Structural Analysis of Specific Environmental Traffic Assignment Problems

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***Abstract*—The goal of this article is to develop a framework for Environmental Traffic Assignment (E-TAP); that is a methodology for allocating traffic flows on a road network with the objective of minimizing objective functions related to energy such as fuel consumption or traffic pollutants. We investigate the underlying minimization problem in E-TAP which we characterize and study for uniqueness. This study is accomplished by exploiting convexity properties of the developed environmental objective functions and obtaining parameter sets for which the objective functions are strictly convex. The considered minimization problem is framed along the well-known Wardrop principles to develop two cases: 1) a User Equilibrium (UE) case that assumes selfish-routing of individuals and 2) a Social Optimal (SO) case that obtains the minimized solution for the entire system. In case of the UE we show uniqueness of solutions. In case of SO numerical studies indicate convexity and thus uniqueness, however a rigorous proof is not developed. We present a case study application of E-TAP for the greater Los Angeles area and compare these results with time-based traffic assignment.**

I. Introduction

The transportation sector represents nearly a third of all U.S energy consumption. It is the second leading source of green-house gas emissions in the U.S, second to just the electricity industry [1]. Worldwide, petroleum and other liquid fuels are the dominant source of transportation energy accounting for 96% of the global share [2]. The use of liquid fuels for transportation has effects beyond climate change and fuel costs, traffic emissions are the leading cause of air pollution estimated to cause 3.7 million premature deaths worldwide [3]. We therefore seek to develop *Traffic Assignment Problem* (TAP) models that can minimize objective functions related to the environment such as fuel consumption and traffic emissions.

TAP models the route choices of travellers on a road transport network using a given relationship between traffic flow and travel cost on a road. These estimates are of use to planners seeking to understand the potential effects of increased demands and/or modifications to a transport system. TAP also enables planners to assess deficiencies in existing transportation systems; evaluate possible effects of improvements and extensions to a system; test alternative transportation system designs, and to develop construction priorities.

In traditional TAP procedures it is usually assumed that travellers desire to minimize their *travel times*. However empirical studies have found additional factors such as travel distance, frequency of traffic signals, presence of tolls, safety, presence of pleasant scenery, and fuel consumption can all impact travellers route choices. By developing TAP models with respect to energy use we aim to enable better consideration of the environment in transportation planning and operation decisions. We term these E-TAP in contrast to the traditional travel-time based TAP which we term T-TAP.

The organization of this article is as follows. We begin in Section II by briefly recapping previous work that addressed environmental objectives in the TAP literature. From the literature we describe a methodology to develop functions that relate traffic flows and environmental objectives for a given link. In Section III we provide the necessary background and notation to formulate a general TAP and discuss its mathematical properties. These are used to ground and confirm the necessary conditions for uniqueness. We describe two cases of E-TAP, User Equilibrium (UE) and Social Optimal (SO) and prove uniqueness in the case of UE. In Section IV we describe an application of E-TAP to a traffic model of Los Angeles where we also discuss and contrast these results with T-TAP. The contributions of this article are as follows:

- By studying the convexity properties of parameter-dependant E-TAP objective functions we show that the solution to the E-TAP minimization problem is unique in case of UE. We accomplish this by obtaining parameter sets for which the E-TAP UE (E-UE) objective function is strictly convex.
- We demonstrate that the parameter conditions for which E-UE is proven to be convex are not overly constricting by applying the framework to a case study using real-world data of the Los Angeles road network.

II. Literature Review

Within the TAP literature, there have been previous efforts to include fuel consumption and/or emissions in the travel cost objective function. Some of the various objective functions and approaches previously proposed are collected in Table I. Tzeng and Chen were among the first to consider the E-TAP and in [4] they proposed an social/system optimal

TABLE I
EMISSIONS/ENERGY FUNCTIONS IN E-TAP LITERATURE.

$p_0 + p_1 \mathcal{Q}_a$	see [4]
$u_a^i = w_t^i \cdot t_a + w_c^i \cdot c_a + w_e^i \cdot e_a$	see [5]
$\frac{A \cdot e^{B \cdot v_{\text{avg}_a}}}{C \cdot v_{\text{avg}_a}}$	see [6]
$e^{\left(A \cdot Z + B \cdot Z^2 - C \cdot Z^3 + D \cdot Z^4 + E\right)} L_a$	see [7]
$A \cdot t_{0a}(v_{\text{avg}_a}) \cdot e^{B \cdot \left(\frac{L_a}{t_a(v_{\text{avg}_a})}\right)}$	see [8]
$\frac{A}{v_{avg_a}} + B - C \cdot v_{\text{avg}_a} + D \cdot v_{\text{avg}_a}$	see [9]
$L_a \left(A + \frac{C \cdot V_{f_a}^2}{\left(\alpha\left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta + 1\right)^2} + \frac{B}{V_{f_a}} \cdot \left(\alpha\left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta + 1\right) \right)$	see [10]

TABLE II
EQUATIONS, CONSTANTS, NOTATIONS

A, B, C, D, E	Constants for best-fit of empirical measurements of different emissions
p_0, p_1	pollutions levels, constant and flow-dependant respectively
$\mathcal{Q}_a$	volume of traffic on link a per unit of time (flow on link a)
t_{0_a}	free flow travel time on link a per unit time
c_a	flow capacity of link a (veh per unit of time)
α	model parameter 0.15 from *Highway Capacity Manual* (HCM)
β	model parameter 4.0 from *Highway Capacity Manual* (HCM)
w_t^i, w_c^i, w_e^i	are respective weights w for time t, cost c, and pollution e for each user class i
t_a, c_a, e_a	are time t, cost c, and pollution e for each link a
V_{f_a}	the free-flow speed on link a
v_{avg_a}	average speed on link a
$Z = V_c - 16$	V_c is the congestion speed on link a. Given by $V_c = \frac{V_{f_a}}{1+\alpha(v_{\text{avg}_a}/c_a)^\beta}$

(SO) TAP where for each link a flow-dependent pollution amount is added to a fixed pollution level. Nagurney et al. [5] proposed a generalized link cost function for each link and traveller class in a transportation network. This generalized cost function composed of a weighted sum of three objectives: travel time, cost, and pollution; each of which was assumed to be flow dependent and continuous. Rilett and Benedek [6], Sugawara and Niemeier [7], and Yin and Lawphongpanich [8] all proposed the use of different exponential functions to model traffic emissions. In [8] Yin and Lawphongpanich showed that *marginal social cost* (MSC) and other first-best congestion pricing schemes do not necessarily guarantee a reduction in traffic emissions. They show that there always exists a pricing scheme which induces a traffic flow distribution with minimum emissions and also provided a bound on the percent reduction of emissions achievable by any such charging scheme. A. Raith and C. Thielen [11] apply a fuel consumption model developed by Song et. al [9] to the E-TAP. They note that since fuel consumption (and emissions) are not strictly increasing functions of speed or traffic flow on a link, E-TAP poses methodical challenges. They propose a limit on the maximum allowable free-flow link speed $V_{f_a}^{\max}$ so as to ensure that the fuel consumption becomes a strictly convex function of the speed and traffic flow on the link.

In this article we seek to develop convexity conditions beyond a limit on the allowable free-flow speed so as to enable E-TAP to be applied to any road network. Therefore we now describe a method to develop an E-TAP objective function for which we will then provide convexity conditions. Because our framework is based on earlier models we first describe their models as building blocks of our approach.

In [10] Patil used the *Comprehensive Modal Emission Model* (CMEM) [12] to model fuel consumption and emissions based on the Highway Fuel Economy Test (HWFET) driving cycle for a single passenger car vehicle type[1]. Equations of best fit were then found for the CMEM's estimates in the form:

$$A + \frac{B}{v_{\text{avg}_a}} + C \cdot v_{\text{avg}_a}^2 \tag{1}$$

For v_{avg_a} in miles per hour the best-fit constants for fuel consumption were found as:

$$A = 39.705188, \ B = 702.856, \ C = 0.0096227 \tag{2}$$

Eq. (1) is later combined with the Bureau of Public Roads (BPR) function in Eq. (3) a commonly used link performance function in traditional T-TAP. Eq. (3) gives the average travel-time S_a for a vehicle on a link a as a function of the flow $\mathcal{Q}_a$. This combination is necessary to connect fuel consumption and traffic-flow via average speed.

$$S_a(\mathcal{Q}_a) = t_{0_a} \cdot \left(1 + \alpha \cdot \left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta\right) \tag{3}$$

From the length of a link L_a, the average speed on a link v_{avg_a} can be computed from the average travel time S_a as in Eq. (4)

$$v_{\text{avg}_a} = \frac{L_a}{S_a(\mathcal{Q}_a)} = \frac{L_a}{t_{0_a} \cdot \left(1+\alpha \cdot \left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta\right)}. \tag{4}$$

Noting that $\frac{L_a}{t_{0_a}}$ is the **free-flow velocity** V_{f_a} on link a, equation Eq. (4) may be written as:

$$v_{\text{avg}_a} = \frac{V_{f_a}}{1+\alpha \cdot \left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta} \tag{5}$$

Equation (5) gives the **average speed as a function of the flow on the link**. Substituting (5) for v_{avg_a} in Eq. (1) results in a function which calculates the consumption in **grams-per-mile-per-vehicle** as a function of the flow on the link. To obtain the link fuel performance function required for E-TAP, it is necessary to multiply by the length of the link L_a to obtain the average fuel consumption in **grams-per-vehicle** for a link as a function of the flow on that link. This is given in Eq. (6).

$$\mathcal{F}_a(\mathcal{Q}_a) = L_a \left(A + \frac{C \cdot V_{f_a}^2}{\left(\alpha\left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta + 1\right)^2} + \frac{B}{V_{f_a}} \left(\alpha\left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta + 1\right) \right) \tag{6}$$

Eq. (6) may now be applied to develop E-TAP.

Remark II.1 (Use of BPR functions)**.** As an archetype of a link performance function we have used BPR functions to

[1]Category 9 tier 1 emissions, high power-to-weight ratio with over 50,000 miles-driven

obtain the fuel functions $\mathcal{F}_a$. Clearly, a similar computation could also be carried out for different link performance functions. However, the provided analysis for the convexity of the corresponding optimization problem in Theorem III.2 would not necessarily hold.

We now briefly describe TAP its mathematical groundings and conditions and apply these to Eq. (6) in Section III.

III. Traffic Assignment Problem Formulation

The TAP is well known in the literature [13]. In this section we briefly describe TAP so as to state the necessary mathematical conditions for the E-TAP we wish to develop. There are two notions of TAP: *User Equilibrium* (UE) and *Social Optimum* (SO) as a result of different traffic assignment principles. These are:

1) the ***principle of equal journey costs***
2) the ***principle of minimal total costs***.

These behavioral principles are usually referred to as the **Wardrop Conditions** [14] attributed to J.G. Wardrop of the Road Research Laboratory. We use these Wardrop Conditions to define our optimization problem in Definition III.2

Principle III.1. Wardrop's first principle: The journey times on all the routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route.

Principle III.2. Wardrop's second principle: The average journey time is a minimum

For further discussion on the development of TAP history and theory refer to [13]. Although many types of TAP models exist e.g. elastic demand, stochastic TAP, dynamic TAP, we restrict our analysis to the fixed-demand static TAP.

A. Optimization formulation of UE

We begin by providing fundamental definitions for the network, link flow, and origins and destinations. Afterwards we formulate the optimization problem in Definition III.2. This has been proven to be equivalent to the social optimum for Wardrop's equilibrium under specific assumptions which we discuss in Remark III.1. We finally extend the minimization problem to E-TAP in Definition III.3.

Notation III.1 ($\mathcal{O}, \mathcal{D}, \mathcal{C}, \mathcal{R}, \boldsymbol{Q}, \delta, \mathcal{A}, \ldots$)**.** Let $\mathcal{O}$ be the set of origins and $\mathcal{D}$ the set of destinations be given, we define $\mathcal{C} \subseteq \mathcal{O} \times \mathcal{D}$ as the set of all origin-destination (O-D) pairs. Let $\mathcal{R}_{od}$ denote all "simple" routes from $o \in \mathcal{O}$ to $d \in \mathcal{D}$. c_{odr} denotes the travel cost on a route $r \in \mathcal{R}_{od}$ from the origin node $o \in \mathcal{O}$ to the destination node $d \in \mathcal{D}$. By setting $\mathbf{c} = (c_{odr})$ we have $\mathbf{c} \in \mathbb{R}^k_{\geq 0}$ with $k := \sum_{(o,d)\in\mathcal{C}} |\mathcal{R}_{od}|$. In addition, we call $\mathbf{h} = (h_{odr}) \in \mathbb{R}^k_{\geq 0}$ the vector of flows on the corresponding routes and $\mathfrak{D} = (\mathfrak{D}_{od}) \in \mathbb{R}^{|C|}_{\geq 0}$ the demand from origin $o \in \mathcal{O}$ to destination $d \in \mathcal{D}$.

A network $\mathcal{G}$ consists of nodes $\mathcal{N}$ and links $\mathcal{A}$. We call $\mathcal{Q}_{aod}$ the link flow on link $a \in \mathcal{A}$ on route $(o,d) \in \mathcal{C}$ and write $\mathbb{R}^{|\mathcal{A}|}_{\geq 0} \ni \boldsymbol{Q}_{od} = (\mathcal{Q}_{oda})_{a\in\mathcal{A}}$ as well as $\mathbb{R}^{|\mathcal{A}|\cdot|\mathcal{C}|}_{\geq 0} \ni \boldsymbol{Q} = (\boldsymbol{Q}_{od})_{(o,d)\in\mathcal{C}}$. Furthermore, we define the entire flow on link $a \in \mathcal{A}$ as $\mathcal{Q}_a := \sum_{(o,d)\in\mathcal{C}} \mathcal{Q}_{aod}$. We call $\boldsymbol{X} = (\mathcal{X}_a)_{a\in\mathcal{A}} \in C\left([0, |\mathfrak{D}|_1]; \mathbb{R}^{|\mathcal{A}|}_{\geq 0}\right)$ the link travel cost.

Definition III.1 (Link flow)**.** Given Notation III.1 and the involved functions the **link flow** is defined as:

$$\mathcal{Q}_{aod} := \sum_{r\in\mathcal{R}_{od}} \delta_{odra} h_{odr}, \quad (o,d) \in \mathcal{C}, a \in \mathcal{A}, \tag{7}$$

where for $(o,d) \in \mathcal{C}$, $a \in \mathcal{A}$ and $r \in \mathcal{R}_{od}$ we set

$$\delta_{odra} := \begin{cases} 1 & \text{if route } r \in \mathcal{R}_{od} \text{ uses link } a \\ 0 & \text{else.} \end{cases} \tag{8}$$

For $(o,d) \in \mathcal{C}$ and $r \in \mathcal{R}_{od}$ we obtain:

$$c_{odr} = \sum_{a\in\mathcal{A}} \delta_{odra} \mathcal{X}_a(\mathcal{Q}_a). \tag{9}$$

The following assumptions regarding the traffic network are made:

Assumption III.1 (Properties of the traffic network)**.**

1) The network is strongly connected, i.e. every O-D pair is at least connected via one route
$$\forall (o,d) \in \mathcal{C}: \ |\mathcal{R}_{od}| \geq 1.$$
2) The demand is non-negative, i.e. $\mathfrak{D} \in \mathbb{R}^{|\mathcal{C}|}_{\geq 0}$.
3) The travel cost function is non-negative and continuous, i.e. $\forall a \in \mathcal{A}$ we have $\mathcal{X}_a \in C(\mathbb{R}_{\geq 0}; \mathbb{R}_{\geq 0})$.

Then, the optimization problem considered in this work, reads as follows:

Definition III.2 (Optimization problem)**.** Given demand $\mathfrak{D} \in \mathcal{R}^{\mathcal{C}}_{\geq 0}$ and a network as in Notation III.1, the considered minimization problem reads as

$$\begin{aligned} &\min_{\boldsymbol{Q},\mathbf{h}} \textstyle\sum_{a\in\mathcal{A}} \int_0^{\mathcal{Q}_a} \mathcal{X}_a(s)\mathrm{d}s \\ &\sum_{r\in\mathcal{R}_{od}} h_{odr} = \mathfrak{D}_{od} \qquad \forall (o,d) \in \mathcal{C}, \ \forall r \in \mathcal{R}_{od} \\ &\mathbf{h} \geq \mathbf{0} \\ &\sum_{(o,d)\in\mathcal{C}} \sum_{r\in\mathcal{R}_{od}} \delta_{odra} h_{odr} = \mathcal{Q}_a \qquad \forall a \in \mathcal{A}. \end{aligned}$$

It has been shown in the literature under the given Assumption III.1 that the user equilibrium and social optimum w.r.t. travel time can be formulated as a minimization problem as defined in Definition III.2. For questions of uniqueness we refer to Section III-B.

Remark III.1 (Separability condition)**.** Usually, an assumption on the symmetry of the objective function is made to guarantee that there exists a potential to that specific function. In our case, due to writing the objective as

$$\sum_{a\in\mathcal{A}} \int_0^{\mathcal{Q}_a} \mathcal{X}_a(s)\mathrm{d}s$$

the cost on a specific link $a \in \mathcal{A}$ **only** depends on the assigned flow $\mathcal{Q}_a$ of this link so that the symmetry condition and in particular the separability condition is satisfied.

The theory here laid out and introduced beforehand for specific travel time minimization is now applied in the context of energy to derive E-TAP. To highlight the differences, we define the specific objective functions for E-TAP and T-TAP in the following which clearly satisfy Item 3 in Assumption III.1.

Definition III.3 (The four objective functions)**.** For $\mathcal{Q}_a \in \mathbb{R}_{\geq 0}$ and $a \in \mathcal{A}$ the different objectives are:

1) **Energy-TA User Equilibrium (E-UE):**
$$\mathcal{X}_a^{\text{E-UE}}(\mathcal{Q}_a) := L_a \left(A + \frac{C \cdot {V_{f_a}}^2}{\left(\alpha \left(\frac{\mathcal{Q}_a}{c_a}\right)^4 + 1\right)^2} + \frac{B \cdot \left(\alpha \left(\frac{\mathcal{Q}_a}{c_a}\right)^4 + 1\right)}{V_{f_a}} \right)$$
where the constants are as in Table I and $\mathcal{X}_a^{\text{E-UE}} \equiv \mathcal{F}_a$ is the fuel function derived in Section II and in [10].
2) **Energy-TA Social Optimum:**
$$\mathcal{X}_a^{\text{E-SO}}(\mathcal{Q}_a) := \mathcal{X}_a^{\text{E-UE}}(\mathcal{Q}_a) + \frac{\mathrm{d}\mathcal{X}_a^{\text{E-UE}}(\mathcal{Q}_a)}{\mathrm{d}\mathcal{Q}_a} \cdot \mathcal{Q}_a$$
3) **Time-TA User Equilibrium (T-UE):**
$$\mathcal{X}_a^{\text{T-UE}}(\mathcal{Q}_a) := t_{0a} \left(\alpha \left(\frac{\mathcal{Q}_a}{c_a}\right)^\beta + 1 \right)$$
where the constants are as in Table II. This function is the widely used BPR function described in [15].
4) **Time-TA Social Optimum (T-SO):**
$$\mathcal{X}_a^{\text{T-SO}}(\mathcal{Q}_a) := \mathcal{X}_a^{\text{T-UE}}(\mathcal{Q}_a) + \frac{\mathrm{d}\mathcal{X}_a^{\text{T-UE}}(\mathcal{Q}_a)}{\mathrm{d}\mathcal{Q}_a} \cdot \mathcal{Q}_a$$

Remark III.2 (Social Optimum vs. User Equilibrium)**.** Note that due to the specific construction of the objective functions for the SO it holds for $a \in \mathcal{A}$ and $\mathcal{Q}_a \in \mathcal{R}_{\geq 0}$ $\sum_{a \in \mathcal{A}} \int_0^{\mathcal{Q}_a} \mathcal{X}_a^{\text{E-SO}}(s)\mathrm{d}s = \sum_{a \in \mathcal{A}} \mathcal{Q}_a \cdot \mathcal{X}_a^{\text{E-UE}}(\mathcal{Q}_a)$. This denotes the summarized travel energy consumption on the entire traffic network. The same computation may be made for travel time. This allows us to replace the objective function for UE by a slightly different one for SO and still having the same structure of an integration in the minimization problem.

Remark III.3 (Convex combination)**.** Obviously, the considered objective function in Definition III.2 can also be a convex combination of the functions introduced in Definition III.3 and we can basically compute a compromise between fuel consumption and travel time (etc.).

B. Uniqueness of equilibrium solutions

In the literature, Theorem III.1 concerning the uniqueness of the solution of the minimization problem is well described in terms of T-TAP with a travel-time based objective function [13]. We state the theorem for a general objective function $X_a,\ a \in \mathcal{A}$ and in Theorem III.2 we then state the conditions for which uniqueness holds for E-TAP.

Theorem III.1 (Uniqueness of the solution)**.** Let the minimization problem as in Definition III.2 be given and let Assumption III.1 hold. Then:

1) The minimal value of the optimization problem is unique.
2) If the objective $\mathcal{X}_a$ is strictly increasing for all $a \in \mathcal{A}$, the link-flows $\boldsymbol{Q}$ are unique.

Theorem III.1 is now applied on the newly introduced objective function for fuel consumption as defined in Definition III.3. Both functions, $\mathcal{X}^{\text{E-SO}}$ and $\mathcal{X}^{\text{E-UE}}$ clearly satisfy the non-negativity and continuity assumptions in Assumption III.1. We thus have to address the question if the functions also satisfy the assumptions in Item 2 of Theorem III.1. This comes down to proving that $\int_0^{\mathcal{Q}_a} \mathcal{X}_a^{\text{E-UE}}(s)\mathrm{d}s,\ a \in \mathcal{A}$, is strictly convex which is due to smoothness of the involved function equivalent to showing that $\mathcal{X}_a^{\text{E-UE}\prime}(s) > 0\ \forall a \in \mathcal{A},\ \forall s \in [0, |\mathfrak{D}|_1]$. For E-SO recalling Remark III.2, one needs to show that $\mathcal{X}_a^{\text{E-UE}\prime\prime}(s) > 0\ \forall a \in \mathcal{A},\ \forall s \in [0, |\mathfrak{D}|_1]$.

Both given inequalities are not necessarily true for every combination of involved parameters, however, for the specific parameter sets which are used in this work the convexity properties hold.

This is detailed in the following Theorem III.2:

Theorem III.2 (Uniqueness of E-UE)**.** Let the objective function as in Definition III.3 with the parameters $A, B, C \in \mathcal{R}_{>0}$ be given. Then, we obtain $\forall a \in \mathcal{A}$

- If $\sqrt[3]{\frac{B}{2C}} > V_{f_a} \implies \mathcal{X}_a^{\text{E-UE}\prime}(s) > 0\ \forall s \in [0, |\mathfrak{D}|_1]$

Proof. In brief, it entails manipulating the first and second derivatives of $\mathcal{X}^{\text{E-UE}}$ to obtain quartic equations which are then checked for non-negativity. □

As pointed out in Remark III.2 and also in Section III-B, the first condition guarantees the uniqueness of a global minimum for the E-UE, while a study of the uniqueness for E-SO is still open. However, the minimization problem in Definition III.2 for the E-SO will still result in an improved situation but not necessarily represent a global minimum, although numerical studies of the specific objective function for SO indicate the needed convexity and thus the uniqueness for SO as well.

As we demonstrate in Section IV, the condition is not restrictive for applying E-TAP for UE in practice. If a condition in Theorem III.2 is found to not hold for a given network V_{f_a} and fit-coefficients B and C, Theorem III.2 can be used to find adjusted fit-coefficients for which uniqueness holds. This is demonstrated in the case study in Section IV and illustrated in Fig. 1.

IV. Case Study: Los Angeles (LA) Basin

For the numerical implementation of the optimization problem we use the well-known Frank-Wolfe algorithm [16] which solves the optimization problem by a series of linear minimization problems. In each step a shortest path algorithm can be used so that it can take advantage from the specific network problem structure. The developed framework was tested for a case study of the Los Angeles (LA) Basin. This traffic model consisted of nearly $15,000$ nodes and over $28,000$ links, with a travel demand of almost $100,000$ Origin-Destination pairs comprising a total demand of approximately $435,000$ vehicles-per-hour. The data for the LA road network was sourced from *Open Street Maps* and the traffic demand was built from the Census Transportation

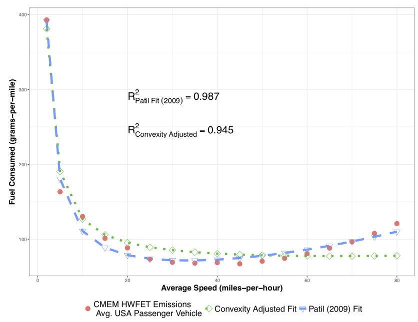

Fig. 1. CMEM Speed-Fuel Relationship.

TABLE III
LA CASE STUDY SUMMARY.

	Energy TA		Time TA	
	System Optimal	User Equilibrium	System Optimal	User Equilibrium
Total Travel Time [hours]	200,846	228,327	187,518	203,482
Fuel Consumption [metric Tons]	577	588	591	602
% > T-SO	7	22	···	9
% > E-SO	···	2	2	4

Planning Products database, and based on 2006-2010 American Community Survey Data ctpp.transportation.org/Pages/5-Year-Data.aspx.

The link free-flow speed on the network ranged from $5-65$miles-per-hour Using Theorem III.2 we find new constants for the fit equation that satisfy convexity for this network. These are listed in (10) and the convexity-adjusted fit equation is plotted in comparison with [10] in Fig. 1.

$$A = 63.20055, \;\; B = 635.8261, \;\; C = 0.001051911 \qquad (10)$$

The TAP was solved for the four objective functions described in Definition III.3. This resulted in four different traffic flow allocations for each of the previously defined objective functions: E-SO, E-UE, T-SO, and T-UE. The resulting four traffic flow assignments were each then used to compute both the resulting fuel consumption and travel time on the network.

The system results are summarized in Table III. By definition, E-SO must have the least fuel consumption and T-SO the least travel time. Although convexity and uniqueness for E-SO have not been explicitly proven, the developed E-SO objective function results in a solution which has the least total fuel consumption among the four traffic assignments.

As a cost metric, travel-time shows far more variability in the non-SO flow allocation costs than fuel. This variability may indicate that travel-time is in a sense a more "flexible" objective than fuel-consumption an insight which can guide planners in developing traffic-control schemes that satisfy both time and energy objectives. However, this could potentially be due to the found E-SO solution being a local minimum. It is interesting to note that the obtained E-SO

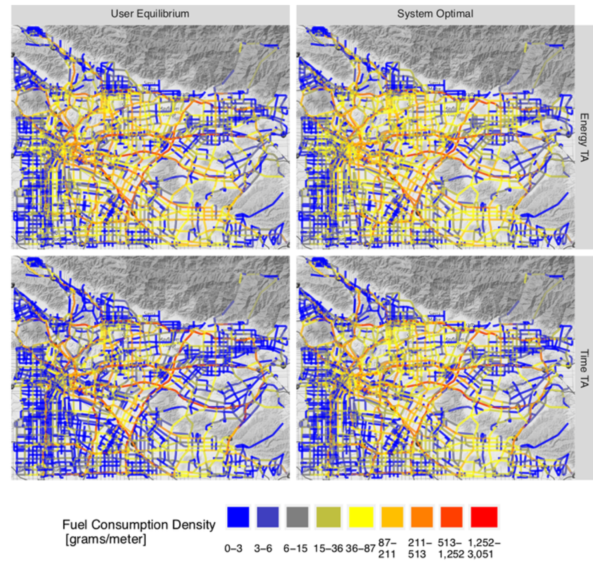

Fig. 2. Fuel Consumption Density (best viewed in color).

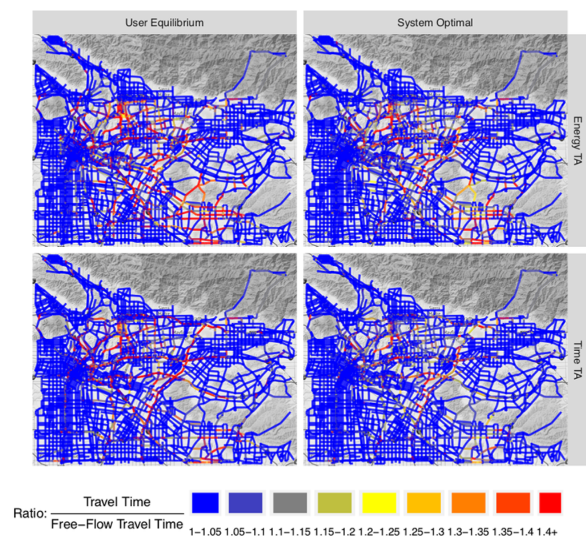

Fig. 3. Normalized Travel Times (best viewed in color).

results in a total travel time that is less than T-UE. This result indicates that given full control of traffic flows on a network, it is feasible to route traffic to minimize energy while keeping travel-times no longer than may be expected under selfish travel-time based routing.

Figure 2 shows the Fuel Consumption *density* in grams per unit length (meters) for each road. This is simply $\frac{\mathcal{F}_a(\mathcal{Q}_a)\times\mathcal{Q}_a}{L_a}$ and highlights the spatial variability in fuel consumption on the network. Figure 3 shows the resulting travel time on each link normalized by the free-flow travel time and Fig. 4 is a subset of Fig. 3 which highlights specifically the roads which have resultant travel times 50% greater than their free-flow times. It is clear that SO TA whether energy or time based is more effective in reducing congestion on a network.

These figures demonstrate the capabilities of the E-TAP

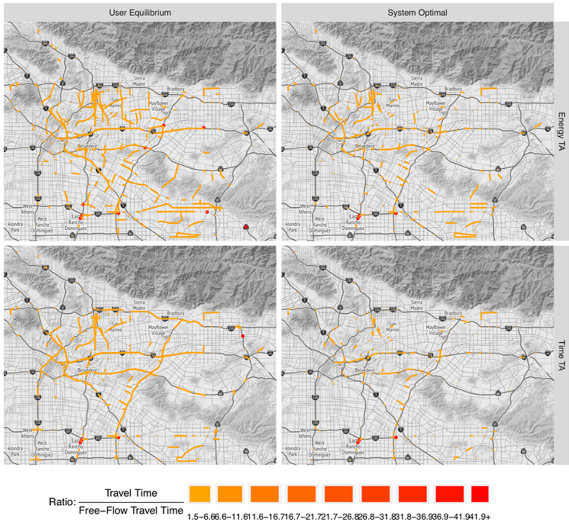

Fig. 4. Congested Roads Highlight (best viewed in color).

framework. Such visualizations allow planners to quickly observe where TAP time and energy objectives conflict. For example, Fig. 4 shows that the area around the merge point of interstates 210 and 710 in Pasedena in the north-west LA area has increased congestion in E-SO in comparison to the T-SO. Imperial Highway between La Mirada and La Habra in the south-east LA area shows similar behavior. The impacts of traffic emissions and resulting air pollution are location specific. For example, it is likely that planners would rather route more traffic and emissions through industrial areas than residential neighborhoods. Therefore, the ability to route traffic with respect to energy and observe and contrast the differences with travel time makes the E-TAP a powerful tool for planners to improve the operation of their transportation system.

V. Conclusions and Future Work

This article developed an E-TAP framework to obtain optimal traffic assignments with respect to fuel consumption under two conditions: Social Optimal and User Equilibrium. We explicitly considered questions of uniqueness and existence for E-TAP solutions and provided conditions for which uniqueness and existence for which the User Equilibrium is guaranteed. We also applied the developed E-TAP framework to the greater Los Angeles area and provided a comparison of the results with T-TAP. In future work we intend to provide conditions that verify the convexity and uniqueness of E-TAP for the Social Optimal case. We also aim to build upon the results of this paper to produce a TAP objective function that is a composite of both energy and time. We would also aspire to extend the E-TAP framework to consider time-dependant TAP.

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