Crowd dynamic simulation

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Introduction

- Goal: simulation of a 2D pedestrian movement flow in specific situations (emergency,…)
- Model: extension of microscopic car following model in 2D.
- Simulations in 2 situations:
  - Free flow of a crowd in a given direction
  - Movement of a crowd toward an exit

Schedule

- Theoretical model
- Numerical simulations
- Results

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Microscopic Model

People adjust their speed based on the speed of the people in front of them.

$$\tau(x_{n-1}(t) - x_n(t))\ddot{x}_n(t) = \dot{x}_{n-1}(t) - \dot{x}_n(t)$$

Macroscopic Model

- Conservation of mass:

$$\rho_t + (\rho v)_x + (\rho u)_y = 0$$

v and u are the components of the velocity along x and y

- Extending the microscopic with the conservation law:

$$v_t + v v_x + u v_y + \rho V'(\rho)\left(v_x + u_y\right) = \frac{V(\rho) - v}{\tau}$$

$$u_t + v u_x + u u_y + \rho U'(\rho)\left(v_x + u_y\right) = \frac{U(\rho) - u}{\tau}$$

V and U are the desired velocities to simulate pedestrian behavior. $\rho V'(\rho)$ is the traffic sound speed at which small traffic disturbances are propagated.

Pedestrian velocity model

We use the Greenshild model for the pedestrian velocity:

$$V(\rho) = v_f \left(1 - \frac{\rho}{\rho_m}\right)$$

PDE system

We obtain an hyperbolic PDE system.

$$Q_t + F(Q)_x + G(Q)_y = 0$$

$$Q_t + A(Q)Q_x + B(Q)Q_y = 0$$

$$Q = \begin{pmatrix} \rho \\ w \\ z \end{pmatrix} = \begin{pmatrix} \rho \\ \rho(v - V) \\ \rho(u - U) \end{pmatrix} \quad F(Q) = \begin{pmatrix} \rho v \\ \rho v(v - V) \\ \rho v(u - U) \end{pmatrix} \quad G(Q) = \begin{pmatrix} \rho u \\ \rho u(v - V) \\ \rho u(u - U) \end{pmatrix}$$

$$A(Q) = \begin{bmatrix} V + \rho V' & 1 & 0 \\ -\frac{w^2}{\rho^2} + wV' & 2\frac{w}{\rho} + V & 0 \\ -\frac{wz}{\rho^2} + zV' & \frac{z}{\rho} & \frac{w}{\rho} + V \end{bmatrix} \quad \text{and} \quad B(Q) = \begin{bmatrix} U + \rho U' & 0 & 1 \\ -\frac{wz}{\rho^2} + wU' & \frac{z}{\rho} + U & \frac{w}{\rho} \\ -\frac{z^2}{\rho^2} + zU' & 0 & 2\frac{z}{\rho} + U \end{bmatrix}$$

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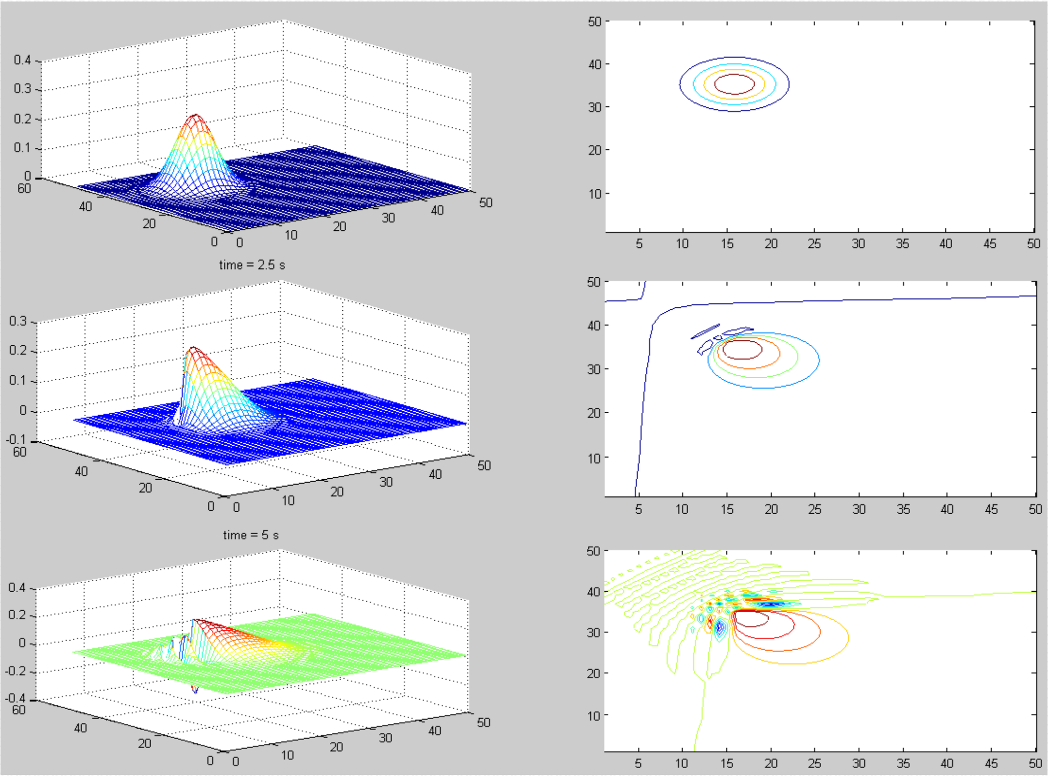
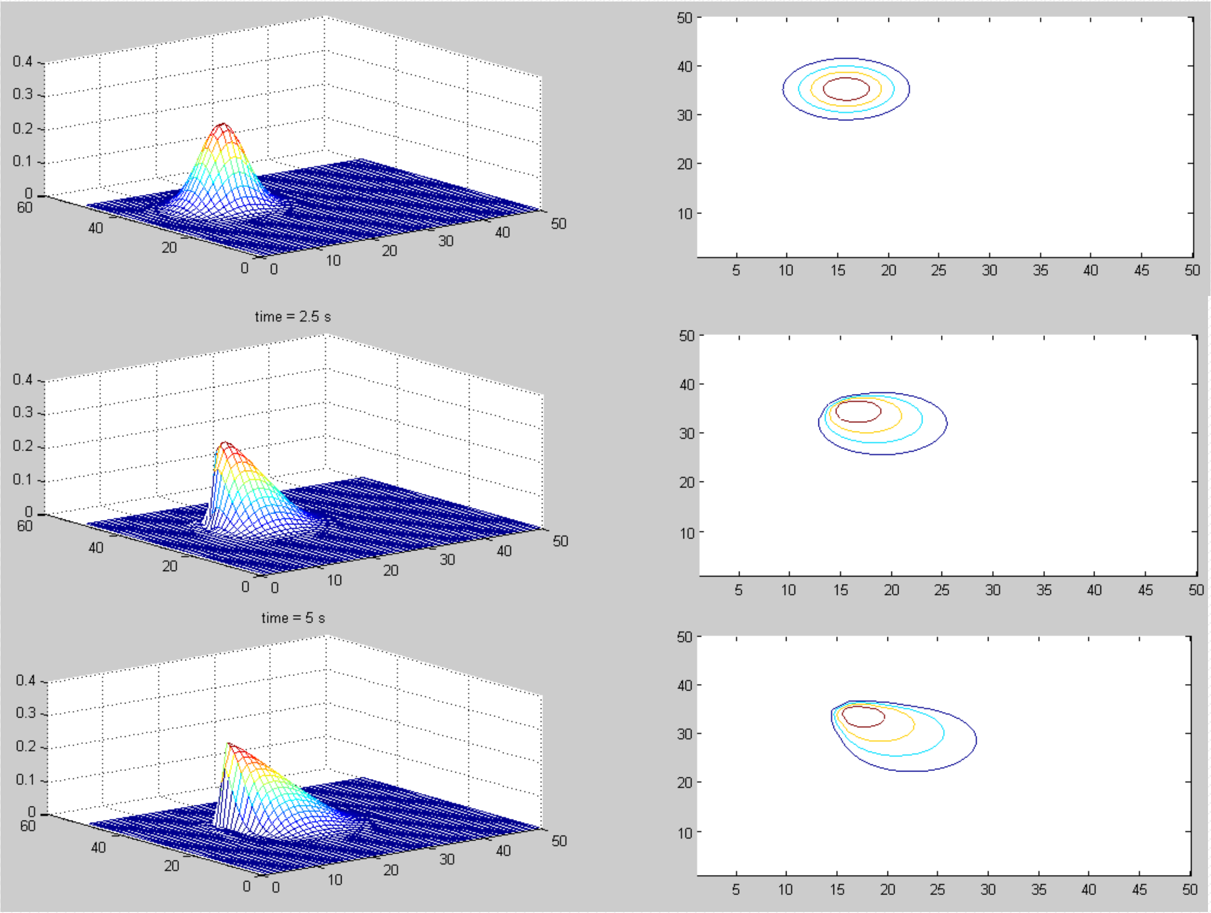
First computation

$$Q(t+dt) = Q(t) - dt\left[A(Q)Q_x + B(Q)Q_y\right]$$

$$Q_x = \frac{Q(x+dx) - Q(x-dx)}{dx}$$



Second computation



Lax Wendroff's model

$$W_t = [F(W)]_x + [G(W)]_y$$

$$W^{n+1/2}_{j+1/2,m+1/2} = \widehat{W}^n_{j+1/2,m+1/2} + \frac{\lambda}{2}[\tilde{F}^n_{j+1,m+1/2} - \tilde{F}^n_{j,m+1/2} + \tilde{G}^n_{j+1/2,m+1} - \tilde{G}^n_{j+1/2,m}]$$

with

$$\begin{cases} \widehat{W}^n_{j+1/2,m+1/2} = [W^n_{j+1,m+1} + W^n_{j+1,m} + W^n_{j,m+1} + W^n_{j,m}]/4 \\ \tilde{F}^n_{j+1,m+1/2} = F\left(\left(W^n_{j+1,m+1} + W^n_{j+1,m}\right)/2\right) \\ \tilde{F}^{n+1/2}_{j+1/2,m} = F\left(\left(W^{n+1/2}_{j+1/2,m+1/2} + W^{n+1/2}_{j+1/2,m-1/2}\right)/2\right) \end{cases}$$

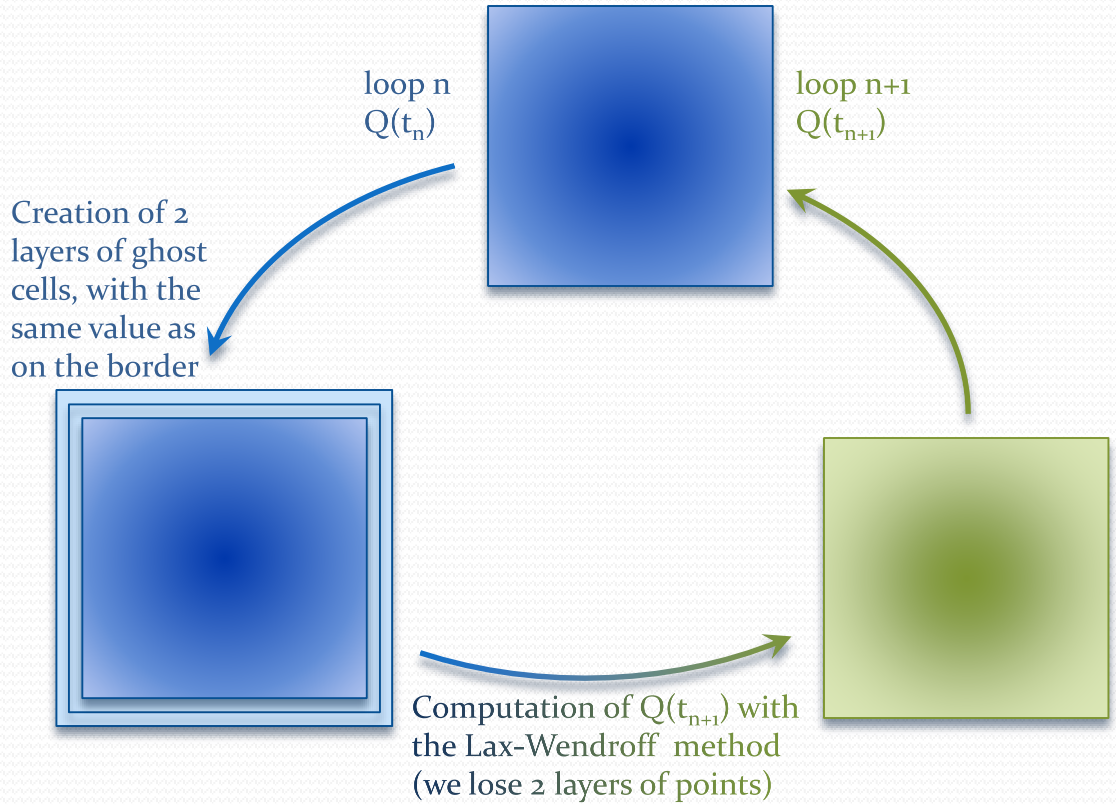
$$W^{n+1}_{j,m} = W^n_{j,m} + \lambda[\tilde{F}^{n+1/2}_{j+1/2,m} - \tilde{F}^{n+1/2}_{j-1/2,m} + \tilde{G}^{n+1/2}_{j,m+1/2} - \tilde{G}^{n+1/2}_{j,m-1/2}]$$

Stability

$$\lambda = \frac{dt}{dx} \leq \frac{1}{\max\limits_i \sqrt{{a_i}^2 + {b_i}^2}}$$

$$\lambda = \frac{dt}{dx} \leq \frac{1}{\sqrt{2}v_f} = 0.52$$

Ghost cells


loop n
Q(t_n)
loop n+1
Q(t_n+1)
Creation of 2 layers of ghost cells, with the same value as on the border
Computation of Q(t_n+1) with the Lax-Wendroff method (we lose 2 layers of points)

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Results - in a closed room

$$\begin{cases} L = 10 \\ dx = 0.2 \\ dt = 0.05 \\ v_f = 1.36\, dt/dx \\ \rho_m = 0.5 \\ \sigma = 1 \\ \rho(x, y, 0) = 0.25 e^{-\left(\frac{x-7}{\sigma}\right)^2 - \left(\frac{y-3}{\sigma}\right)^2} \end{cases}$$

Results - toward an exit

$$\begin{cases} L = 10 \\ dx = 0.2 \\ dt = 0.05 \\ v_f = 1.36\, dt/dx \\ \rho_m = 1 \\ \sigma = 1 \\ \rho(x, y, 0) = 0.25 e^{-\left(\frac{x-5}{\sigma}\right)^2 - \left(\frac{y-7}{\sigma}\right)^2} \end{cases}$$

Xe=3 , Ye=10

$$V(\rho) = \pm v_f \left(1 - \frac{\rho}{\rho_m}\right) \frac{x - x_e}{\sqrt{(x - x_e)^2 + \left(y - y_e\right)^2}}$$

Discussion

- The results follow the expectations in such situations.
- The model predicts jam density near the exit which seems realistic in an emergency situation.
- Some numerical imperfections.

Conclusion

- An attempt to simulate the crowd dynamic flow in different situations
- Macroscopic model developed on microscopic behavior assumptions
- Model ➔ hyperbolic system of PDEs
- Realistic results, follow expectations