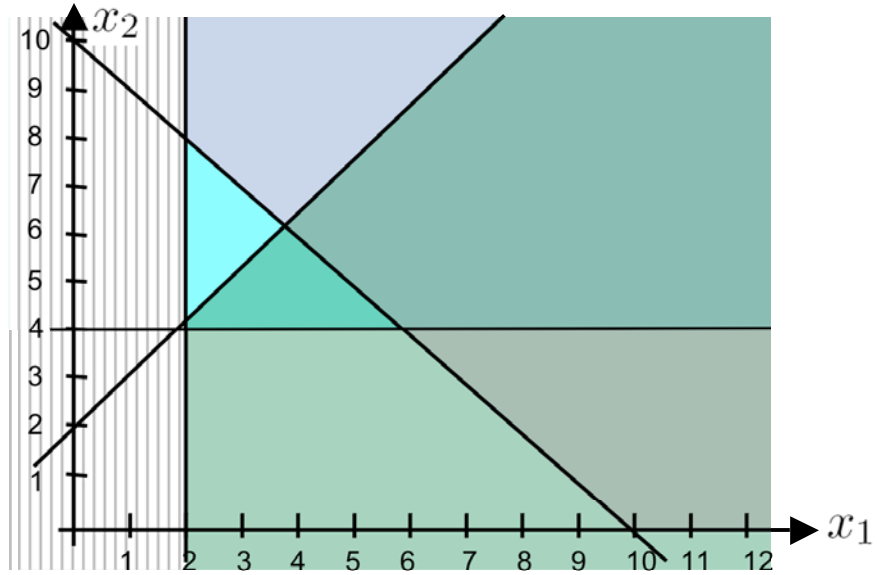

$x_1 + x_2 \leq 10$

Feasible set might be empty

Feasible set might be empty

Feasible set might be empty

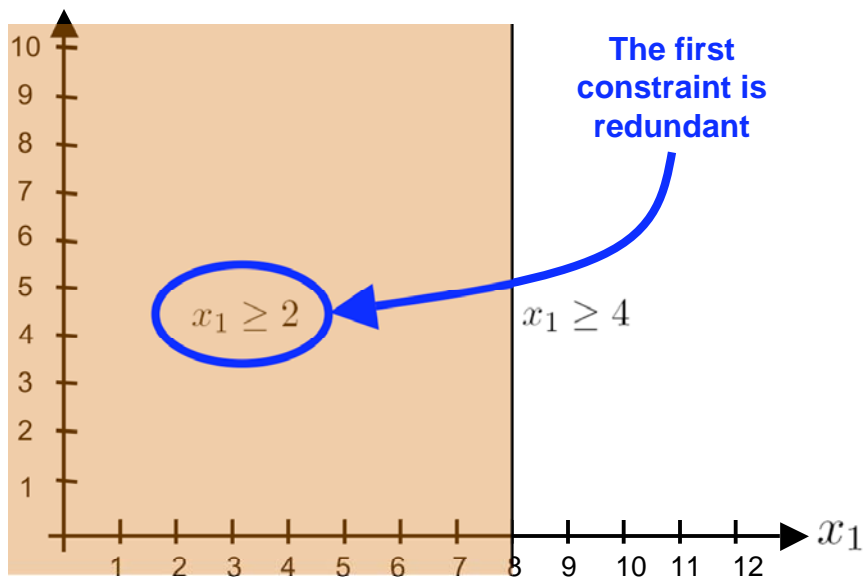## Redundant constraints

$$x_1 \geq 2$$
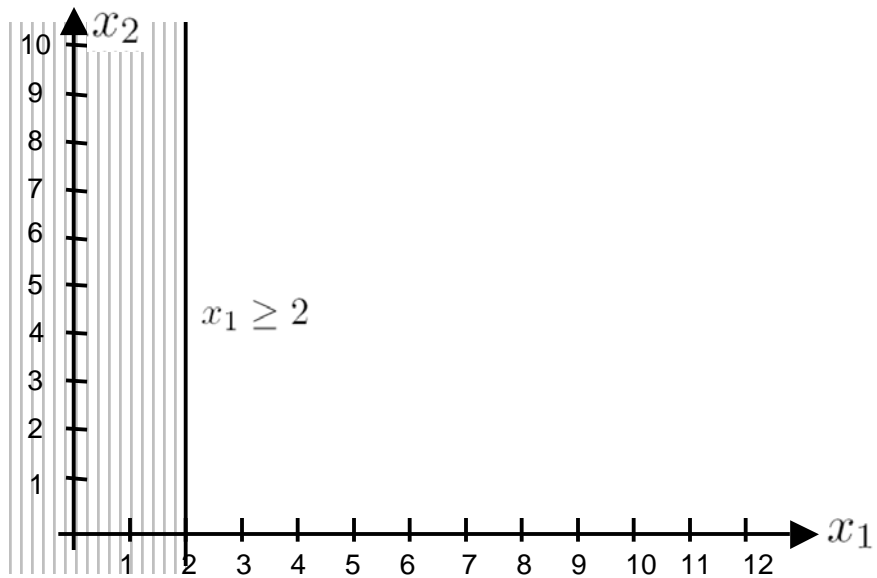
Redundant constraints

Redundant constraints

The first constraint is redundant
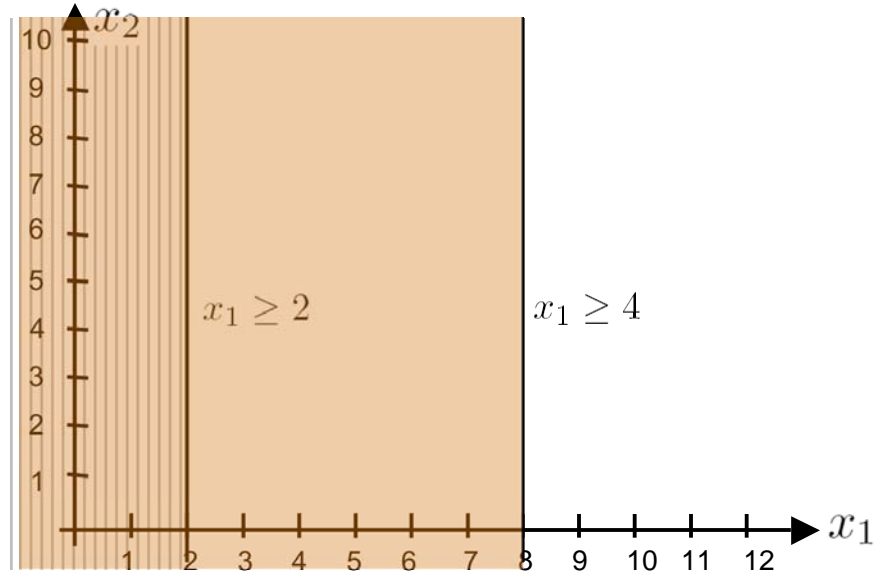
Graphical solution of LPs: general method

- Write your LP
- Successively eliminate half spaces corresponding to your constraints
- Is the feasible set empty?

    YES → problem infeasible

    NO → is the feasible set bounded?

        NO → is solution finite?

            NO: → finished

            YES → is there a unique solution?

                YES → corner point → finished

                NO → face →finished

        YES → is there a unique solution?

            YES → corner point → finished

            NO → face →finished