Inversion of Rayleigh Wave Dispersion Measurements in Geological Media

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CE 291F

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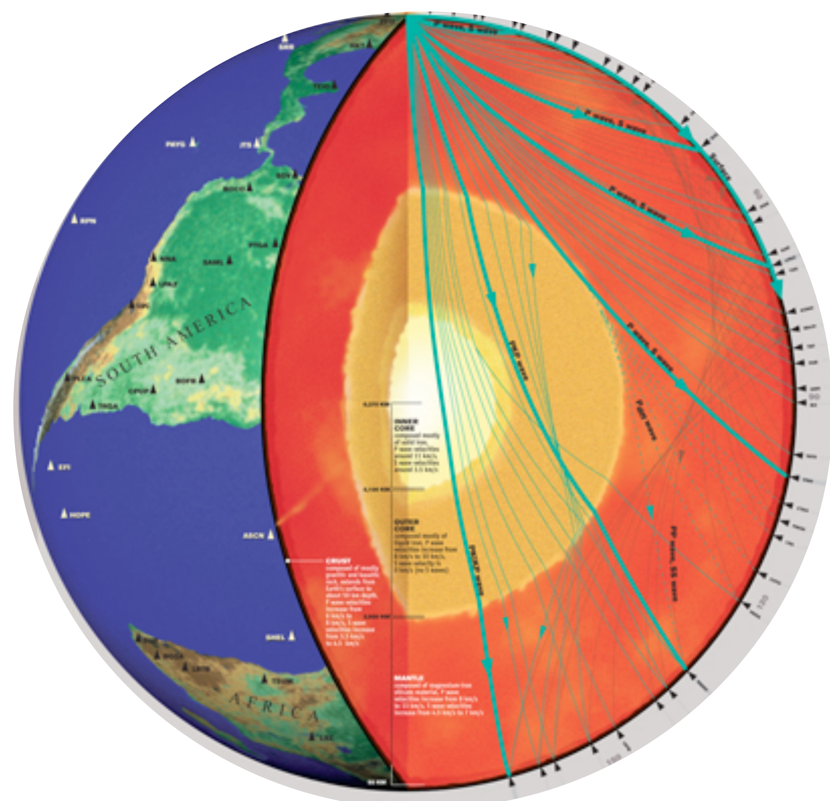
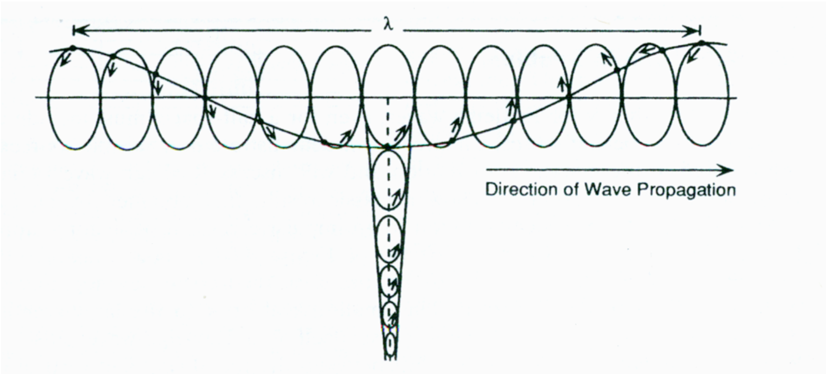
Introduction - Seismology

- Body Waves
  - Compressional (P) wave
  - Shear (S) wave
- Surface Waves
  - Rayleigh (R) Wave
  - Love (L) Wave
- Terminology:
  - Angular frequency, $\omega = 2\pi/T$
  - Wavenumber, $k = 1/\lambda$
  - Material velocity, $V = f(G,K,\rho)$
  - Phase velocity, $V^* = \omega/k$

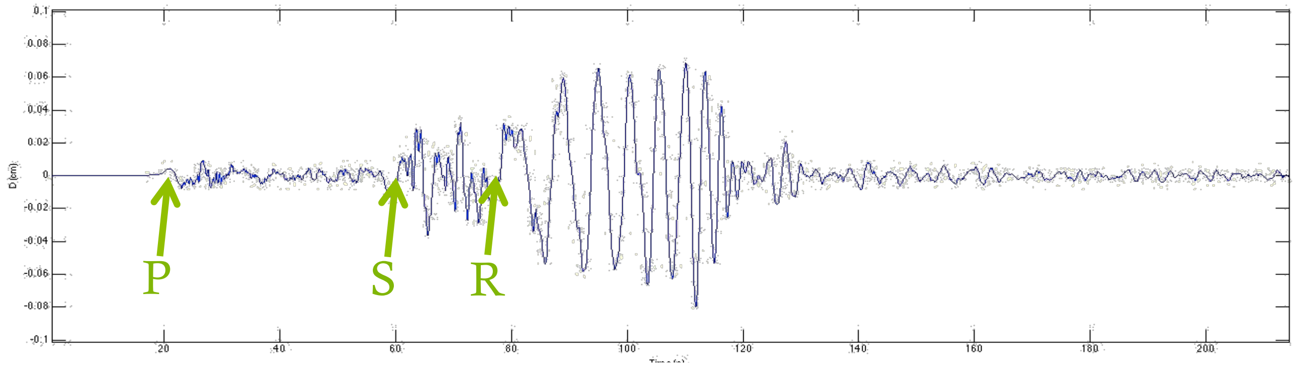


Introduction - Seismology

- Rayleigh Waves:
  - Interaction between P and S_v waves at a free surface
  - Vertically polarized
  - Retrograde elliptical particle motion
  - Amplitude decays exponentially with depth
  - Dispersion ($V^* = \omega / k \neq$ constant)



2011 Virginia Earthquake:

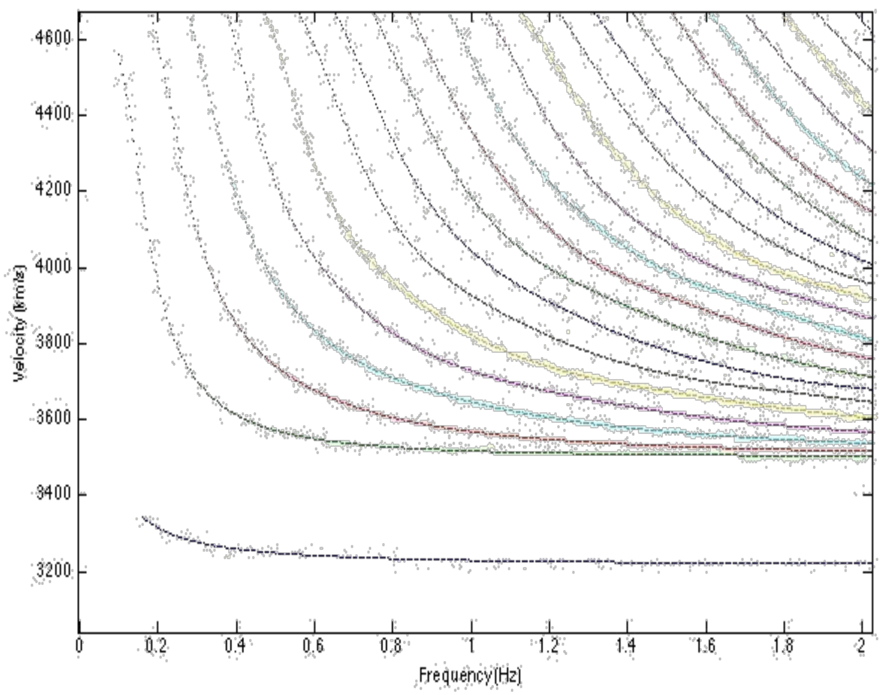


Surface Wave Inversion

- The set of V* for all possible k and ω is called the dispersion relationship, **D**
  - Fundamental mode vs. Overtones

- Goal of analysis:
  - Measure **D** for an unknown system <u>or</u> simulate **D** for a set of assumed (forward) models
  - Search for the best fit (inverted) model

- Applications:
  - Seismology
  - Geophysics
  - Non-destructive materials testing



1D Surface Wave Model, General Solution

$$\rho\ddot{\mathbf{u}}_j(\boldsymbol{x},t) = (\lambda_j + 2\mu_j)\nabla(\nabla\cdot\mathbf{u}_j(\boldsymbol{x},t)) - \mu_j\nabla\times\nabla\times\mathbf{u}_j(\boldsymbol{x},t)$$

$$\frac{d}{dz}\begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{pmatrix} = \begin{pmatrix} 0 & k & 1/\mu & 0 \\ -k\psi & 0 & 0 & \psi \\ k^2\xi - \omega^2\rho & 0 & 0 & k\lambda\psi \\ 0 & -\omega^2\rho & -k & 0 \end{pmatrix}\begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{pmatrix}$$

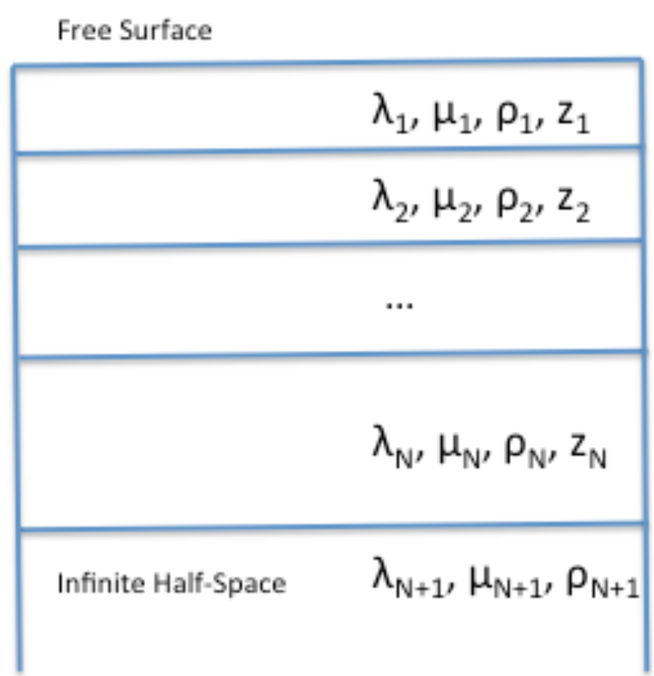
$$\psi = \frac{1}{\lambda+2\mu}$$

$$\xi = 4\mu(\lambda+\mu)\psi$$

$$u = r_1(k,z,\omega)e^{i(kr-\omega t)}$$

$$v = ir_2(k,z,\omega)e^{i(kr-\omega t)}$$

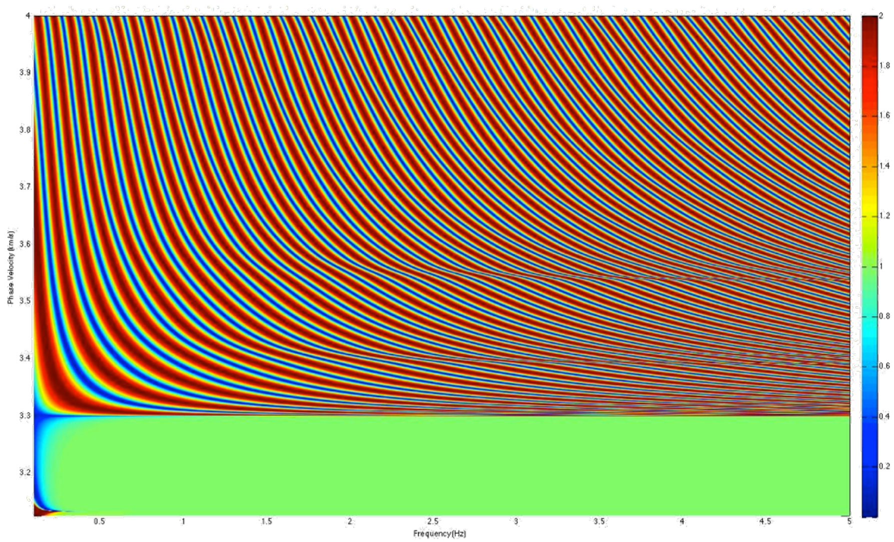
$$w = 0$$



FK Integration, Cost Function

- Use FK integration to simulate **D** (Haskell, 1964; Chen, 1993)
- Non-trivial solutions for model V_k
  - Exist only for particular values of ω_i and k_{ij}
  - Located at the real roots of the secular function, **S**=f(ω_i, k_{ij}, V_k)
- Cost Function:

$$M_k = \left\| S_k \right\| = \sum_{i=1}^{n} \sum_{j=1}^{m} \mathrm{Re}\left\{ S\left(\omega_i, k_{ij}, \tilde{V}_k \right) \right\}$$



Assumptions

- The geological system includes N homogeneous layers, with thickness L, over an infinite half-space
- **D** is most sensitive to the shear wave velocity (V_s) of each layer

$$V_{s,i} = \sqrt{\frac{\mu_i}{\rho_i}}$$

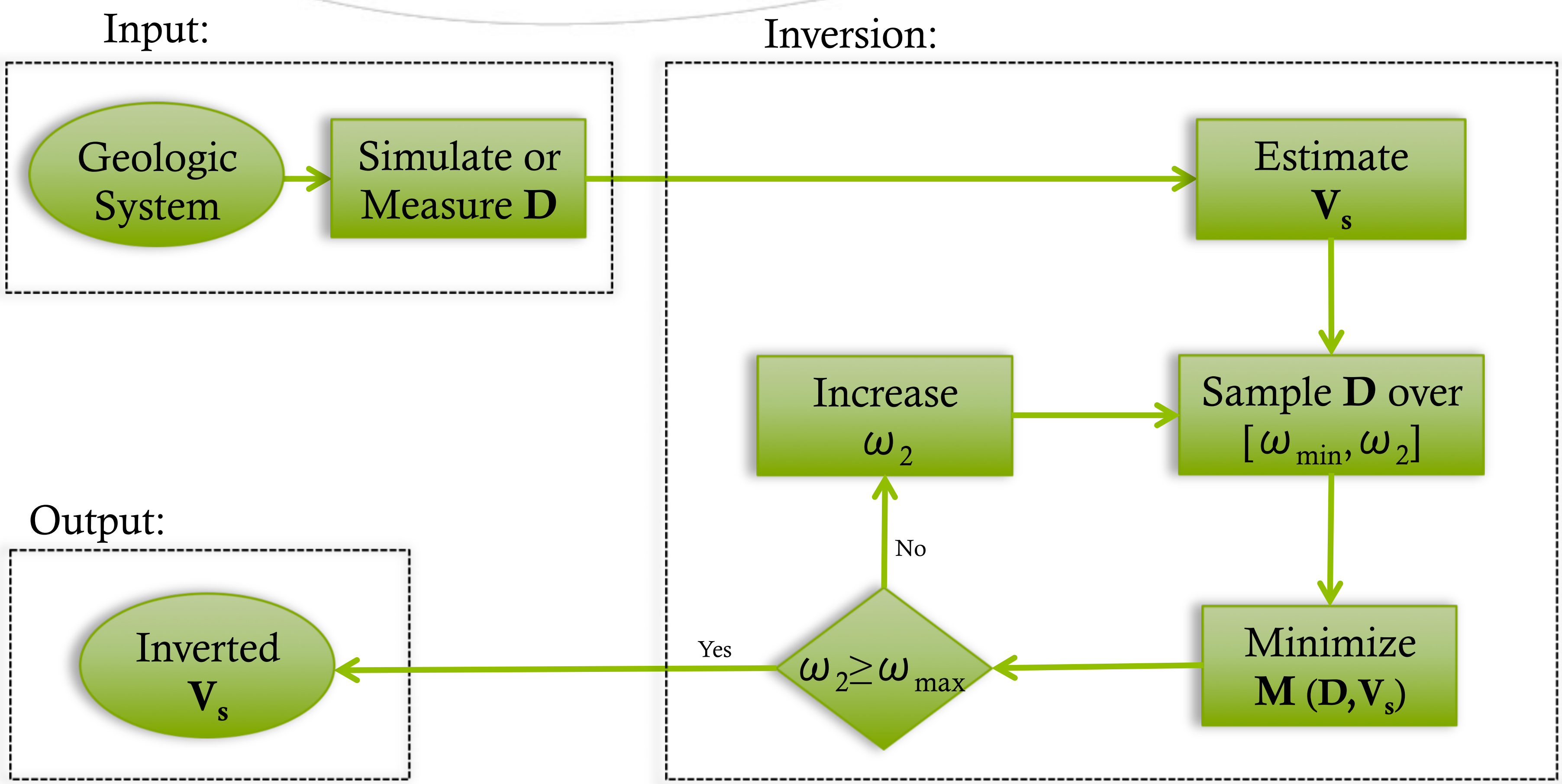
- Rayleigh waves sample layers up to 2-3 λ deep

$$\dot{u}(t,z) \sim \sin(\omega t)\exp(-z/\lambda)$$

- The range of acceptable V_s is limited, and increases monotonically with depth

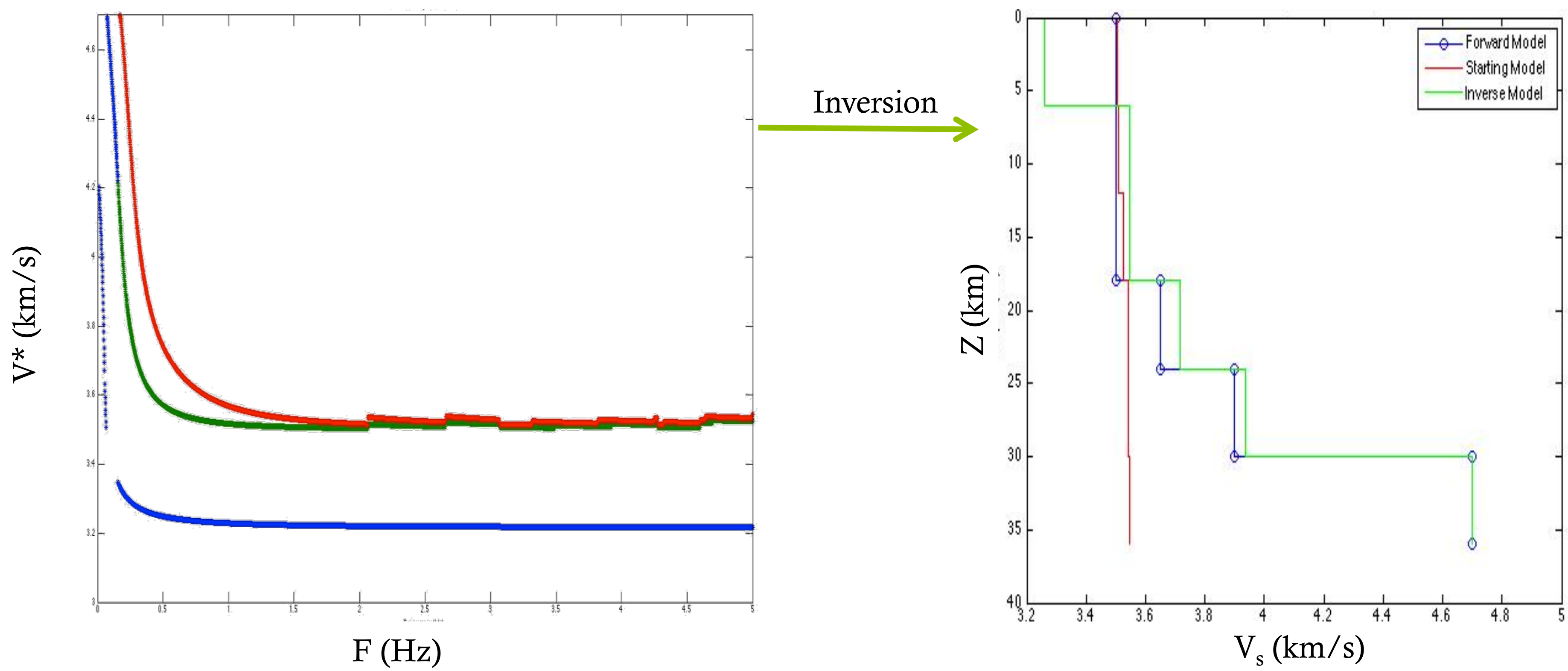
$$v_i \in [v_{min}, v_{max}]$$
$$v_i \leq v_{i+1}$$

Inversion Method



Results

Measured Dispersion Relationship:



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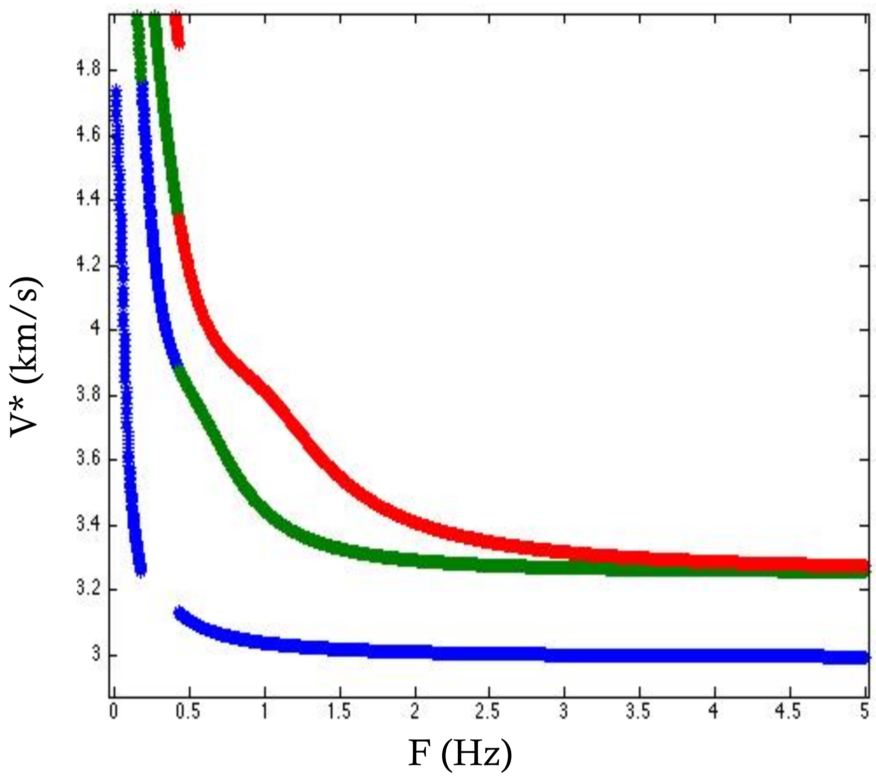
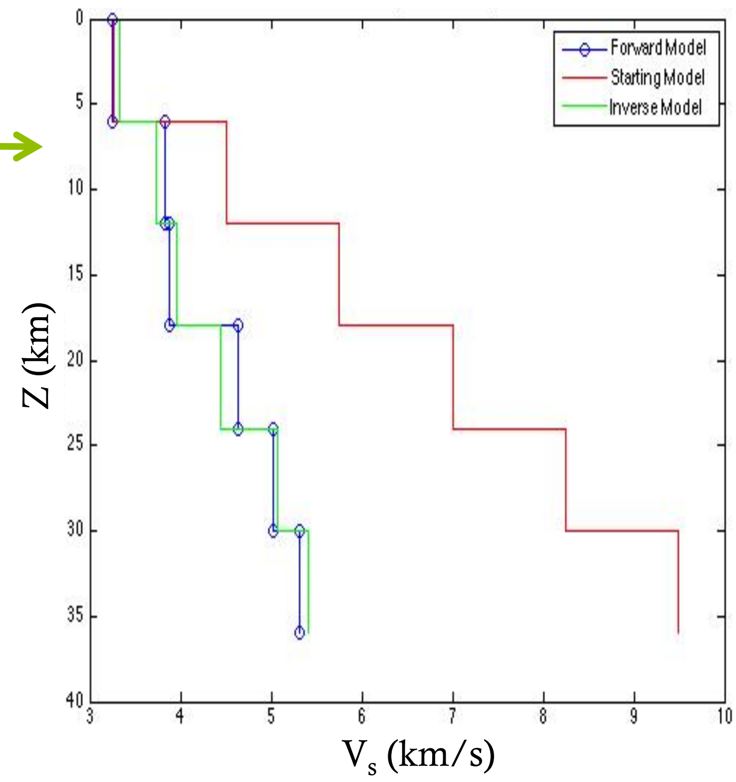


Inversion



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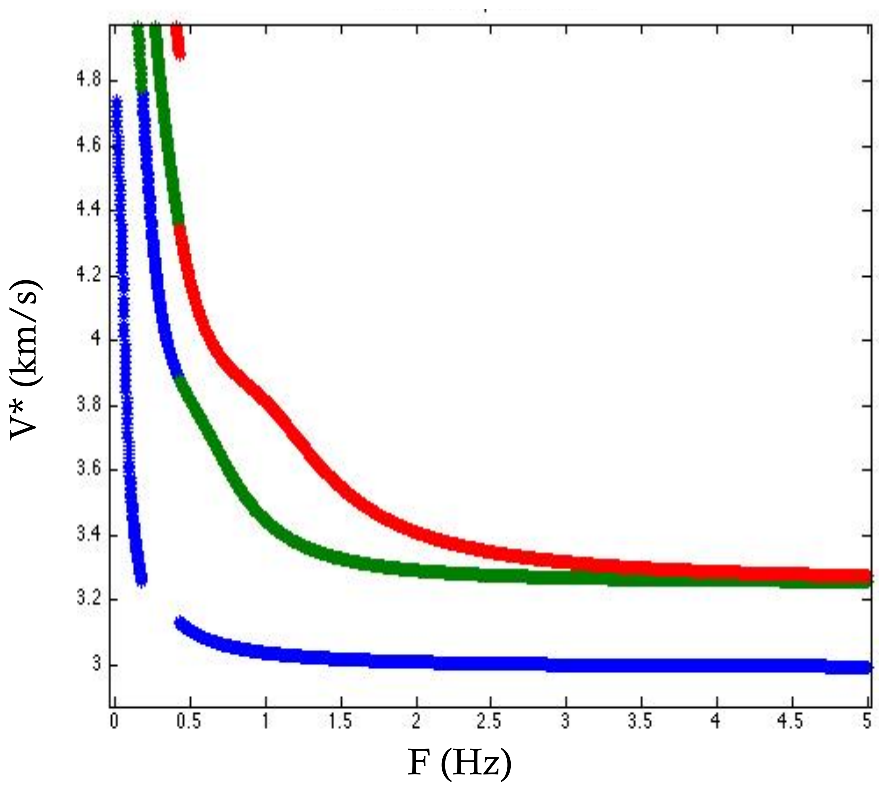

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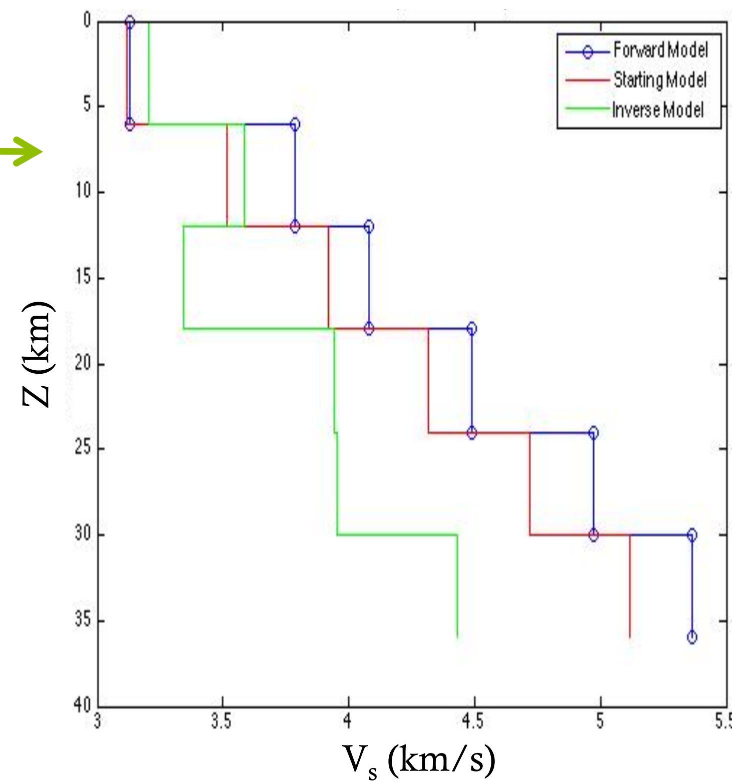
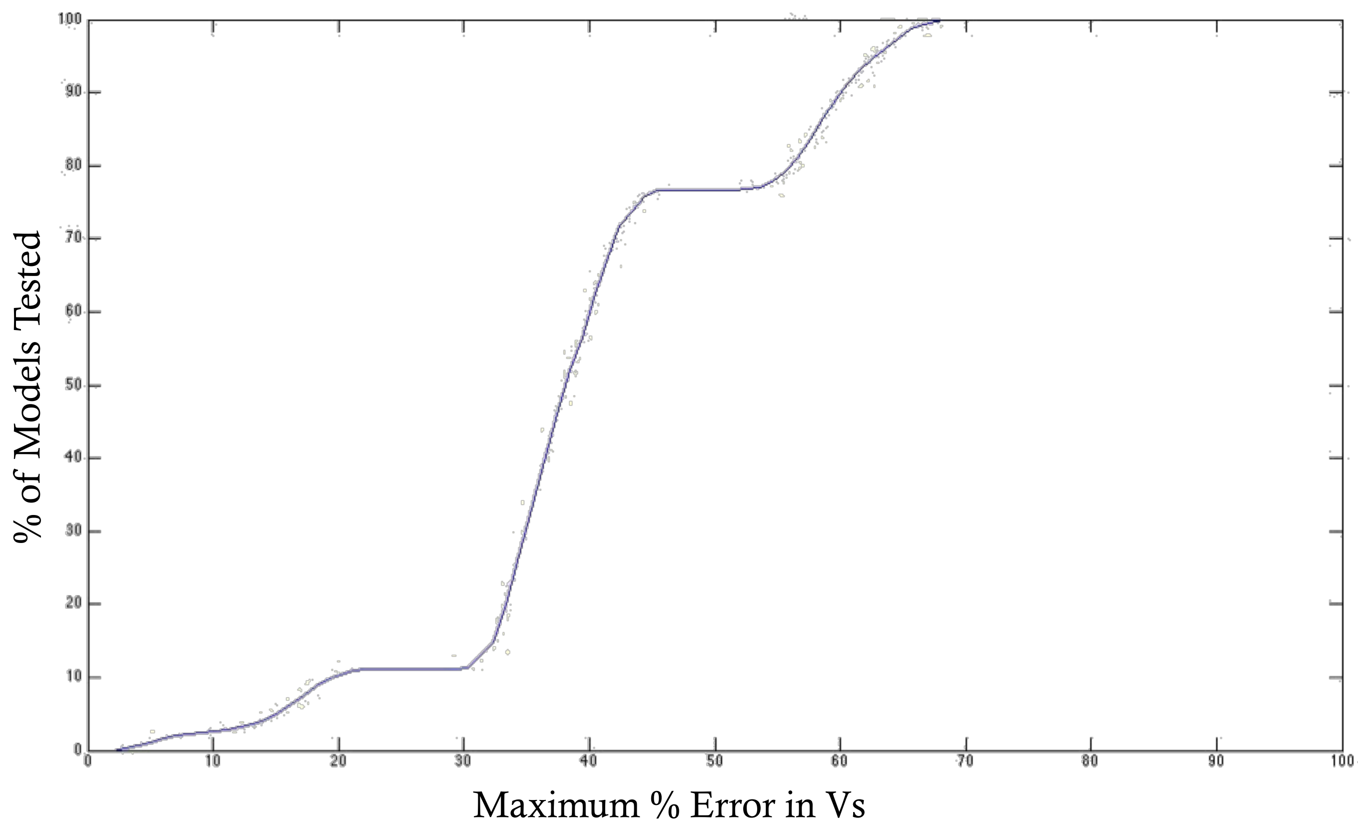


Evaluation of Inversion Strategy



Future Work

- Explore the effect of the starting point upon the final (inverted) model
- Estimate how reliable inverted results are, based upon the final value of the cost function **(M)**
- Develop and evaluate other cost functions
- Implement a global optimization method for the inversion
  - Simulated Annealing or Fast Simulated Annealing?