Creating a Visual, Interactive Representation of Traffic Flow

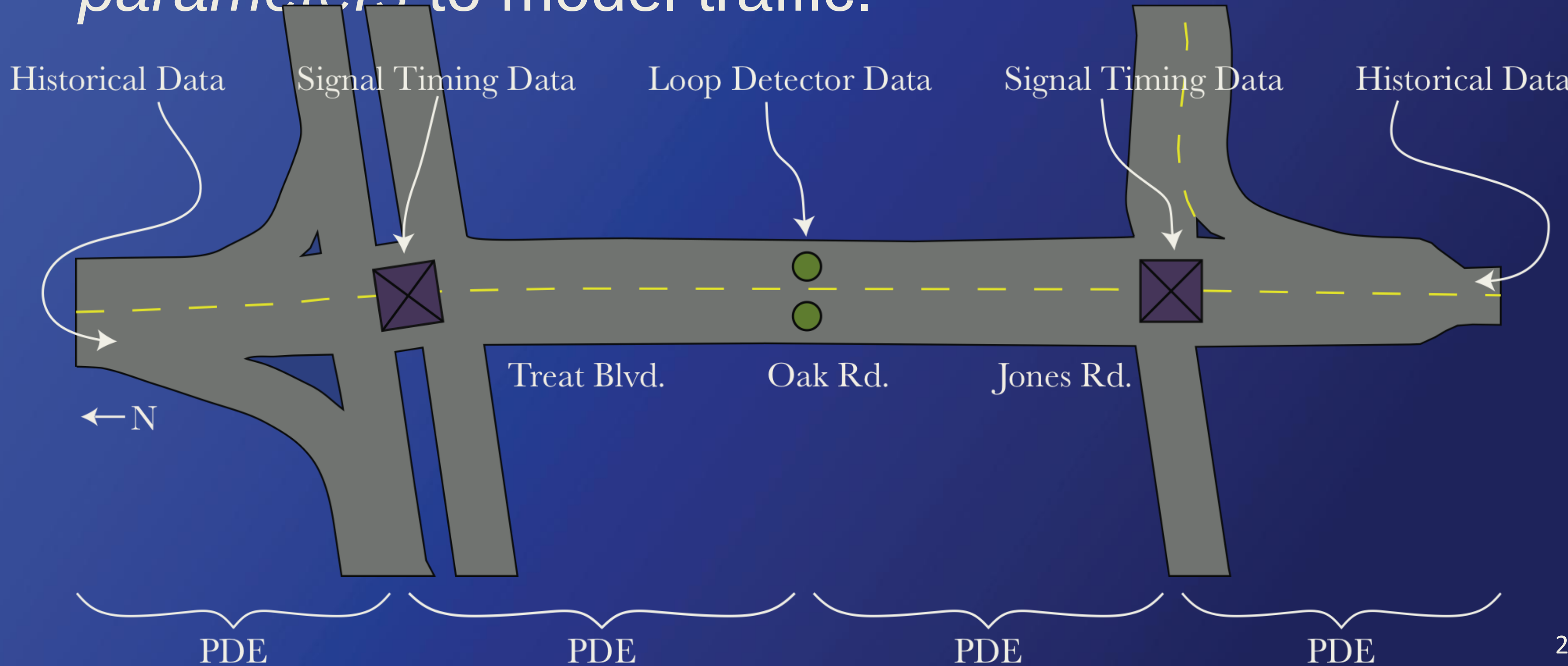
Eric Mai • CE 291F • May 4, 2009

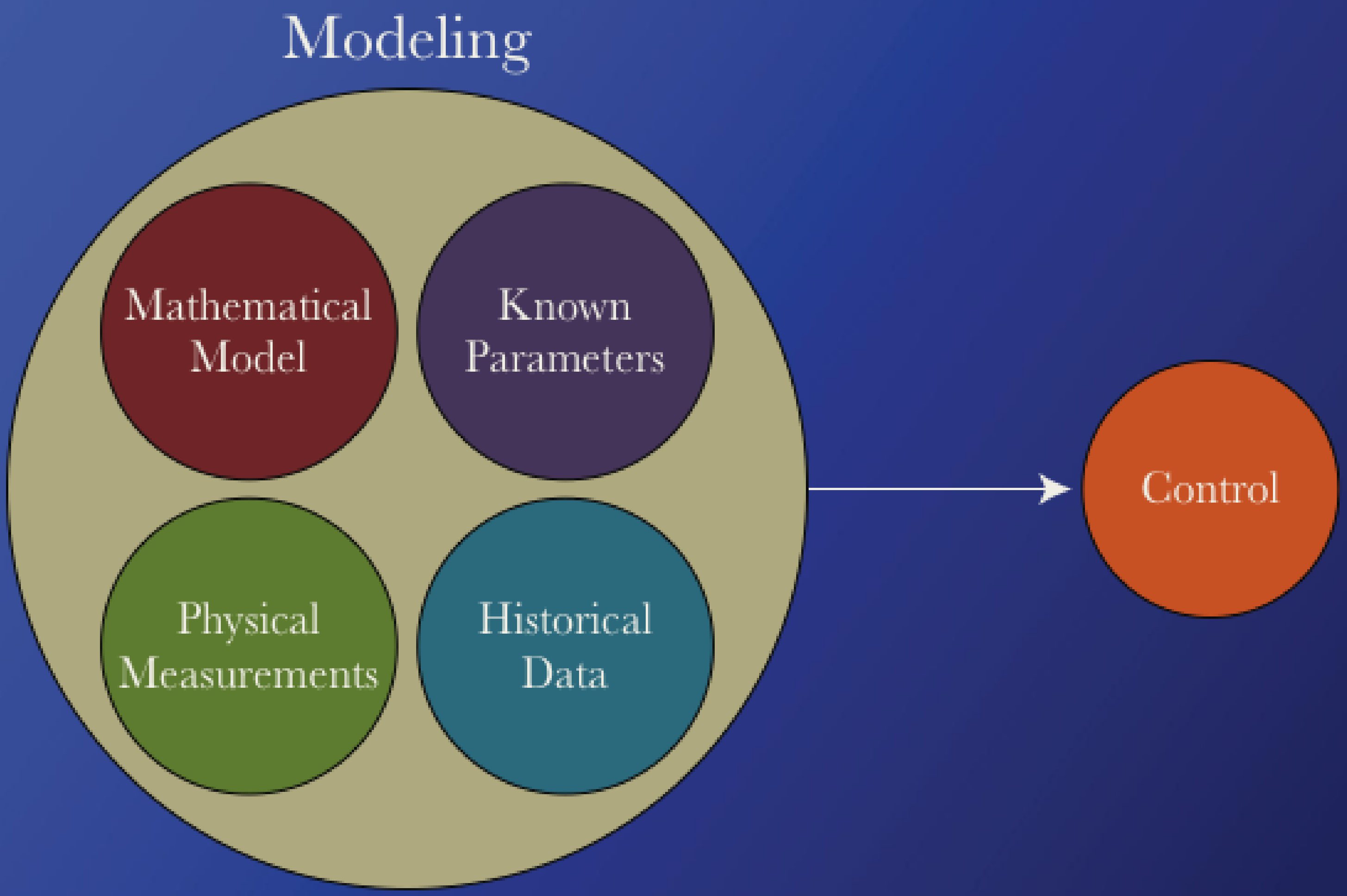
Project Criteria

- Apply PDEs to a real world system
- Implement PDEs in computer code, to improve my programming skills
- Do something related to my research

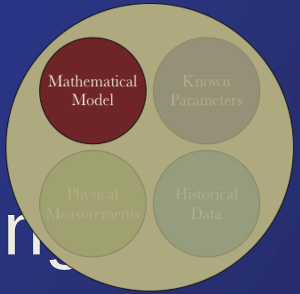
Project Description

Create an interactive Java applet to visualize how a *PDE* model can be integrated with *loop detector data*, *historical data*, and *known system parameters* to model traffic.




Modeling
Mathematical Model
Known Parameters
Physical Measurements
Historical Data
Control

LWR PDE



Consider a control volume of length dx along highway and write conservation laws for it:

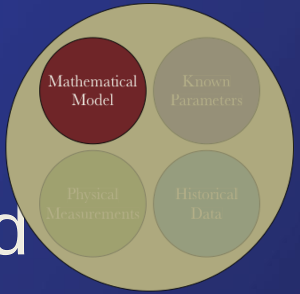
$$(\text{vehicles entering at } t) - (\text{vehicles entering at } t+dt) =$$
$$(\text{vehicles entering between } t \text{ and } t+dt) - (\text{vehicles exiting between } t \text{ and } t+dt)$$

which, as has been seen in the course notes, leads to the LWR PDE:

$$\frac{\partial \rho(x,t)}{\partial t} + q'(\rho(x,t))\frac{\partial(\rho(x,t))}{\partial x} = 0$$

for $\rho(x,t)$ = density and $q(\rho(x,t))$ = flux, as dx and dt go to zero.

LWR PDE, cont.



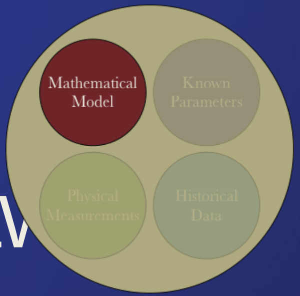
It has been seen that using the Greenshield function for $q(\rho(x,t))$ provides good results. The resulting equation follows:

$$q(\rho) = v\rho(1 - \frac{\rho}{\rho^*})$$

$$\frac{\partial \rho(x,t)}{\partial t} + v(1 - \frac{2\rho(x,t)}{\rho^*})\frac{\partial \rho(x,t)}{\partial x} = 0$$

for $\rho^* =$ jam density and $v =$ free flow velocity.

Godunov Scheme [9]

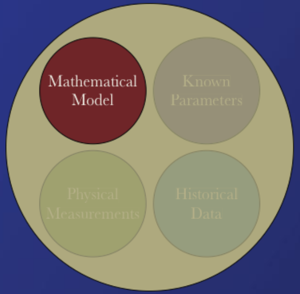


In order to obtain a (weak) solution of the LW PDE for bounded domains, we examine the convergence of the Godunov Scheme to the entropy solution of the PDE. Here, ρ^{n+1} is computed from ρ_i^n as follows:

$$\begin{cases} \rho^n_{i+\frac{1}{2}} & \text{is an element } k \text{ of } I(\rho_i^n, \rho_{i+1}^n) \text{ such that } sg(\rho_{i+1}^n - \rho_i^n)q_k \text{ is minimal} \\ \rho_i^{n+1} = & \rho_i^n - r(q(\rho^n_{i+\frac{1}{2}}) - q(\rho^n_{i-\frac{1}{2}})) \end{cases}$$

where the subscripts correspond to spatial coordinates, superscripts to time steps, $I(x,y) = [\inf(x,y), \sup(x,y)]$, and sg is the sign function

Godunov Scheme, cont. [9]



The weak boundary conditions (necessary because of the problems with strong boundary conditions in this PDE) are formulated as follows:

$$L(\rho(a,t),\rho_a(t)) = 0 \text{ and } R(\rho(b,t),\rho_b(t)) = 0$$

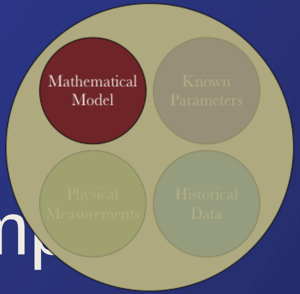
where

$$L(x,y) = \sup_{k \in I(x,y)} (sg(x-y)(q(x)-q(k)) \text{ and}$$

and

$$R(x,y) = \inf_{k \in I(x,y)} (sg(x-y)(q(x)-q(k)) \text{ for } x,y \in R$$

Godunov Scheme, cont. [9]



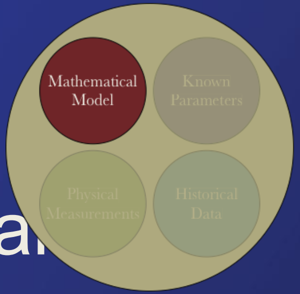
The Godunov Scheme can be practically imp
mented in this application as follows:

$$\rho_i^{n+1} = \rho_i^n - r\left(q_G(\rho_i^n, \rho_{i+1}^n) - q_G(\rho_{i-1}^n, \rho_i^n)\right)$$

Flux is often concave in traffic monitoring, and we call the density at which it reaches its maximum ρ_c.

$$q_G = \begin{cases} \min(\rho_1, \rho_2) & \text{if } \rho_1 \le \rho_2 \\ q(\rho_1) & \text{if } \rho_2 < \rho_1 < \rho_c \\ q(\rho_c) & \text{if } \rho_2 < \rho_c < \rho_1 \\ q(\rho_2) & \text{if } \rho_c < \rho_2 < \rho_1 \end{cases}$$

Godunov Scheme, cont. [9]



"Ghost cells" are inserted at the beginning a end of the domain ($i = 0$ and $i = M$) to handle the boundary conditions:

$$\rho_0^{n+1} = \rho_0^n - r(q_G(\rho_0^n, \rho_1^n) - q_G(\rho_{-1}^n, \rho_0^n))$$

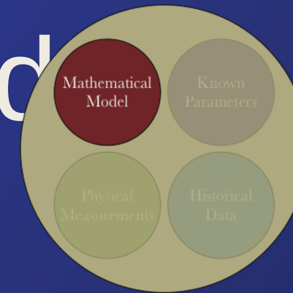
$$\rho_M^{n+1} = \rho_M^n - r(q_G(\rho_M^n, \rho_{M+1}^n) - q_G(\rho_{M-1}^n, \rho_M^n))$$

with:

$$\rho_{-1}^n = \frac{1}{rh}\int_{J_n} \rho_a(t)dt, \; 0 \le n \le N \qquad \rho_{M+1}^n = \frac{1}{rh}\int_{J_n} \rho_b(t)dt, \; 0 \le n \le N$$

This allows us to obtain highway densities numerically.

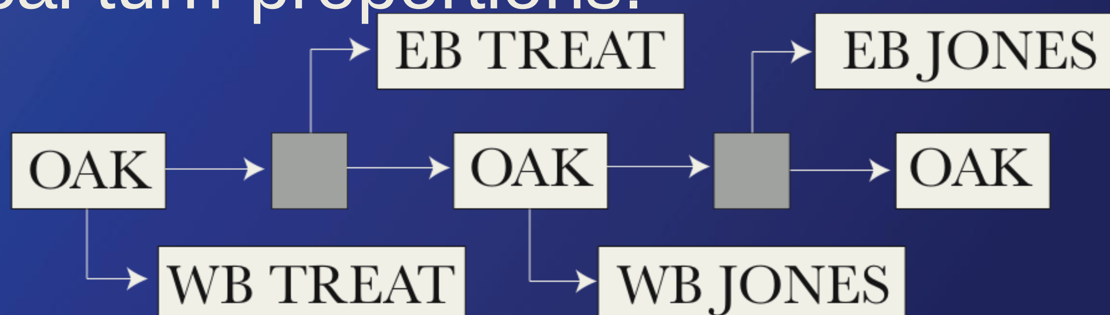
Alternative Numerical Methods



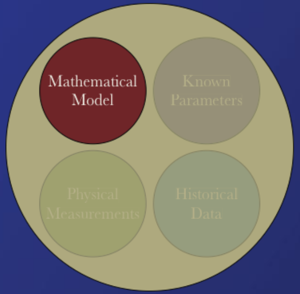
Cell Transmission Model: Discretely approximates the LWR model by finding flows on links based on flow & storage capacities of that link and adjacent links under discrete lengths and time steps.

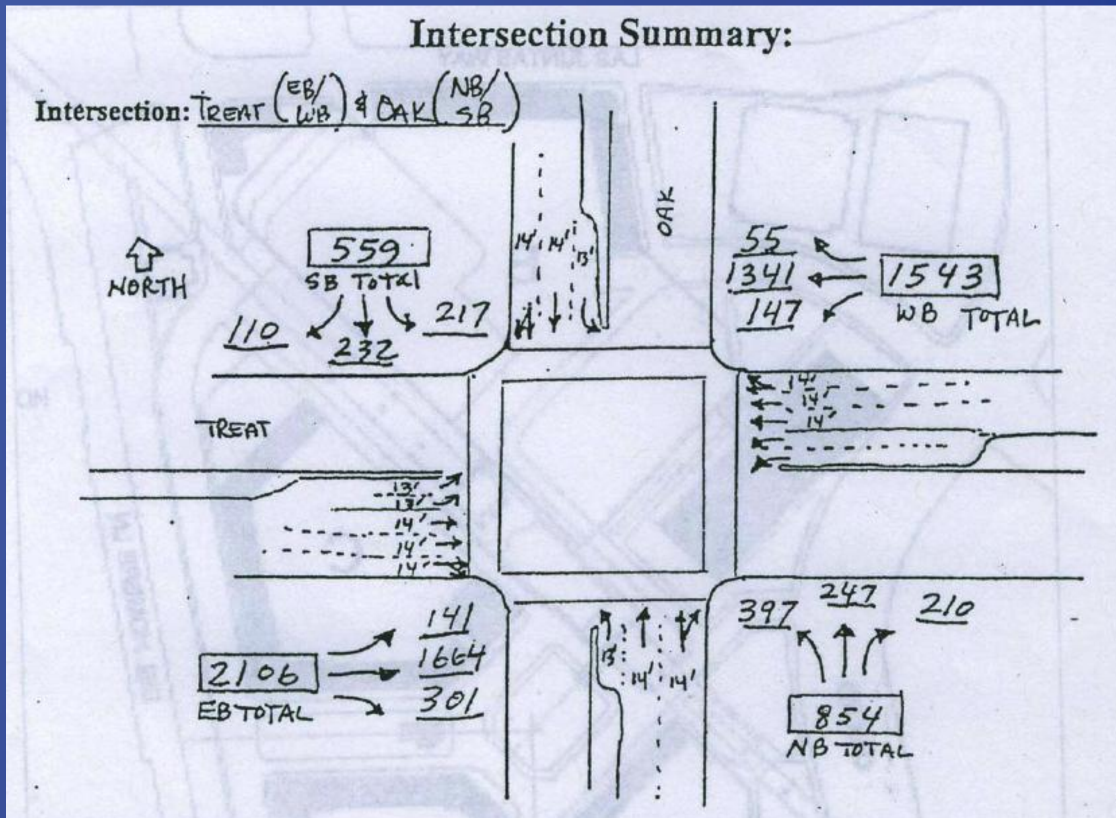
Needs historical turn-proportions.

CTM for SB Oak:



CTM, cont. [2]



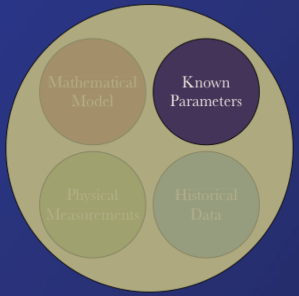


Approach	Peds	Pkg/Nm	Arr. Type	PHF	%HV	No. Buses	Grade
EB	30	No/o	5	.90	0	0	0
WB	20	No/o	5	.90	0	0	0
NB	10	No/o	5	.90	0	0	2
SB	10	No/o	5	.90	0	0	0

Signal Phasing:

	Direction	Movement	Seconds of: Green	Y+AR	Type:
Phase 1	EB-WB	Left Turn	22	3	Pre-timed
Phase 2	EB-WB	Thru-Right	47	3	"
Phase 3	NB-SB	Left Turn	22	3	"
Phase 4	NB-SB	Thru-Right	17	3	"

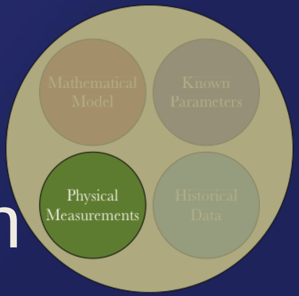
Known System Parameters



Need boundary conditions and physical characteristics of system for PDEs to work:

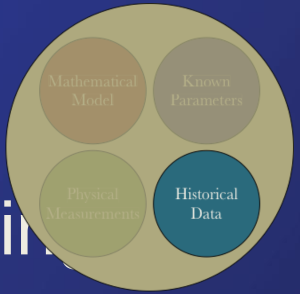
- Signal timing
- Lane configuration
- Expected turning proportions (for CTM)

Physical Measurements



- Loop detector data – detect changes in system and inform the PDEs (boundary conditions)

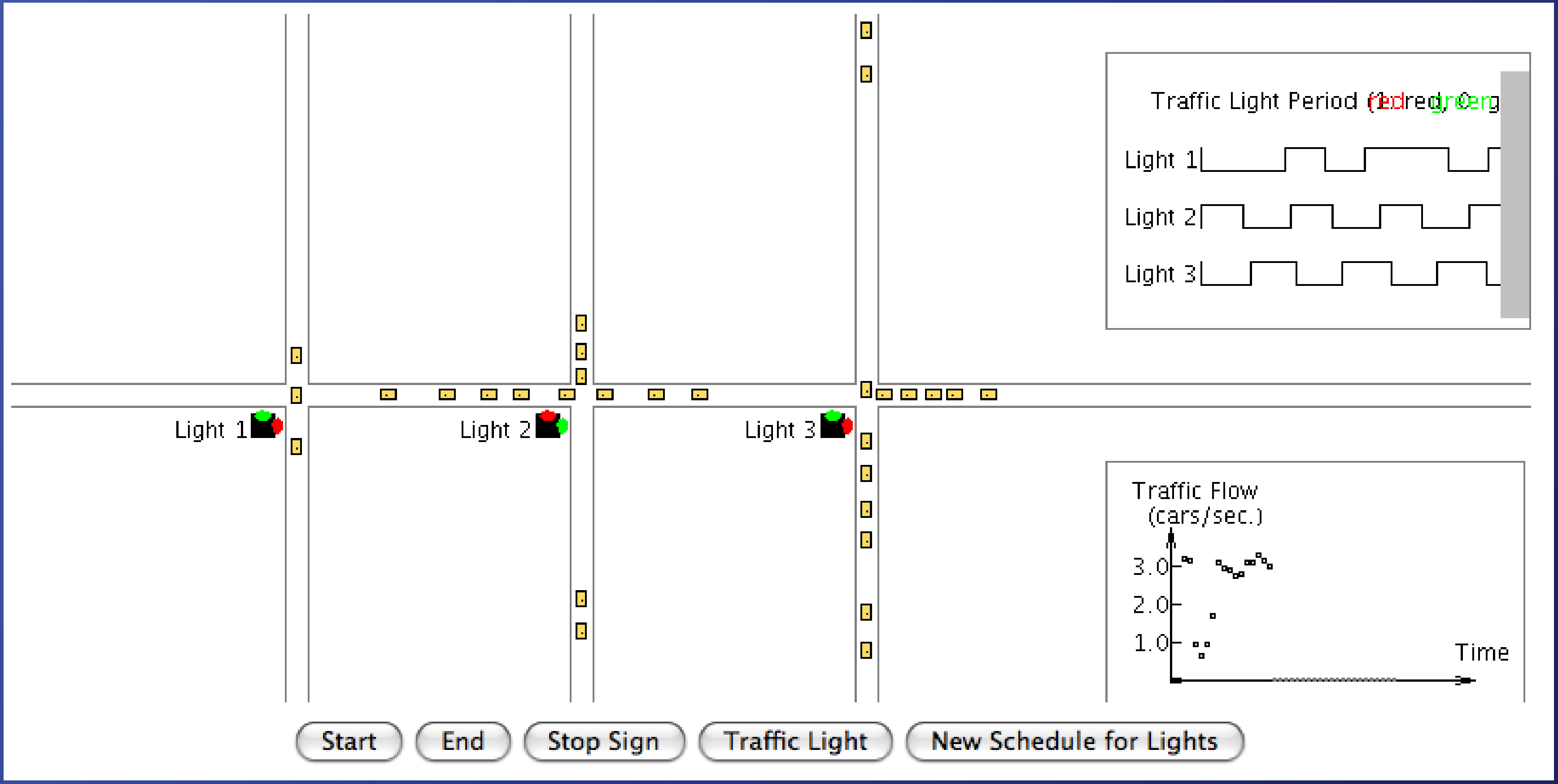
Historical Data



Traffic counts – assume that vehicles entering the system form a Poisson process at the historical flow rate, Expected proportion of vehicles turning (for CTM).

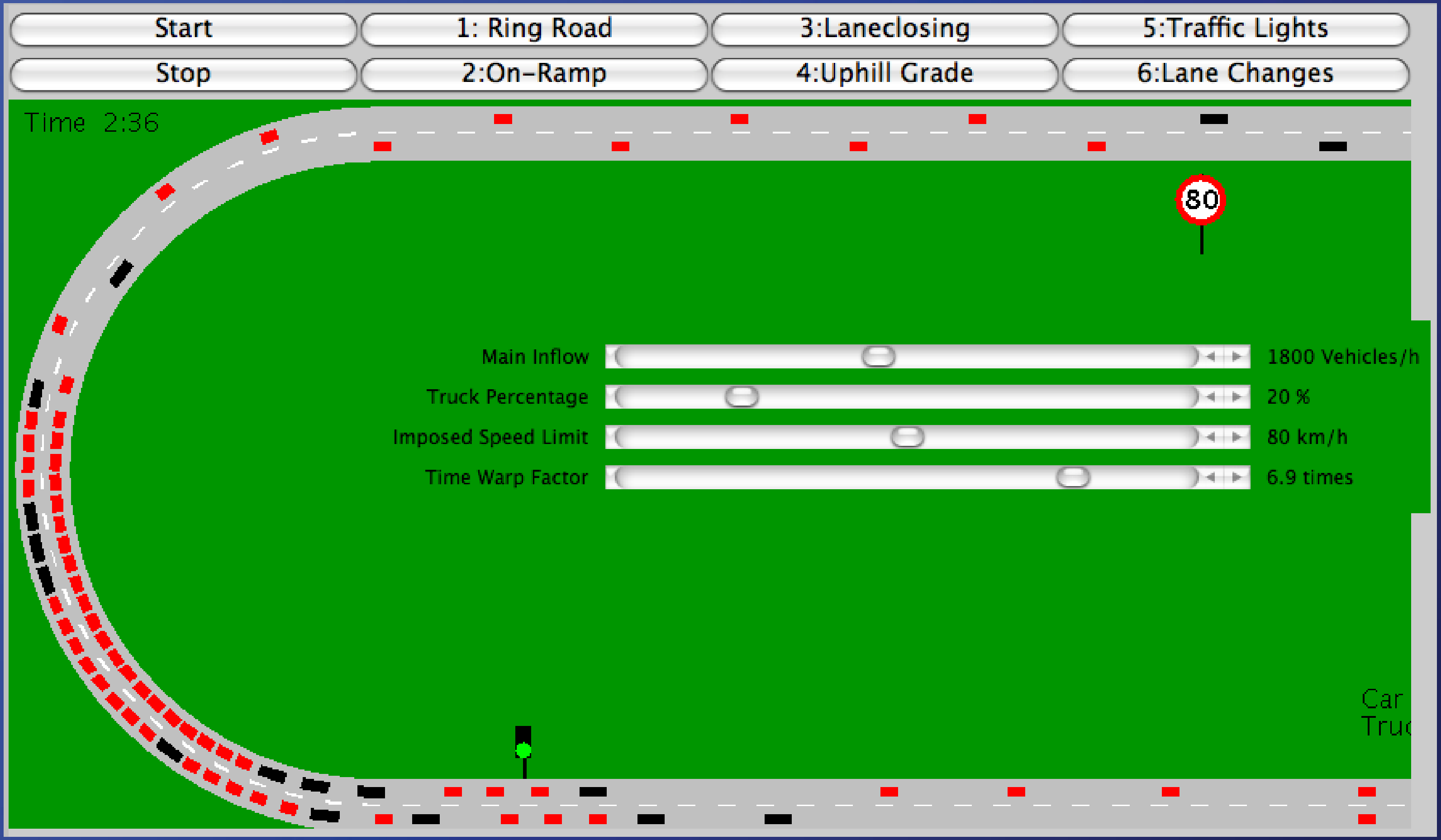
Historical data, physical measurements, and known parameters can be combined aid the numerical approximation given by the Godunov scheme for the LWR PDE to give it more meaning.

Inspiration: Signaling Application



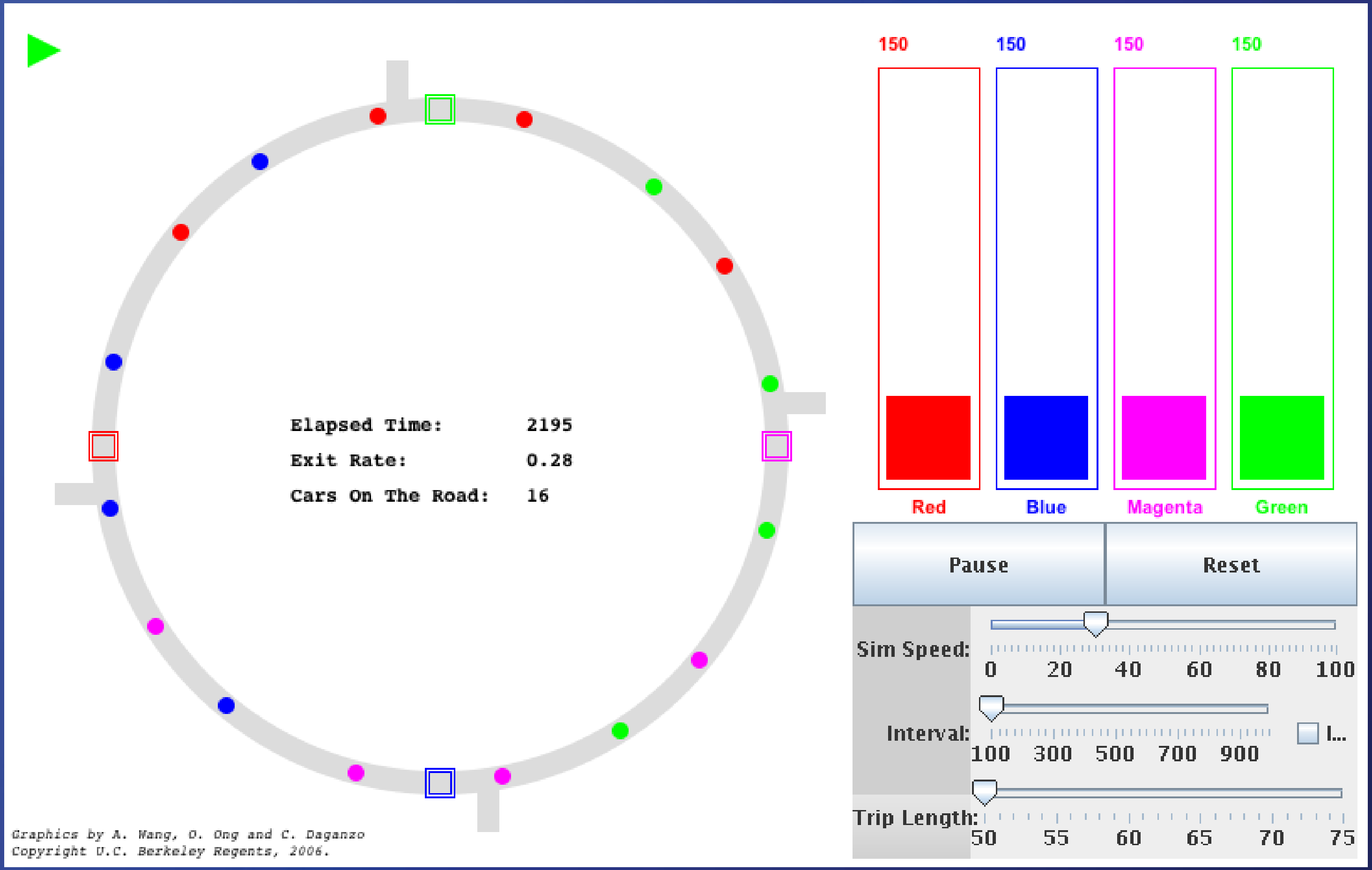
Java application by Kelly Liu
http://www.phy.ntnu.edu.tw/oldjava/Others/trafficSimulation/applet.html

Inspiration: Signal on Loop



Java application by Martin Treiber
http://www.traffic-simulation.de/

Inspiration: Urban Gridlock



Java application by A. Wang, O. Ong, and C. Daganzo
http://www.its.berkeley.edu/volvocenter/gridlock/

Remaining Tasks:

- Finalize a meaningful combination of the data with the numerical model
- Further explore the possibility of including CTM
- Obtain loop detector data
- Finishing implementing this in Java

```
class ChangeLight implements Runnable {
   int signal;
   int pauss, redpauss, greenpauss;
   Thread lighter;

   ChangeLight(){
      signal=1;
      redpauss=6000;
      greenpauss=6000;
  }

public void run() {
              signal=1;
   while (true) {
       if (signal==1){
                      signal=0;
                      pauss=greenpauss;
                  }
                  else {
                      signal=1;
                      pauss=redpauss;
                  }
       try {
       Thread.sleep(pauss);
       } catch (InterruptedException e) {
       break;
       }
   }
   }
```

References

1. C.G. Claudel, A. Hofleitner, N. Mignerey and A. M. Bayen. Guaranteed bounds on highway travel times using probe and fixed data. To appear in the 88th TRB Annual Meeting Compendium of Papers DVD, Washington D.C., January 11-15 2009. Transportation Research Board.
2. Daganzo, Carlos F., The Cell Transmission Model: Network Traffic, *California PATH*, 1994.
3. Hong K. Lo, A novel traffic signal control formulation, Transportation Research Part A: Policy and Practice, Volume 33, Issue 6, August 1999, Pages 433-448, ISSN 0965-8564, DOI: 10.1016/S0965-8564(98)00049-4.
4. J. C. Herrera and A. M. Bayen. Eulerian versus Lagrangian Sensing in Traffic State Estimation. To appear in the 18th symposium on Transportation and Traffic Theory, Hong Kong, 2009.
5. J.C. Herrera and A. M. Bayen. Traffic flow reconstruction using mobile sensors and loop detector data. In 87th TRB Annual Meeting Compendium of Papers DVD, Washington D.C., January 13-17 2008. Transportation Research Board.
6. Jun-Seok Oh, R. Jayakrishnan, and Will Recker, "Section Travel Time Estimation from Point Detection Data" (August 1, 2002). Center for Traffic Simulation Studies. Paper UCI-ITS-TS-WP-02-15.
7. Lo, H.K.; Chan, Y.C.; Chow, A.H.F., "A new dynamic traffic control system: performance of adaptive control strategies for over-saturated traffic," *Intelligent Transportation Systems, 2001. Proceedings. 2001 IEEE , vol., no., pp.404-409, 2001*
8. Lin, W.H., Ahanotu, D., 1995. Validating the basic cell transmission model on a single freeway link. University of California, Berkeley. Technical Note, UCB-ITS-PATH-TN-95-3.
9. Strub, I and A. Bayen, "Mixed Initial-boundary Value Problems for Scalar Conservation Laws: Application to the Modeling of Transportation Networks," Hybrid Systems: Computation and Cotnrol (J. Hespanha, A. Tiwari, Eds), Lecture Notes in Computer Science 3927, Springer-Verlag, March 2006, pp. 552-567