A general phase transition model for vehicular traffic

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Abstract

We propose an extension of the Colombo 2×2 phase transition model. We define the notion of equilibrium fundamental diagram and establish conditions to derive a perturbed fundamental diagram to model more accurately transport phenomena observed on highways. The solution of the 2×2 system of partial differential equations is built through the definition of a Riemann solver, and a modified Godunov scheme is used to construct the numerical solution.

Keywords

Distributed parameters systems, Riemann solver, Godunov scheme.

1 Introduction

Scalar conservation laws are a traditional way of modeling traffic flow on highways [1]. The state of the system at a location (x, t) can be defined by its density $\rho(x, t)$ which satisfies the mass conservation equation or *LWR* equation, and by an empirical relation $v = V(\rho)$ called the *fundamental diagram* in transportation engineering. Experimental data shows that two essentially different dynamics rule in *free-flow* and in *congestion*. In congestion the variable density is not sufficient to describe the state of the system since for a given density the velocity of vehicles is not single-valued. In [2], Colombo introduces distinct hyperbolic systems to take this property into account. He uses the so-called *Newell-Daganzo* flux with the mass conservation equation in free-flow, and in congestion introduces a 2×2 system of conservation law with a given flux function. Here, we propose to extend his 2×2 phase transition model to improve his initial framework.

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2 General phase transition model

We use a system similar to the one introduced in [2]:

$$\begin{cases} \partial_t \rho + \partial_x (\rho\, v) = 0 & \text{in free-flow} \\ \begin{cases} \partial_t \rho + \partial_x (\rho\, v) = 0 \\ \partial_t q + \partial_x (q\, v) = 0 \end{cases} & \text{in congestion} \end{cases} \tag{1}$$

where the second equation in the congested regime describes the flow of the variable q, considered as a perturbation around the *equilibrium* described by the first equation in congestion and $q = 0$. The 2×2 system in congestion models the fact that the state of the congested regime is not always at equilibrium. For scalar conservation laws the velocity v is a function of the density ρ only, but here, we have:

$$v = \begin{cases} v_f(\rho) & \text{in free-flow} \\ v_c(\rho, q) & \text{in congestion.} \end{cases} \tag{2}$$

Since q is a perturbation, we propose to write the fundamental diagram in congestion as $v = v_{\rm c}(\rho, q) = v_{\rm c}^{\rm eq}(\rho)\,(1+q)$ where $v_{\rm c}^{\rm eq}(\cdot)$ is the fundamental diagram in congestion at equilibrium. In this article we define the conditions under which the congested system of (1) with the fundamental diagram $v_{\rm c}(\cdot, \cdot)$ is hyperbolic, and under which conditions it yields a physically acceptable model. The benefits of the framework proposed here is the incorporation of a set-valued fundamental diagram into the model, in order to capture effects observed experimentally.

The fundamental diagram for free-flow is chosen to be $v_f(\rho) = V$ which is a standard assumption among the traffic community. This choice is shown to yield a simple definition of the Riemann solver and thus of the Godunov scheme for the phase transition system.

This model is instantiated with several specific equilibrium fundamental diagrams. In each case, a Riemann solver is derived. The numerical solution is built by a modified Godunov scheme [3], justified by the fact that the state-space is not convex. Extensions to this work include the evaluation of the accuracy of the model introduced, compared to usual scalar models, and its implementation in practice. This will be part of *Mobile Millenium.* This traffic information system collects traffic data from GPS-equipped mobile phones and feeds it to an inverse modeling algorithm based on the ensemble Kalman filtering applied to a scalar conservation law. An estimate of traffic conditions is processed and broadcasted back to the users' mobile phones.

References

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