Computation of the Violin Front Response to an Excitation Generated by String Vibrations

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CE 291: Control and Optimization of Distributed Systems and Partial Differential Equations

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Outlines

- **Introduction**
- **Definition of the Studied System**
- **Splitting Method**
- **Calculation of Eigensolutions**
- **Experimental results**
- **Next steps**
- **References**

Front page image: http://neureither-luthier.com/instruments.htm

Introduction

String vibration

↓

Bridge

↙ ↘

Bass bar — Sound post

↓ ↓

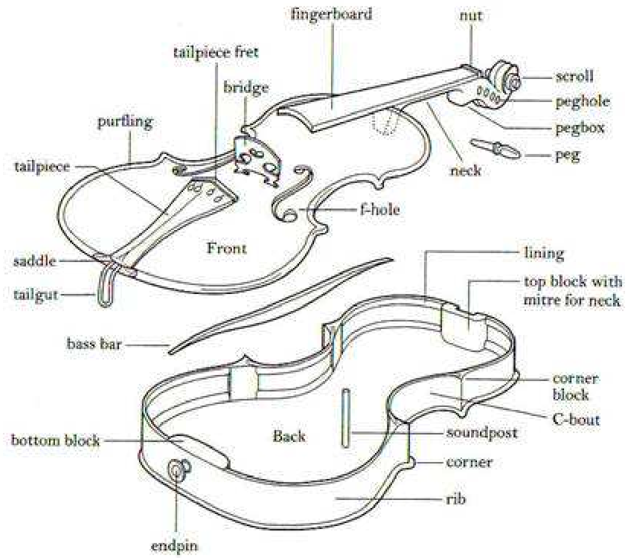
Front — Back

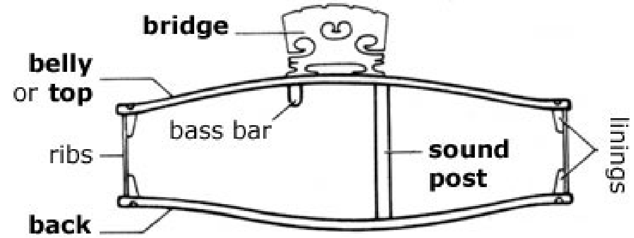
↘ ↙

Vibration of the body

↓

Sound we hear





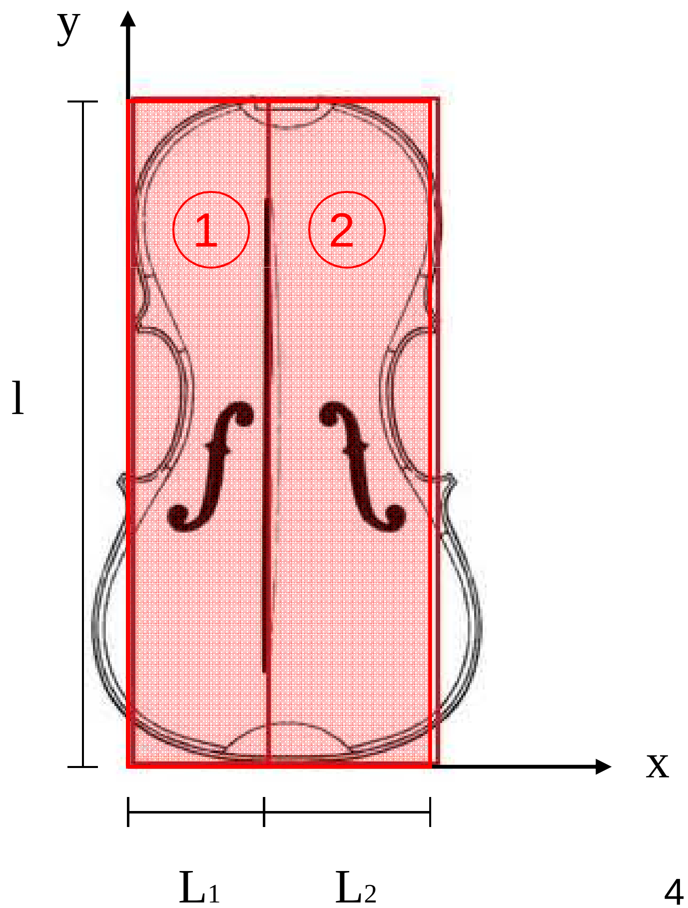
Source: http://library.thinkquest.org/27178/en/section/2/2.html

Front Table Vibration

Bass Bar

Source: http://www.chismviolins.com/photos.htm

Simplified Problem



Definition of the System

Wave equation *u(x,y,t)* through the front table:

$$a^2 \Delta u - u_{tt} = 0$$

With ***a*** *the speed of propagation of the wave (in wood)*

Boundary Conditions:

$$u(x,0,t) = 0$$

$$u(x,l,t) = 0$$

Initial Conditions:

$$u(x,y,0) = 0$$

Separation of Variables

SOV: $u(x,y,t) = U(x,y).T(t)$

The equation becomes: $a^2 \frac{\Delta U}{U} = \frac{T''}{T} = -\omega^2$

Solution of the time component: $T(t) = A\sin(\omega t) + B\cos(\omega t)$

from the initial condition

$B = 0$

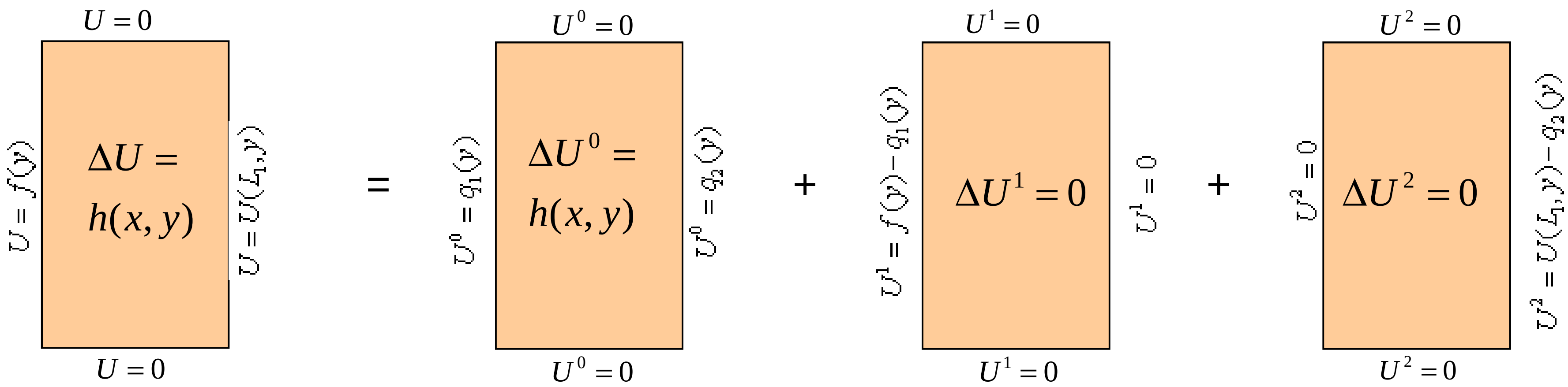
$$T(t) = A\sin(\omega t)$$

with $\boldsymbol{\omega}$ *the oscillation frequency*

Splitting Method

$$U(x,y) = U^0(x,y) + U^1(x,y) + U^2(x,y)$$

Domain ①



Sub-set 0 Sub-set 1 Sub-set 2

Sub-problem 1

SOV: $U^1(x,y) = X(x)Y(y)$

BC: $$\begin{cases} U^1(x,0) = 0 \\ U^1(x,l) = 0 \\ U^1(L_1,y) = 0 \\ U^1(0,y) = f(y) - q_1(y) = s_1(y) \end{cases}$$

Then $\Delta U^1 = U^1_{xx} + U^1_{yy} = 0$

The PDE becomes: $X''Y + XY'' = 0$

$$\frac{X''}{X} = -\frac{Y''}{Y} = \lambda^2$$

$$\begin{cases} X'' - \lambda^2 X = 0 \\ Y'' + \lambda^2 Y = 0 \end{cases}$$

Eigenfunctions

Eigenfunction in y * $Y(y) = K_1 \sin(\lambda y) + K_2 \cos(\lambda y)$

Boundary Conditions
$$\begin{cases} U^1(x,0) = 0 \;\Rightarrow\; K_2 = 0 \\ U^1(x,l) = 0 \;\Rightarrow\; \lambda_n l = n\pi \end{cases}$$

Eigenfunction in x * $X(x) = K_3 \sinh(\lambda(L_1 - x)) + K_4 \cosh(\lambda(L_1 - x))$

Boundary Conditions $U^1(L_1, y) = 0 \;\Rightarrow\; K_4 = 0$

Therefore
$$\boxed{U_n^1(x,y) = K \sin(\frac{n\pi}{l} y).\sinh(\frac{n\pi}{l}(L_1 - x))}$$

Eigensolution

Solution of the Sub-problem 1

$$U^1(x,y)=\sum_{n=1}^{+\infty}A_n^1(y).K\sin(\frac{n\pi}{l}y).\sinh(\frac{n\pi}{l}(L_1-x))$$

Boundary Condition and Orthogonality of the Y eigenfunctions

$$U^1(0,y)=s_1(y)=\sum_{n=1}^{+\infty}A_n^1(y).K\sin(\frac{n\pi}{l}y).\sinh(\frac{n\pi}{l}L_1)$$

if n ≠ m

$$\int_0^l Y_n Y_m dy=\int_0^l \sin(\frac{n\pi}{l}x)\sin(\frac{m\pi}{l}x)dy=0$$

Coefficients

$$A_n^1(y)=\frac{\int_0^l s_1(y).\sin(\frac{n\pi}{l}y)dy}{\sinh(\frac{n\pi}{l}L_1)\int_0^l \sin^2(\frac{n\pi}{l}y)dy}$$

Particular Solution

We expand the function h(x,y) in Fourier series in x

$$h(x,y) = \sum_{n=0}^{+\infty} a_n(y)\cos(\frac{2n\pi}{L_1}x) + \sum_{n=1}^{+\infty} b_n(y)\sin(\frac{2n\pi}{L_1}x)$$

with a_n and b_n coefficients:

$$\begin{cases} a_0(y) = \frac{1}{L_1}\int_0^{L_1} h(x,y)dx \\ a_n(y) = \frac{2}{L_1}\int_0^{L_1} h(x,y)\cos(\frac{2n\pi}{L_1}x)dx \\ b_n(y) = \frac{2}{L_1}\int_0^{L_1} h(x,y)\sin(\frac{2n\pi}{L_1}x)dx \end{cases}$$

Particular Solution

$$U^0(x,y) = \sum_{n=0}^{+\infty} A_n(y)\cos(\frac{2n\pi}{L_1}x) + \sum_{n=1}^{+\infty} B_n(y)\sin(\frac{2n\pi}{L_1}x)$$

A_n and B_n coefficient verify

$$\begin{cases} A_0'' = a_0(y) \\ A_n'' - (\frac{2n\pi}{L_1})^2 A_n = a_n(y) \\ B_n'' - (\frac{2n\pi}{L_1})^2 B_n = b_n(y) \end{cases}$$

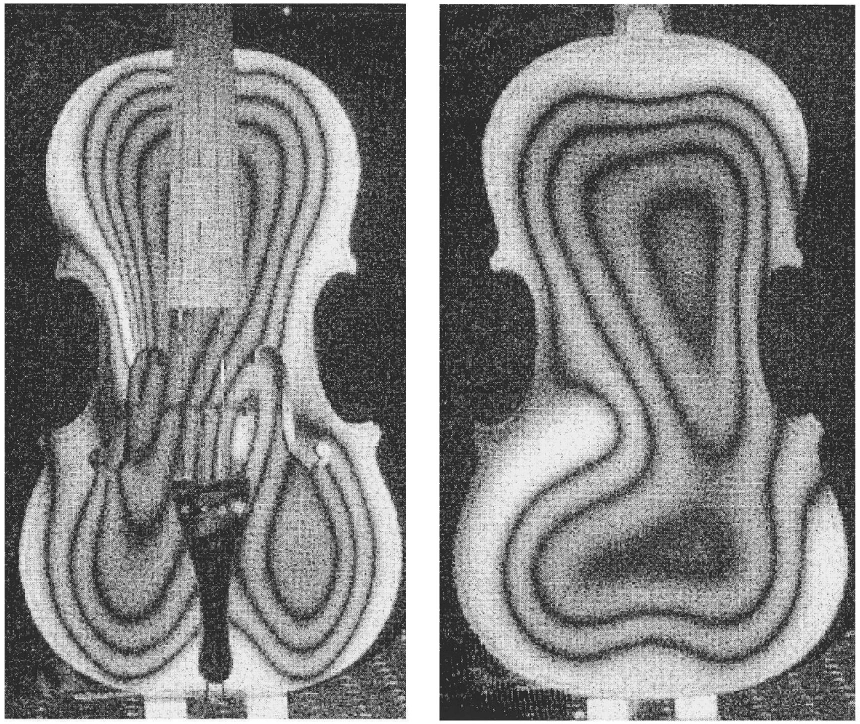
Final Solution

Final Solution of the PDE in the domain 1

$$u(x,y,t)=A\sin(\omega t)\left[\begin{aligned}&\sum_{n=0}^{+\infty}A_n(y)\cos(\frac{2n\pi}{L_1}x)+\sum_{n=1}^{+\infty}B_n(y)\sin(\frac{2n\pi}{L_1}x)\\&+\sum_{n=1}^{+\infty}A_n^1(y).K\sin(\frac{n\pi}{l}y).\sinh(\frac{n\pi}{l}(L_1-x))\\&+\sum_{n=1}^{+\infty}A_n^2(y).K'\sin(\frac{n\pi}{l}y).\sinh(\frac{n\pi}{l}x)\end{aligned}\right]$$

Experimentation

Interferograms of wall vibrations of the violin with soundpost at the second air mode A1 at 460 Hz of (c) the top plate and (d) the back plate. A loudspeaker was used to excite the violin

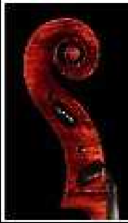


(c) (d)

Source: Vibration modes of the violin forced via the bridge and action of the soundpost (2)

Next Steps

- **Finish MATLAB code**
- **Calculate the eigensolution for the second domain**
- **Implement the eigensolution in the code and modeling the node lines**



References

1) ***Solution of Partial Differential Equations for Engineers***, (Chapter 4) Eigenfunction expansion in linear problems, W. C. Reynolds, Stanford University, Preliminary Edition – Winter 1999 version

2) ***Vibration modes of the violin forced via the bridge and action of the soundpost***, Henrik O. Saldner and Nils-Erik Molin, Lulea° University of Technology, Sweden, Erik V. Jansson, Royal Institute of Technology, Sweden, accepted 29 March 1996

3) ***On the "Bridge Hill" of the Violin***, J. Woodhouse, Cambridge University Engineering Department, accepted 7June 2004

4) ***Body Vibration of the Violin – What Can a Maker Expect to Control?***, J. Woodhouse, Cambridge University Engineering Department, May 2002

5) ***Physical modeling by directly solving wave PDE***, Marco Palumbi, Lorenzo Seno, Centro Ricerche Via Lamarmora, Roma, Italy

Questions

Thank you

Violin Concerto No1 g-moll op26 (Final), Max Bruch
Violin: Frederieke Saeijs