Topics covered in this presentation

- Advantages of FR techniques over RL
- Define FR
- Define Bode & Nyquist plots
- Relation between poles & zeros to Bode plots (slope, etc.)
- Features of 1st- & 2nd-order system Bode plots
- Define Nyquist criterion
- Method of dealing with OL poles & zeros on imaginary axis
- Simple method of dealing with OL stable & unstable systems
- Determining gain & phase margins from Bode & Nyquist plots
- Define static error constants
- Determining static error constants from Bode & Nyquist plots
- Determining TF from experimental FR data
10 Frequency response techniques

10.1 Introduction
10.2 Asymptotic approximations: Bode plots
10.3 Introduction to Nyquist criterion
10.4 Sketching the Nyquist diagram
10.5 Stability via the Nyquist diagram
10.6 Gain margin and phase margin via the Nyquist diagram
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Advantages of frequency response (FR) methods, [1, p. 534]

In the following situations

- When modeling TFs from physical data
- When designing lead compensators to meet a steady-state error requirements
- When finding the stability of NL systems
- In settling ambiguities when sketching a root locus
The concept of FR, [1, p. 535]

- At steady-state, sinusoidal inputs to a linear system generate sinusoidal responses of the same frequency with different amplitudes and phase angle from the input, each of which are a function of frequency.

- **Phasor** – complex representation of a sinusoid
  - $||G(\omega)||$ – amplitude
  - $\angle G(\omega)$ – phase angle
  - $M \cos(\omega t + \phi)$ … $M \angle \phi$

**Figure**: Sinusoidal FR: a. system; b. TF; c. IO waveforms
The concept of FR, [1, p. 535]

- **Steady-state output sinusoid**

\[
M_o(\omega) \angle \phi_o(\omega) = M_i(\omega) M(\omega) \angle (\phi_i(\omega) + \phi(\omega))
\]

- **Magnitude FR**

\[
M(\omega) = \frac{M_o(\omega)}{M_i(\omega)}
\]

- **Phase FR**

\[
\phi(\omega) = \phi_o(\omega) - \phi_i(\omega)
\]

- **FR**

\[
M(\omega) \angle \phi(\omega)
\]

**Figure:** Sinusoidal FR: a. system; b. TF; c. IO waveforms
Analytical expressions for FR, [1, p. 536]

- **General input sinusoid**
  \[
  r(t) = A \cos(\omega t) + B \sin(\omega t) = \sqrt{A^2 + B^2} \cos(\omega t - \tan^{-1}\left(\frac{B}{A}\right))
  \]

- **Input phasor forms**
  - **Polar,** \( M_i \angle \phi_i \)
    \[
    M_i = \sqrt{A^2 + B^2} \\
    \phi_i = -\tan^{-1}\left(\frac{B}{A}\right)
    \]
  - **Rectangular,** \( A - jB \)
  - **Euler’s,** \( M_i e^{j\phi_i} \)

---

**Figure:** System with sinusoidal input
**Analytical expressions for FR, [1, p. 536]**

- **Forced response**
  \[ C(s) = \frac{A{s} + B\omega}{s^2 + \omega^2} G(s) \]

- **Steady-state forced response after partial fraction expansion**
  \[ C_{ss}(s) = \frac{\frac{1}{2}M_i M_G e^{-j(\phi_i - \phi_G)}}{s + j\omega} + \frac{\frac{1}{2}M_i M_G e^{j(\phi_i - \phi_G)}}{s - j\omega} \]
  Where \( M_G = ||G(j\omega)|| \) and \( \phi_G = \angle G(j\omega) \)

- **Time-domain response**
  \[ c(t) = M_i M_G \cos(\omega t + \phi_i + \phi_G) \]

- **Time-domain response in phasor form**
  \[ M_o \angle \phi_o = (M_i \angle \phi_i)(M_G \angle \phi_G) \]

- **FR of system**
  \[ G(j\omega) = G(s)|_{s \rightarrow j\omega} \]
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History interlude

History (Hendrik Wade Bode)

- 1905 – 1982
- American engineer
- 1930s – Inventor of *Bode plots, gain margin, & phase margin*
- 1944 – WWII anti-aircraft (including V-1 flying bombs) systems
- 1947 – Cold War anti-ballistic missiles
- 1957 – Served on NACA (now NASA) with Wernher von Braun (inventor of V-1 flying bombs & V-2 rockets)

*Figure: Hendrik Wade Bode*
General Bode plots, [1, p. 540]

\[ G(j\omega) = M_G(\omega) \angle \phi_G(\omega) \]

- Separate magnitude and phase plots as a function of frequency
  - Magnitude – decibels (dB) vs. \( \log(\omega) \), where \( dB = 20 \log(M) \)
  - Phase – phase angle vs. \( \log(\omega) \)
10 FR techniques

10.2 Asymptotic approximations: Bode plots

Bode plots approximations, [1, p. 542]

- **TF**
  \[ G(s) = s + a \]

- **Low frequencies**
  \[ G(j\omega) \approx a \angle 0^\circ \]

- **High frequencies**
  \[ G(j\omega) \approx \omega \angle 90^\circ \]

- **Asymptotes** – straight-line approximations
  - **Low-frequency**
  - **Break frequency**
  - **High-frequency**

**Figure:** Bode plots of \( s + a \): a. magnitude plot; b. phase plot
Simple Bode plots, [1, p. 542]

Figure: Bode plot of $\frac{s+a}{a}$

Figure: Bode plot of $\frac{a}{s+a}$
10 FR techniques

10.2 Asymptotic approximations: Bode plots

Simple Bode plots, [1, p. 545]

**Figure:** Bode plot of $s$

**Figure:** Bode plot of $\frac{1}{s}$
Simple Bode plots, [1, p. 549]

Figure: Bode plot of $\frac{s^2 + 2\zeta \omega_n s + \omega_n^2}{\omega_n^2}$

Figure: Bode plot of $\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$
Detailed $2^{nd}$-order Bode plots, [1, p. 550]

Figure: Bode plot of $\frac{s^2 + 2\zeta \omega_n s + \omega_n^2}{\omega_n^2}$

Figure: Bode plot of $\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$
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History interlude

History (Harry Theodor Nyquist)

- 1889 – 1976
- American engineer
- 1917 – 1934 AT&T
- 1934 – 1954 Bell Telephone Labs
- 1924 – Nyquist-Shannon sampling theorem
- 1926 – Johnson–Nyquist noise
- 1932 – Nyquist stability criterion

Figure: Harry Theodor Nyquist
Introduction, [1, p. 559]

- Relates the stability of a CL system to the OL FR and the OL poles and zeros
  - # CL poles in RHP
- Provides information on the transient response and steady-state error

Figure: CL control system
Derivation concepts, [1, p. 560]

\[ G(s) = \frac{N_G}{D_G} \quad \text{and} \quad H(s) = \frac{N_H}{D_H} \]

\[ G(s)H(s) = \frac{N_GH_G}{D_GD_H} \]

\[ F(s) = 1 + G(s)H(s) = \frac{D_GD_H + N_GN_H}{D_GD_H} \]

\[ T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_GN_H}{D_GD_H + N_GN_H} \]

- Poles of \( 1 + G(s)H(s) \) are the same as the poles of the OL system, \( G(s)H(s) \)
- Zeros of \( 1 + G(s)H(s) \) are the same as the poles of the CL system, \( T(s) \)
Derivation concepts, [1, p. 560]

- Map – function
- Contour – collection of points
- For our particular scenario, assume

\[ F(s) = \frac{(s - z_1)(s - z_2)\ldots}{(s - p_1)(s - p_2)\ldots} \]

and a clockwise direction for mapping the points on the contour A

**Figure:** Mapping contour A through function \( F(s) \) to contour B
If $F(s)$ has only zeros or only poles that are not encircled by the contour then contour $B$ maps in a *clockwise* direction.

**Figure:** Contour mapping – without encirclements
Derivation concepts, [1, p. 561]

- If $F(s)$ has only zeros that are encircled by the contour then contour $B$ maps in a clockwise direction.
- If $F(s)$ has only poles that are encircled by the contour then contour $B$ maps in a counterclockwise direction.
- If $F(s)$ has only poles or only zeros that are encircled by the contour then contour $B$ map does encircle the origin.

Figure: Contour mapping – with encirclements
If $F(s)$ has $\#\text{poles} = \#\text{zeros}$ that are encircled by the contour then contour $B$ map does not encircle the origin.

Figure: Contour mapping – with encirclements
Each pole or zero of \( 1 + G(s)H(s) \) whose vector undergoes a complete rotation of contour \( A \) must yield a change of 360\(^\circ\) in the resultant, \( R \), or a complete rotation of contour \( B \).

A zero inside a CW contour \( A \) yields a CW rotation of contour \( B \).

A pole inside a CW contour \( A \) yields a CCW rotation of contour \( B \).

\[
N = P - Z
\]

- \( N \), \# CCW rotations of contour \( B \) about the origin
- \( P \), \# poles of \( 1 + G(s)H(s) \) inside contour \( A \)
- \( Z \), \# zeros of \( 1 + G(s)H(s) \) inside contour \( A \)
Derivation concepts, [1, p. 562]

**Adjustment** – extend the contour $A$ to include the entire RHP

- $Z$, # RHP CL poles
  - CL stability!
- $P$, # RHP OL poles
  - Easy
- $N$, # CCW rotations of contour $B$ about origin
  - Difficult

**Adjustment** – map $G(s)H(s)$ instead of $1 + G(s)H(s)$

- $N$, # CCW rotations of contour $B$ about $-1$
  - Less difficult

**Figure:** Contour enclosing RHP to determine stability
Definition, [1, p. 563]

Definition (Nyquist stability criterion)

- If a contour, \( A \), that encircles the entire RHP is mapped through the OL system, \( G(s)H(s) \), then the \# of RHP CL poles, \( Z \), equals the \# of RHP OL poles, \( P \), minus the \# of CCW revolutions, \( N \), around \(-1\) of the mapping.

\[
Z = P - N
\]

- The mapping is called the Nyquist diagram of \( G(s)H(s) \).

- FR technique because the mapping of points on the positive \( j\omega \)-axis through \( G(s)H(s) \) is the same as substituting \( s = j\omega \) into \( G(s)H(s) \) to form the FR function \( G(j\omega)H(j\omega) \).
Applying the Nyquist stability criterion, [1, p. 563]

- No RHP CL poles
  - \( P = 0 \)
  - \( N = 0 \)
  - \( Z = 0 \)
  - CL system is stable

- 2 RHP CL poles
  - \( P = 0 \)
  - \( N = -2 \)
  - \( Z = 2 \)
  - CL system is unstable

\[ \text{Figure: Mapping examples – with encirclement: a. contour does not enclose CL poles; b. contour does enclose CL poles} \]
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Method
Example (general)

\[ G(s) = \frac{500}{(s+10)(s+3)(s+1)} \]

Figure: Vector evaluation of the Nyquist diagram: a. vectors on contour at low frequency, b. vectors on contour around \( \infty \); c. Nyquist diagram
Example (poles on contour)

\[ G(s) = \frac{s+2}{s^2} \]

Figure: a. contour, b. Nyquist diagram
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Example, [1, p. 569]

Example (general)

\[ G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)} \]

Figure: a. system; b. contour, c. Nyquist diagram
Example (general)

\[ G(s) = \frac{K}{s(s+3)(s+5)} \]

**Figure:** a. contour; b. Nyquist diagram
Stability via mapping only the positive $j\omega$-axis, [1, p. 571]
10.6 Gain margin & phase margin via the Nyquist diagram
Definitions, [1, p. 574]

Two quantitative measures of how stable a system is

- **Gain margin,** $G_M$ – the change in OL gain, expressed in $dB$, required at $180^\circ$ of phase shift to make the CL system unstable

- **Phase margin,** $\Phi_M$ – the change in OL phase shift required at unity gain to make the CL system unstable

**Figure:** Nyquist diagram showing gain and phase margins
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Stability via Bode plots, [1, p. 576]

Method

- Draw a Bode log-magnitude plot
- Determine the range of the gain that ensures that the magnitude is less than 0 $dB$ (unity gain) at that frequency where the phase is $\pm 180^\circ$
Example, [1, p. 577]

Example (general)

Use Bode plots to determine the range of $K$ within which the unity FB system is stable.

$$G(s) = \frac{k}{(s + 2)(s + 4)(s + 5)}$$

Figure: Bode log-magnitude and phase diagrams
Gain & phase margin via Bode plots, [1, p. 578]

Method

- **Gain margin**
  - Phase plot →
    \[ \omega_{G_M} = \omega\big|_{\Phi=180^\circ} \]
  - At \( \omega_{G_M} \), magnitude plot → gain margin, \( G_M \), which is the gain required to raise the magnitude curve to 0 dB

- **Phase margin**
  - Magnitude plot →
    \[ \omega_{\Phi_M} = \omega\big|_{G=0dB} \]
  - At \( \omega_{\Phi_M} \), phase plot → phase margin, \( \Phi_M \), which is the difference between the phase value and 180°

Figure: Gain and phase margins on the Bode diagrams
Example (general)

If $K = 200$, find the gain and phase margins.

$$G(s) = \frac{k}{(s + 2)(s + 4)(s + 5)}$$

Figure: Bode log-magnitude and phase diagrams
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Damping ratio & CL FR, [1, p. 580]

Peak magnitude of the CL FR

\[ M_p = \frac{1}{2\zeta \sqrt{1 - \zeta^2}} \]

Frequency of the peak magnitude

\[ \omega_p = \omega_n \sqrt{1 - 2\zeta^2} \]

Figure: 2\textsuperscript{nd}-order CL system

Figure: CL FR peak vs. \%OS for a 2 pole system
Response speed & CL FR, [1, p. 581]

- Bandwidth of a 2-pole system

\[ \omega_{BW} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \]

- \( \omega_n - T_s \) relation

\[ \omega_n = \frac{4}{T_s \zeta} \]

- \( \omega_n - T_p \) relation

\[ \omega_n = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} \]

- \( \omega_n - T_r \) relation
  - Found using look-up table

**Figure:** Representative log-magnitude plot
Response speed & CL FR, [1, p. 582]

**Figure:** Normalized bandwidth vs. damping ratio for:

a. $T_s$, b. $T_p$; c. $T_r$
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Constant $M$ circles & constant $N$ circles, [1, p. 583]

Skip for now
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Damping ratio from $M$ circles, [1, p. 589]

Skip for now
Damping ratio from phase margin, [1, p. 589]

Skip for now
Response speed from OL FR, [1, p. 591]

Skip for now
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Position constant, [1, p. 593]

Type 0 system

\[
G(s) = K \frac{\prod_{i=1}^{n}(s + z_i)}{\prod_{j=1}^{m}(s + p_j)}
\]

Initial log-magnitude value

\[
20 \log M = 20 \log K_p
\]

Position constant

\[
K_p = K \frac{\prod_{i=1}^{n} z_i}{\prod_{j=1}^{m} p_j}
\]

Figure: Typical unnormalized and unscaled Bode log-magnitude plots showing the value of static error constants
Velocity constant, [1, p. 594]

Type 1 system

\[ G(s) = K \frac{\prod_{i=1}^{n}(s + z_i)}{s \prod_{j=1}^{m}(s + p_j)} \]

Initial log-magnitude value

\[ 20 \log M = 20 \log \frac{K_v}{\omega_0} \]

Velocity constant

\[ K_v = K \frac{\prod_{i=1}^{n} z_i}{\prod_{j=1}^{m} p_j} \]

Frequency axis intersect

\[ \omega = K_v \]

Figure: Typical unnormalized and unscaled Bode log-magnitude plots showing the value of static error constants
10 FR techniques

10.11 Steady-state error characteristics from FR

**Acceleration constant, [1, p. 595]**

Type 2 system

\[ G(s) = K \frac{\prod_{i=1}^{n} (s + z_i)}{s^2 \prod_{j=1}^{m} (s + p_j)} \]

Acceleration constant

\[ K_a = K \frac{\prod_{i=1}^{n} z_i}{\prod_{j=1}^{m} p_j} \]

Initial log-magnitude value

\[ 20 \log M = 20 \log \frac{K_a}{\omega_0^2} \]

Frequency axis intersect

\[ \omega = \sqrt{K_a} \]

**Figure:** Typical unnormalized and unscaled Bode log-magnitude plots showing the value of static error constants
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Time delay – delay between the commanded response and the start of the output response

\[ G'(s) = e^{-sT} G(s) \]

FR

\[ G'(j\omega) = e^{-j\omega T} G(j\omega) \]

\[ = |G(j\omega)| \angle [-\omega T + \angle G(j\omega)] \]

Figure: Effect of delay upon FR
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Skip for now
1. Estimate the pole-zero configuration from the Bode diagrams
   - Initial slope → system type
   - Phase excursions → \#poles & \#zeros

2. Look for obvious 1\textsuperscript{st} - & 2\textsuperscript{nd} -order pole or zero FR characteristics

3. Peaking & depressions → underdamped 2\textsuperscript{nd}-order pole & zero, respectively

4. Extract 1\textsuperscript{st} - & 2\textsuperscript{nd}-order characteristics
   - Overlay ±20 or ±40 dB/decade lines on magnitude curve & ±45°/decade lines on the phase curve
   - Estimate break frequencies
   - For 2\textsuperscript{nd}-order poles & zeros, estimate \( \zeta \) & \( \omega_n \)

5. Form a TF of unity gain using the poles & zeros found
   - Subtract the FR of the model from the measured FR and repeat the process if necessary