Differential Games among Major Fix-wing UAVs

Reachable Sets analysis using Hamilton-Jacobi Formulation

Zhengyin Qian
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Abstract




- Objective: Compare ***maneuverability*** of major fixed-wing unmanned aerial vehicles (UAV).

- Technique: Calculate the viscosity solution of the ***Hamilton-Jacobi*** formulation using ***level set*** method.

- Result: ***Reachability*** plot of different UAV pairs.

Outline

- Abstract
- Problem Statement
- Math Formulation
- Simulation
- Remaining Work

Problem Statement

- An evader UVA (player 1) and a pursuer (player 2) are flying in the **same altitude**.
- The pursuer tries to **hit** the evader while the evader tries its best to **escape.**
- The evader and the pursuer are assumed to have **fixed cruising speeds** v_a and v_b .

Problem Statement

- **Inputs** of the two UAVs are Rate of Turn.

  Evader: $\omega_a \in \mathcal{A} = [\underline{\omega_a}, \ \overline{\omega_a}]$

  Pursuer: $\omega_b \in \mathcal{B} = [\underline{\omega_b}, \ \overline{\omega_b}]$

- The pursuer has an **initial position** of $\begin{bmatrix} x_r & y_r & \psi_r \end{bmatrix}$ relative to the evader.

- The hit event is said to happen once the pursuer reaches ***d*** **radius** circle of evader.

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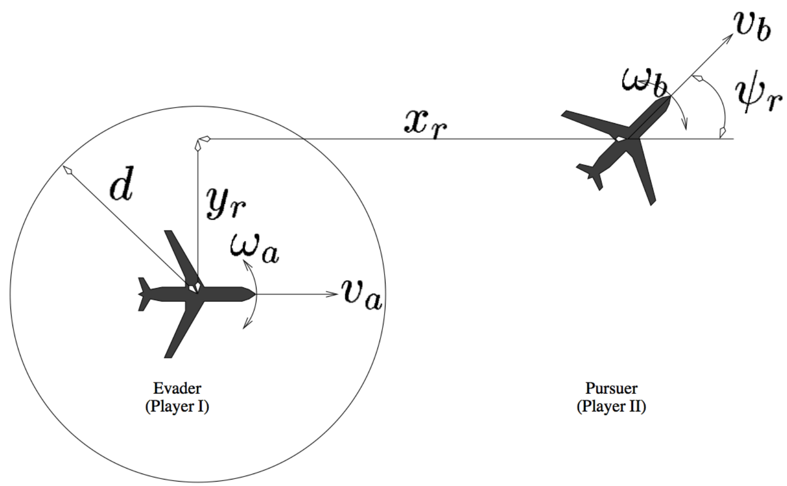
Math Formulation



Var.	Meaning
x_r	Relative position in direction of evader's flight
y_r	Relative position in perpendicular to direction of evader
ψ_r	Relative heading ($0 \leq \psi_r < 2\pi$)
v_a	Translational speed of evader
v_b	Translational speed of pursuer
ω_a	Angular velocity and input of evader ($\omega_a \in \mathcal{A}$)
ω_b	Angular velocity and input of pursuer ($\omega_b \in \mathcal{B}$)
d	Minimum safe separation distance

Math Formulation

- Define the state vector: $z = [x_r \quad y_r \quad \psi_r]^T$
- Individual UAV dynamic:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \psi \end{bmatrix} = \begin{bmatrix} v\cos\psi \\ v\sin\psi \\ \omega \end{bmatrix}$$

- Based on the individual dynamic, it is easy to observe:

$$\dot{z} = \frac{d}{dt}\begin{bmatrix} x_r \\ y_r \\ \psi_r \end{bmatrix} = \begin{bmatrix} v_b\cos\psi - v_a + \omega_a y_r \\ v_b\sin\psi - \omega_a x_r \\ \omega_b - \omega_a \end{bmatrix} = f(z, \omega_a, \omega_b)$$

Math Formulation

- Set of states that represents a "hit" event:

$$\mathcal{G}_0 = \{z \in \boldsymbol{R}^3 | x_r^2 + y_r^2 \leq d^2\}$$

- If we define: $\phi_0(z) = \sqrt{{x_r}^2 + y_r^2} - d$
- The above set is equivalent as:

$$\mathcal{G}_0 = \{z \in \boldsymbol{R}^3 | \phi_0(z) \leq 0\}$$

- Define the relative distance between the UAVs as the cost function:

$$J(z, \omega_a, \omega_b, t)$$

Math Formulation

- Optimum Control:
  - Pursuer: $\omega_b^* = \arg \min_{\omega_b \in \mathcal{B}} \max_{\omega_a \in \mathcal{A}} J(z, \omega_a, \omega_b, t)$
  - Evader: $\omega_a^* = \arg \max_{\omega_a \in \mathcal{A}} \min_{\omega_b \in \mathcal{B}} J(z, \omega_a, \omega_b, t)$
- From Mitchell, Bayen, & Tomlin' 2005 paper, $\phi(z,t)$ is the solution to of ***Hamilton-Jacobi-Isaacs PDE***:

$$\frac{\partial \phi(z,t)}{\partial t} + \min[0, H^*(z, \nabla\phi(z,t))] = 0$$

Math Formulation

$$\frac{\partial \phi(z,t)}{\partial t} + \min\left[0, H^*\left(z, \nabla\phi(z,t)\right)\right] = 0$$

- Terminal Condition: $\phi(z,0) = \phi_0(z)$
- Where the Hamiltonian is:

$$H^*(z,p) = \max_{\omega_a \in \mathcal{A}} \min_{\omega_b \in \mathcal{B}} H(z,p,\omega_a,\omega_b)$$

- Where:

$$H(z,p,\omega_a,\omega_b) = [p_1 \quad p_2 \quad p_3] \begin{bmatrix} v_b \cos\psi - v_a + \omega_a y_r \\ v_b \sin\psi - \omega_a x_r \\ \omega_b - \omega_a \end{bmatrix}$$

$$= -p_1 v_a + p_1 v_b \cos\psi_r + p_2 v_b \sin\psi_r + \boxed{\omega_a}\boxed{(p_1 y_r - p_2 x_r - p_3)} + \boxed{\omega_b}\boxed{p_3}$$

- Where: $p = [p_1 \quad p_2 \quad p_3]^T$ is the co-state.

Math Formulation

$$H^*(z,p) = \max_{\omega_a \in \mathcal{A}} \min_{\omega_b \in \mathcal{B}} \left(C + \omega_a(p_1 y_r - p_2 x_r - p_3) + \omega_b p_3\right)$$

- The optimum control to achieve *H** can therefore be obtained as:

$$\omega_a^* = \begin{cases} \omega_a & if\ p_1 y_r - p_2 x_r - p_3 > 0 \\ -\omega_a & if\ p_1 y_r - p_2 x_r - p_3 \le 0 \end{cases}$$

$$\omega_b^* = \begin{cases} \omega_b & if\ -p_3 > 0 \\ -\omega_b & if\ -p_3 \le 0 \end{cases}$$

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Simulation

Global Hawk

Wing-Span: 35.4 m

Length: 13.5 m

Cruising Speed:

575 km/h

US Air Force,

US Navy, NASA,

German Air Force

Predator

Wing-Span: 14.8 m

Length: 8.22 m

Cruising Speed:

130-165 km/h

US Air Force

ALTAIR

Wing-Span: 13.0 m

Length: 8.8 m

Cruising Speed:

300 -350 km/h

NASA

MARINER

Wing-Span: 26.2 m

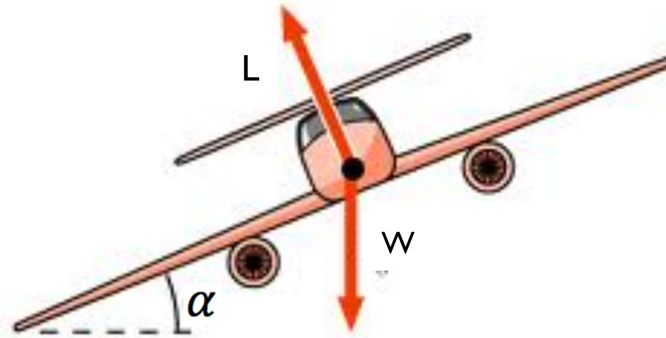
Length: 11.0 m

Cruising Speed:

400 km/h

US Navy, Dept of Homeland Security

Simulation

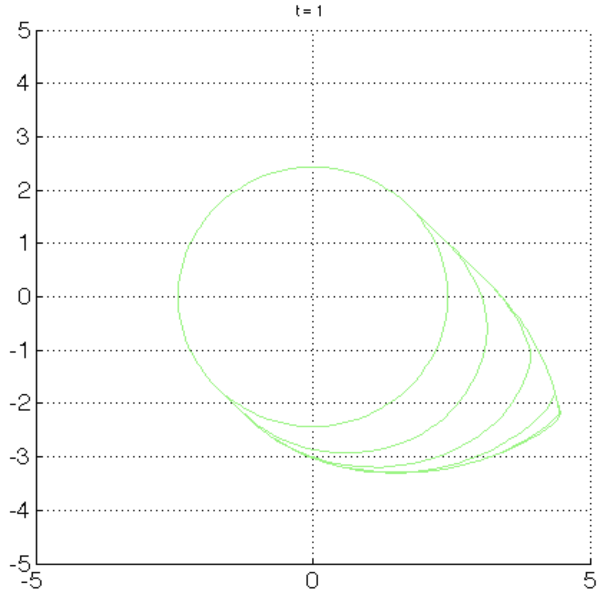


- Units:
  - Nautical Mile (1 nm = 1.852 km);
  - Knot (1 kt = 1.852km/hr)
- From cruising speed to Rate
  - Radius of tern$_{\min}$ = $R_{\min}$ = $v_a^2/g \tan \alpha_{max}$
  - Rate of turn$_{\max}$ = $w_{max} = v_a/R_{min}$
  - $\alpha_{max} = 30^{\circ}$
- Safe distance d (which determines $\phi_0(z)$):
  - Minimum Radius of turn

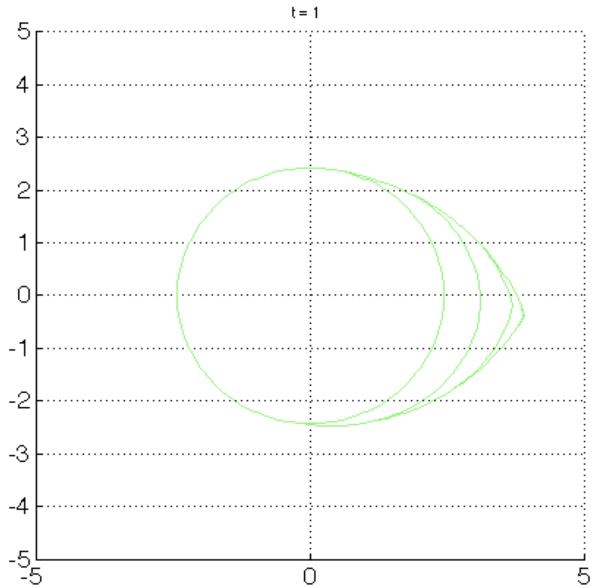
Simulation

Evader: Global Hawk
Escape Radius: 2.43 nm

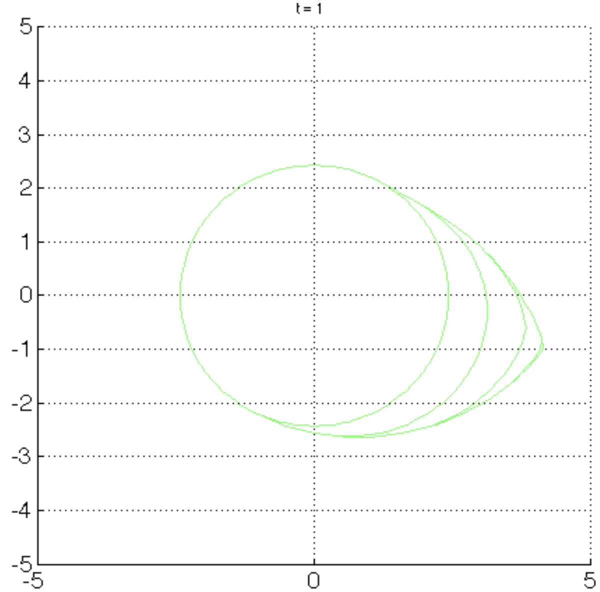
Global Hawk



Predator



ALTAIR



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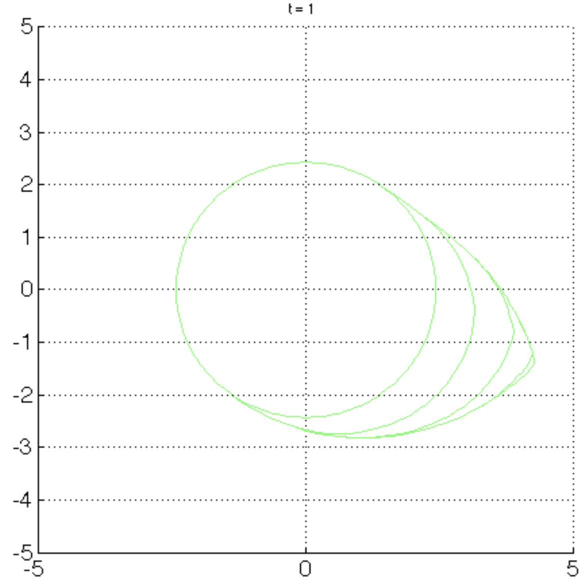
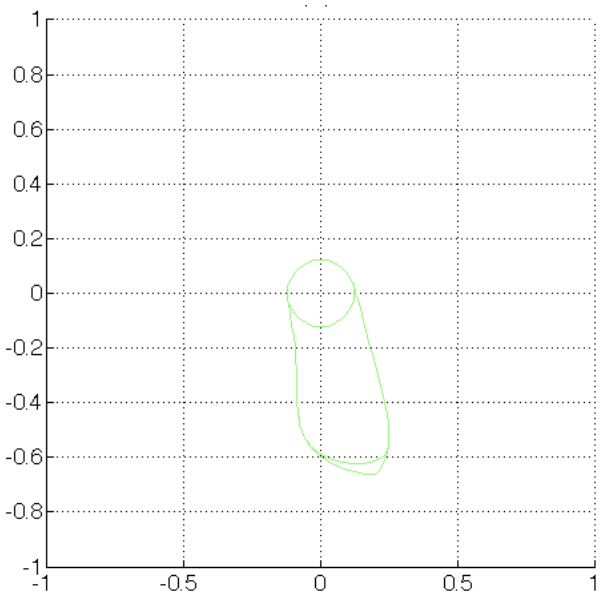


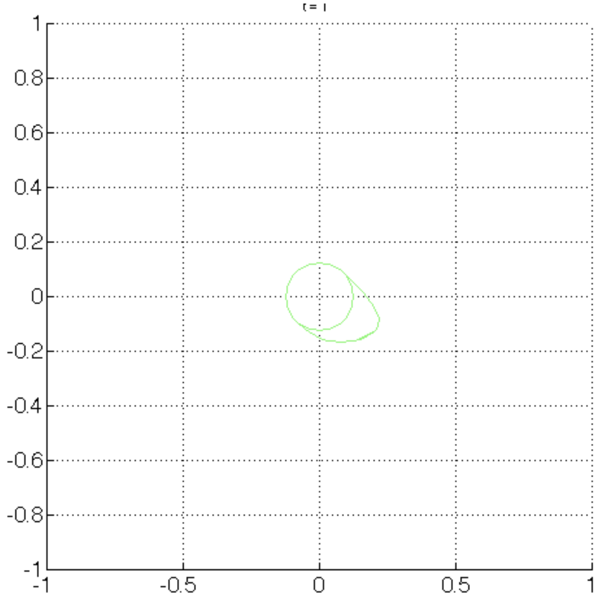
Simulation

Evader: Predator
Escape Radius: 0.124 nm

Global Hawk



Predator



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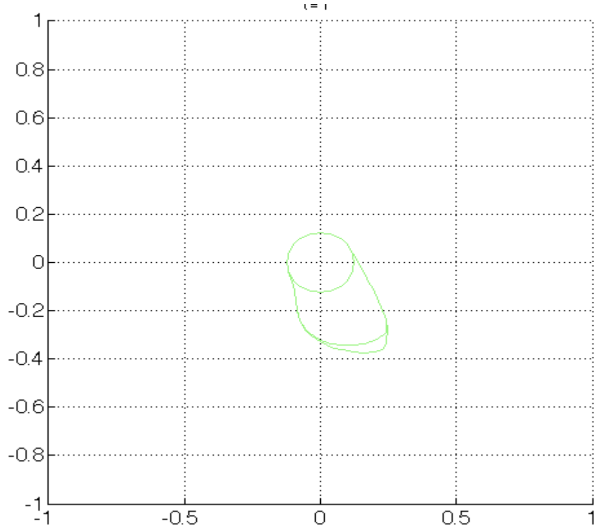
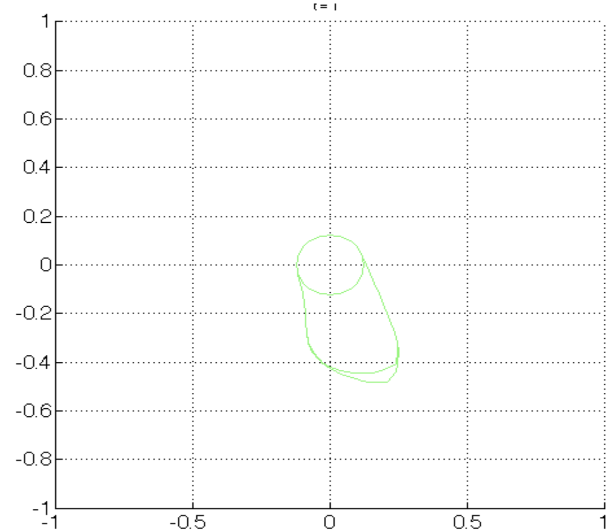


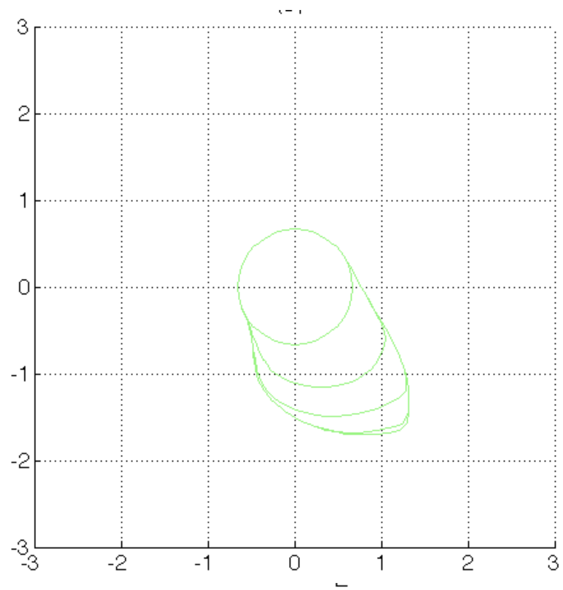
MARINER



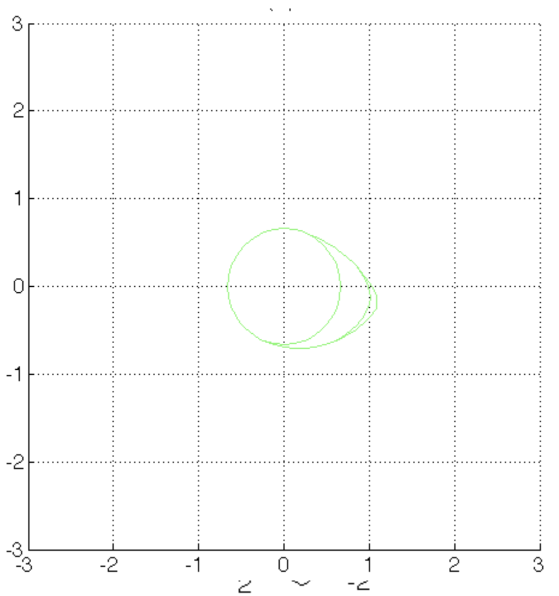
Evader: ALTAIR
Escape Radius: 0.663nm

Simulation

Global Hawk



Predator



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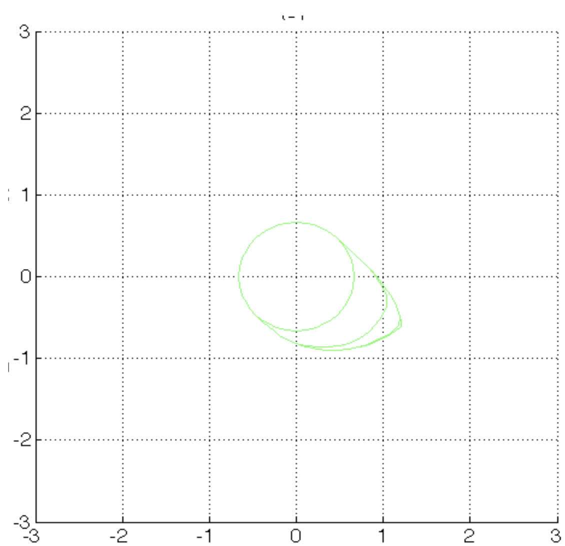
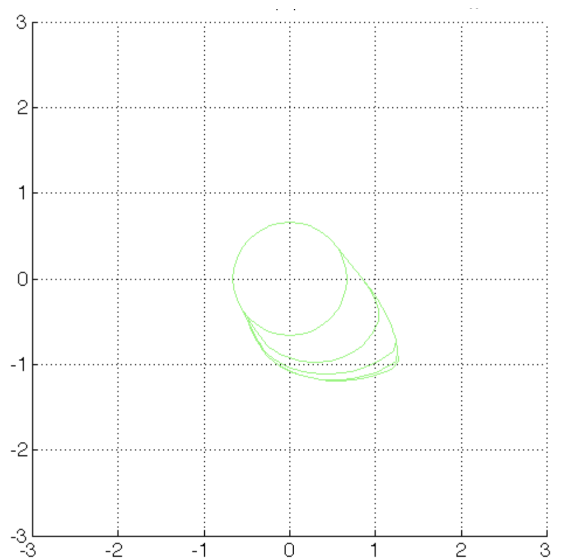


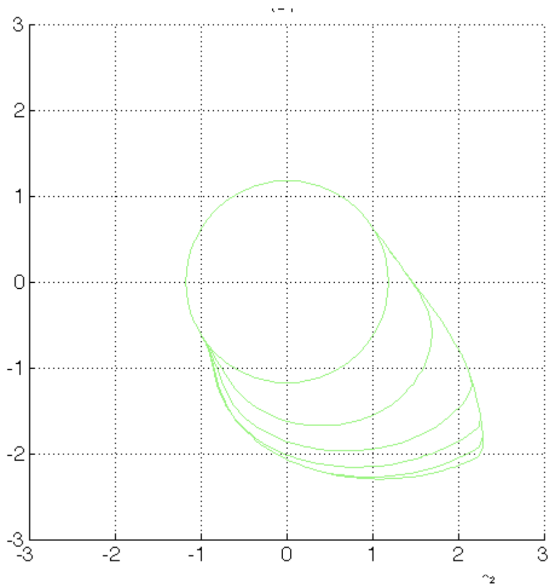
MARINER



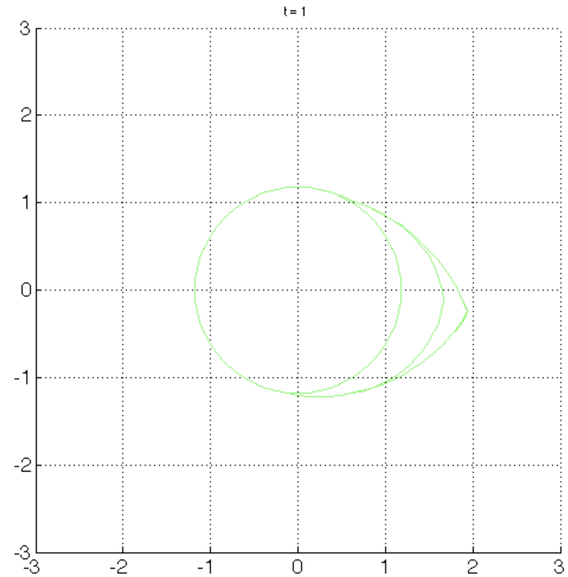
Simulation

Evader: MARINER
Escape Radius: 1.178nm

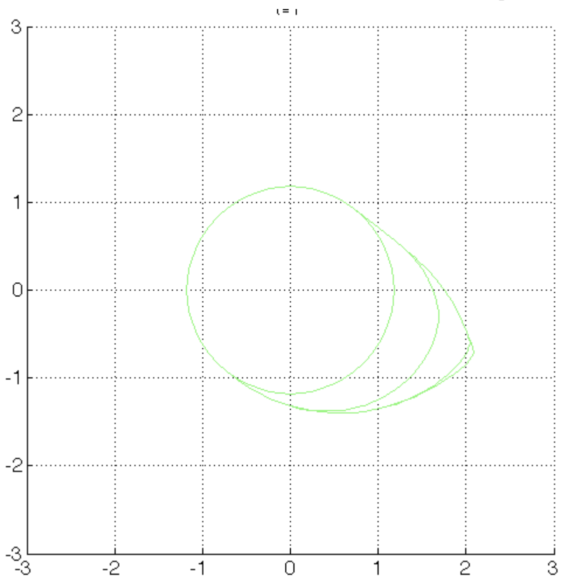
Global Hawk



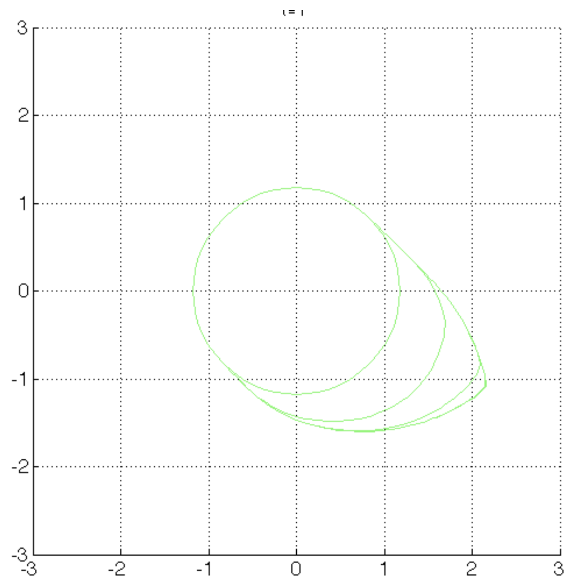
Predator



ALTAIR



MARINER



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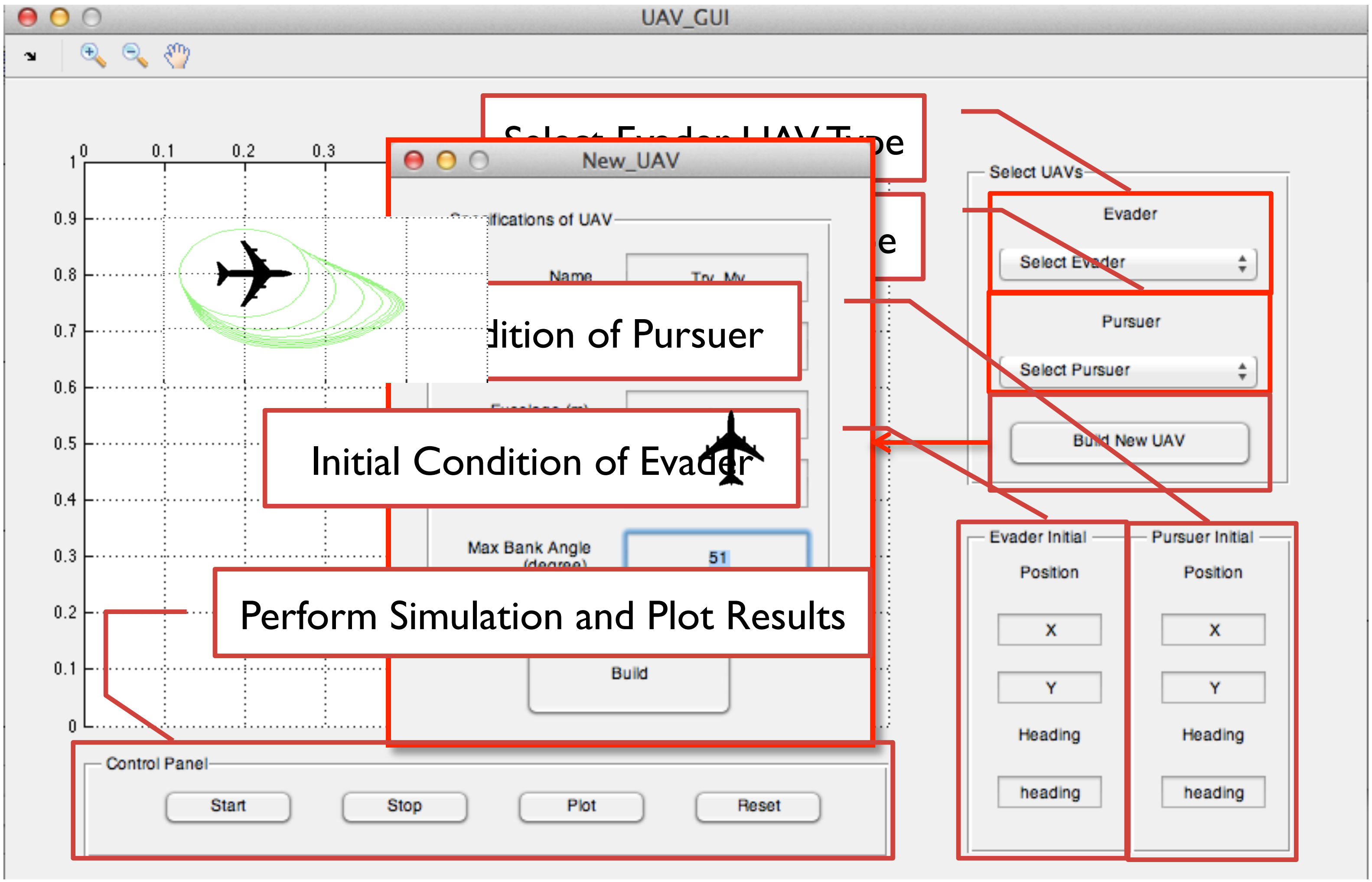

Thank You