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Modeling and Estimation of the Humans' Effect on the CO_2 Dynamics Inside a Conference Room

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***Abstract*— We develop a data-driven, *Partial Differential Equation-Ordinary Differential Equation* (PDE-ODE) model that describes the response of the *Carbon Dioxide* (CO_2) dynamics inside a conference room, due to the presence of humans, or of a user-controlled exogenous source of CO_2. We conduct two controlled experiments in order to develop and tune a model whose output matches the measured output concentration of CO_2 inside the room, when known inputs are applied to the model. In the first experiment, a controlled amount of CO_2 gas is released inside the room from a regulated supply, and in the second, a known number of humans produce a certain amount of CO_2 inside the room. For the estimation of the exogenous inputs, we design an observer, based on our model, using measurements of CO_2 concentrations at two locations inside the room. We perform several simulation studies for the illustration of our design.**

I. INTRODUCTION

A. Motivation

The knowledge of occupancy levels in discrete zones within a building offers the potential of significant energy savings when coupled with zonal control of building services [7], [11], which is a motivation for the work presented in the present article.

A relatively unexplored approach for estimating the number of humans occupying discrete zones of office spaces, such as, for example, a conference room within a larger office space, is to model and estimate the effect of the CO_2 that is produced from humans on the total CO_2 concentration in the specific discrete zone (i.e., the conference room). The reason is that humans are the primary producers of CO_2 inside a building [25] and that CO_2 sensors are widely deployed in smart buildings (since CO_2 is an important quantity to observe in order to manage occupant comfort [25] and since this quantity can be measured using cheap sensors).

Modeling CO_2 dynamics is challenging, due to the complexity of air dynamics. Most recently, two categories of models are used: Zonal models and *Computational Fluid Dynamics* (CFD) models. CFD models provide the most rich and detailed view of air motion in a space, however, they are beset by arduous work in modeling the physical space (e.g. providing locations of all walls, furniture, and occupants) and identifying all parameters that are needed for the model. CFD models also suffer from lengthy computation times to solve the necessary PDEs at a high resolution, especially near boundaries [18], [24]. Zonal models relate the movement of air between discrete and well-mixed spaces, such as rooms and parts of rooms. Generally, zonal models rely on ODE mass-balance laws between these spaces, which, in comparison to CFD models, can be solved very quickly [18]. However, this comes at the expense of not modeling the distributed nature of airborne contaminant transfer within a single space, and complex local phenomena such as jets of air coming from a vent [19].

Yet, for designing and implementing estimation algorithms for the CO_2 concentration, one has to develop a simple, and at the same time, accurate PDE-based model that retains the distributed character of the system. Based on this model, one can then design an observer for estimating the unknown CO_2 input that is produced from humans. The observer design has to be developed using the minimum number of sensors, in order to reduce cost and increase reliability.

B. Literature

Our modeling and estimation efforts for the CO_2 produced by humans in a room lie in the general study of airborne contaminant modeling and estimation in indoor spaces. There is a wide variety of models from mass-conservation-based ODE models [22] to highly detailed CFD models for indoor airflow [8], [16]. The choice of model is often dependent on the end application and what information is available. For instance, ODE models may not operate at fine enough spatial resolution to be useful, whereas a detailed CFD model may be too complex for designing estimation algorithms. Techniques for the estimation of the concentration of contaminants emitted from a source in indoor environments exist in the literature [2], [6], [17], [23], [26], [30]. In particular [15], [28], are dealing with the estimation of CO_2 emitted by humans for the purposes of occupancy detection. Our method is unique in that we derive a simple, data-driven PDE-ODE coupled system, and recast the problem of identification of the unknown CO_2 input produced by humans as a problem of state estimation of the PDE-ODE system.

Boundary observers for some classes of PDEs are constructed in [9], [10], [12], [13], [27] via backstepping. In [20], this methodology is applied for the estimation of the state-of-charge of batteries. Observer designs for time-delay systems with unknown inputs are presented in [1].

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C. Results

We conduct two experiments (Section II). In the first, a regulated amount of CO_2 gas is released in the conference room for specific time periods while CO_2 concentrations are measured at seven different locations in the room. We use the measurements of the CO_2 concentration in order to develop a model that reproduces the measured CO_2 concentrations at the seven different locations, given the known CO_2 release. We also use the measurements from this experiment in order to identify in which sensing locations the measured CO_2 concentrations are more sensitive to an external CO_2 source. In the second experiment, we monitor the evolution of the CO_2 concentrations at three different locations in the room (the ventilation system's input and output, and at a table located in the conference room) as two researchers enter and exit the room at recorded times. The purpose is to verify the model that we develop in the first experiment under a CO_2 input that is generated by humans.

By conducting the two experiments, we aim to develop a data-driven model whose output matches the output of the actual system, when the same inputs are applied to both the model and the system, which is simple enough for estimation and identification purposes. We do not attempt to modeling the exact physical phenomena govern the dynamics of the CO_2. Yet, our model provides some insight on the actual spatially distributed dynamics of the CO_2 concentration since it is a PDE model.

We model the dynamics of the CO_2 concentration in the room using a convection PDE with a source term which is the output of a first-order ODE system driven by an unknown input which models the human's emission rate of CO_2 (Section III). The source term represents the effect of the humans on the CO_2 concentration in the room. In our experiments, we observe a delay in the response of the CO_2 concentration in the room to changes in the human's input. For this reason, the source term is a filtered version of the unknown input rather than the actual input. We assume that the unmeasured input from the humans has the form of a piecewise constant signal. This formulation is based on our experimental observation that humans contribute to the rate of change of the CO_2 concentration of the room with a filtered version of step-like changes in the rate of CO_2.

The value of the PDE at the one boundary of its spatial domain indicates the CO_2 concentration inside the room at the location of the air supply. At this location, incoming air is entering the room, and hence, one can view the CO_2 concentration of the fresh incoming air as an input to the system. The value of the PDE at the other boundary of its spatial domain indicates the CO_2 concentration at the air return of the ventilation system. The air at this point is mixed with CO_2 that convects from the air supply towards the air return, and with CO_2 that is produced from humans. We consider the CO_2 concentration at this point as the output of our system. Any value of the PDE on an interior point of its spatial domain is an indicator of the concentration of CO_2 at the ceiling in a (non-ratiometric) normalized distance along an axis from the supply to the return vent.

Fig. 1. The office space under study.

We design an observer for the overall PDE-ODE system using boundary measurements, at the air supply and the air return (Section IV). The observer estimates the unknown input from the humans, as well as the overall PDE state of our model. Our observer design and the proof of exponential stability of the observation error is based on the observer design from [4] for linear systems with distributed sensor delays.

Notation: The spatial $L_2(0,1)$ norm is denoted by $\|\cdot\|$.

II. Experimental Set-Up

Our experimental work takes place in a $44\,\mathrm{m}^3$ conference room, shown in Fig. 1. The room is completely interior within the building and has no outside walls. On the ceiling there is one air supply vent with a diffuser and protective grate, and an air return vent with a protective grate.

We measure CO_2 concentration using the K-30 Sensor Module [14] which comes with specifications of $\pm 30\,\mathrm{ppm} \pm 3\%$ accuracy and repeatability of $\pm 20\,\mathrm{ppm} \pm 1\%$. We expect the nominal CO_2 concentrations of the room to be no more than $1500\,\mathrm{ppm}$, which gives a repeatability error bound of $\pm 35\,\mathrm{ppm}$.

Constant errors in CO_2 readings are corrected by a method called *Baseline Correction*. Essentially, sensor readings are taken over a time period and the lowest concentration seen during that period is assumed to be $400\,\mathrm{ppm}$, corresponding the outdoor air concentration (i.e., the steady-state value if the room is ventilated and no humans are present). The sensor itself performs this correction automatically over a seven and a half day interval. We manually perform this correction by operating all of the sensors overnight, then subtracting an offset from each data set so that the minimum readings from each sensor equaled each other. This ensures that all of our sensor readings are using the same baseline, even if this baseline is up to $(-30\,\mathrm{ppm} - 400\,\mathrm{ppm}\cdot 3\%) = -32\,\mathrm{ppm}$ away from the real value.

A. Experiment I: Controlled CO_2 Release

In the first experiment, we have two goals:

- The first goal is to examine the spatial dependence of CO_2 concentration in the room, in particular how well-mixed the air is. If there is a spatial dependence, we would like to identify the sensor which exhibits the most dependence on CO_2 generation in the room.

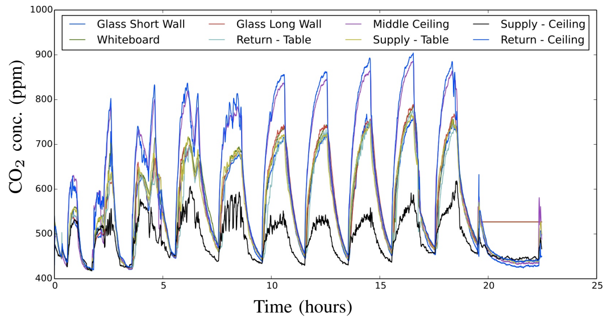

Fig. 2. CO_2 concentrations during Experiment I. Showing measurements from all 7 locations over the approximately 22 hour experiment.

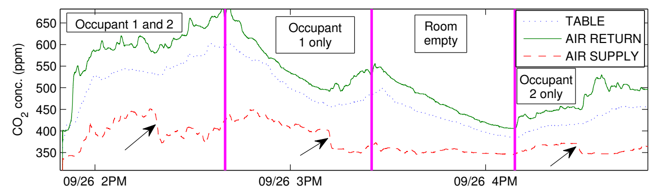

Fig. 3. CO_2 concentrations during Experiment II. Showing measurements from 3 locations in the conference room over 3 hour experiment. Magenta lines indicate when occupancy changes occurred. The arrows indicate the time instants at which the ventilation rate increases[2].

- The second goal is to collect data that can be used for manual or automatic identification of the parameters of a model whose output matches the measured data, when the same CO_2 input is applied to the model and the conference room.

Therefore, our testing methodology is to add a controlled disturbance of CO_2 into the room and measure the resulting response on the sensors placed in the room.

The disturbance input consists of beverage grade (99.9% purity) CO_2 gas being released via a flow regulator at approximately 2 CFM, and passed through a small 200 W personal heater, to simulate warm breath. A mechanical timer is used to switch the regulator and heater on and off with a 2 hr period (1 hr on, 1 hr off).

We deploy a total of fifteen CO_2 sensors in the conference room at eight different locations. At seven of the eight locations, two CO_2 sensors are co-located for redundancy in case hardware failure made a reading invalid. We do not encounter any hardware failures during the experiment, so we instead take the mean of redundant measurements.

Fig. 2 gives the sensor readings from this test. When the CO_2 injection is turned off, all of the measurements settle to a steady-state value, which is almost the same for all sensors. However, when CO_2 injection begins, we see clear spatial differences in CO_2 concentrations. During injection, the highest concentrations of 900 ppm are seen by sensors placed at the air return vent and sensors placed on the ceiling at the midpoint between the supply and return vents. The lowest concentrations are seen at the supply vent, which stays below 600 ppm. All of the other sensors, which are placed between chest and waist level in the room, exhibit similar behavior in response to the CO_2 injection. In general, besides transient behavior due to ventilation turning on and off, the CO_2 concentrations from different points in the room react the same, albeit with different magnitudes.

From this experiment, we conclude that, when CO_2 is being generated in the room, the concentration of CO_2 local to the air supply represents a mixture of the room's CO_2 concentration and that of the fresh air (about 400 ppm). Other than at the supply vent, we observe that there are large variations on the CO_2 concentration between points at the ceiling and points at table height. This is explained by the fact that a warm breath from a human occupant acts as a "bubble" of gas that rises to the ceiling, since it is more buoyant than the ambient, cooler air. Also, we observe that there are smaller variations on the CO_2 concentration between different points at the ceiling.

We also conclude that, of all the sensors, the measurements most affected by the production of CO_2 are those taken at the air return vent. Therefore, these measurements will be most useful to observe and perform system identification with.

B. Experiment II: Release of CO_2 from Humans

In the second experiment, the goal is to determine the effect of real human occupancy on the concentration of CO_2 in the room. For this experiment, three CO_2 sensors are deployed: one each at the air supply and return vents, and one on the conference table at the center of the room. Our excitation procedure consists of adding or removing one of two participants of the experiment, and noting the time that the occupancy changes. Fig. 3 is a plot of the data gathered from this experiment in which occupancy transitions occur.

From this plot, we can see the general trend that CO_2 concentration at the conference room table and at the return vent increases when occupants arrive and decreases when occupants leave the room. We also conclude that the concentration at the air supply vent is much less dependent on occupancy. This can be attributed to the constant fresh air ventilation that is provided by building services, so that fresh air concentration dominates the concentration in the area near the vent.

We can also see an interesting effect starting at the approximate times of 1:30PM, 2:20PM and 3:40PM, where the CO_2 measurement at the air supply sharply drops and corresponds to a rise in CO_2 in the other two measurements. We hypothesize this is due to the ventilation rate increasing. Near to the supply vent, a greater quantity of fresh air would mix with the air near the sensor, driving the concentration down. A higher air velocity in the room will also impart more turbulent mixing of pockets of CO_2 concentration within the room, pushing them out of the air return and increasing the concentration at that point. The mixing of these pockets also causes an increase in the CO_2 concentration near the table.

III. Model of the CO_2 Dynamics

Our model consists of a PDE and an ODE part. The ODE part is given by

$$\dot{X}(t) = -aX(t) + V(t) \quad (1)$$
$$\dot{V}(t) = 0, \quad (2)$$

where, X, in ppm, models the source term of human CO_2 production on the relative concentration (in ppm) of the room in the local vicinity of the human (the evolution of which is described later on by a PDE), and V is a step-valued function, in ppm $\cdot$ s^{-1}, representing the level of the human CO_2 production rate within the vicinity of humans. Parameter, $\frac{1}{a}$, in 100s, represents a time constant specifying how fast changes in occupancy affect the CO_2 concentration in the room, in the local vicinity of the human.

The ODE is coupled with a PDE that models the CO_2 concentration in the room given by

$$u_t(x,t) = bu_x(x,t) + b_X X(t) \quad (3)$$
$$u(0,t) = U_\mathrm{e} - \Delta U(t), \quad (4)$$

with $\Delta U(t) = U(t) - U_\mathrm{e}$, where $u(x,t)$, in ppm, is the concentration of CO_2 in the room at a time $t \geq 0$ s and for $0 \leq x \leq 1$, $-b > 0$, in $\frac{1}{100\mathrm{s}}$, represents the rate of air movement in the room, and $b_X > 0$, in $\frac{1}{10^4\mathrm{s}}$, specifies the rate of diffusion of CO_2 from the local vicinity of the human to the room. The spatial variable x is unitless and represents a normalized distance along a horizontal axis that connects the air supply and air return. The air supply and air return are located at $x = 0$ and $x = 1$ respectively. Therefore, $u(0,t)$ is the CO_2 concentration inside the room at the location of the air supply and $u(1,t)$ is the CO_2 concentration inside the room at the location of the air return. The input $U(t)$ is the measured ppm concentration of the fresh incoming air. We do not simply specify the boundary condition at $x = 0$ as $u(0,t) = U(t)$. The reason is that during our experiments we observe that a sudden drop in the measured CO_2 concentration at the air supply results in an increase of the CO_2 concentration at the air return. Our explanation for this effect is that a drop in CO_2 concentration at the supply from its equilibrium value corresponds to increased airflow at the vent, i.e. more fresh air gets mixed in the local vicinity. The increased airflow has the effect of pushing pockets of CO_2 air out of the return vent. One way to capture this effect is to multiply the difference of the CO_2 concentration from its equilibrium value $\Delta U(t) = U(t) - U_\mathrm{e}$ with minus one, where U_e, in ppm, is the steady state input CO_2 concentration at the supply ventilation.

In Fig. 4, we illustrate the geometrical representation of our model. The PDE portion of the model, u, represents convection of air from the air supply to the air return vent near the ceiling. Note the absence of a diffusive term, which we have omitted since it plays a relatively minor role in dispersing indoor pollutants [3]. We model the CO_2 concentrations near the ceiling since this is where we see most effect from human-generated CO_2 (Section II). This is explained by the fact that a warm breath from a human occupant acts as a "bubble" of gas that rises to the ceiling, since it is more buoyant than the ambient, cooler air. Thus, the air coming from lower in the room is modeled as a source term on the PDE across its entire length. The ODE portion of the model is intended to model the fact that this bubble of air does not immediately rise to the ceiling but only gradually.

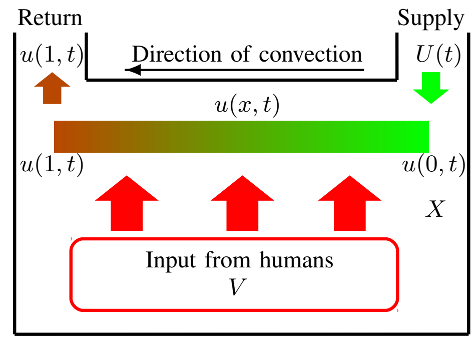

Fig. 4. Geometrical representation of our model. Fresh air (U) enters the room from the supply vent. Air near the ceiling (u) convects from the air supply to the air return. Humans produce CO_2 (V) rising (X) to the ceiling.

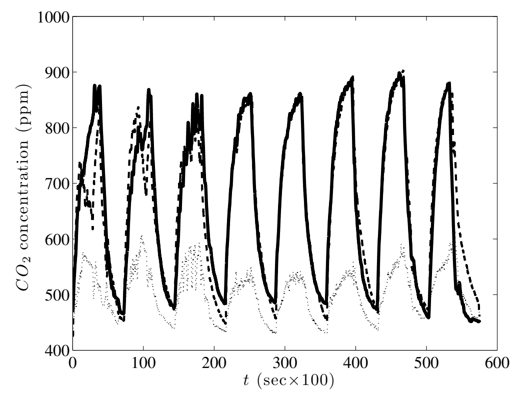

Fig. 5. Solid: The simulated concentration of CO_2 at the air return $u(1,t)$ given by the model (1)–(4) for Experiment I. Dashed: The concentration of the CO_2 at the air return measured by the CO_2 sensor. Dotted: The concentration of CO_2 at the air supply measured by the CO_2 sensor.

In Fig. 5 we show the CO_2 concentration at the air return and air supply, measured by the CO_2 sensors for Experiment I, in which we periodically release CO_2 every one hour. We also show the model's output $u(1,t)$ with parameters from Table I[3] and initial condition $u(x,0) = 400$ ppm. The input V to our model, with which we emulate the behavior of the CO_2 released from the pump, is the square wave in Fig. 6.

In Fig. 7 we show the CO_2 concentration from Experiment II measured from the CO_2 sensor and predicted from

[2] We assume that the air ventilation rate increased at the marked points because an increased proportion of fresh air (typically 400 ppm) would cause a drop in the supply CO_2 concentration. The validity of this hypothesis could be verified by measuring the flow of incoming air at the supply with an anemometer.

[3] In this section we manually tune the parameters of model (1)–(4) in order to match the measured CO_2 concentration at the air return with $u(1,t)$.

TABLE I

PARAMETERS OF THE MODEL (1)–(4) FOR EXPERIMENTS I, II.

Physical Paramater	Model parameter	Value (I)	Value (II)
Convection coefficient $\left(\frac{1}{100\mathrm{s}}\right)$	$-b$	0.8	0.16
Source term coefficient $\left(\frac{1}{10^4\mathrm{s}}\right)$	b_X	0.2	0.2
Time constant of the human's effect (100s)	$\frac{1}{a}$	10	10
Equilibrium concentration at the air return (ppm)	U_e	450	370

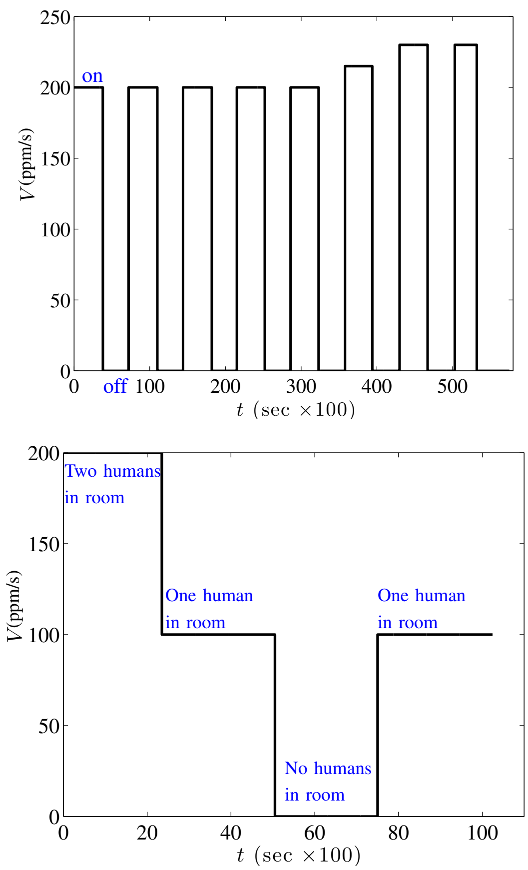

Fig. 6. Top: The input V to the model (1)–(4) from Experiment I modeling the concentration of CO_2 that is released from the pump. When $V = 0$ the CO_2 pump is turned off and when $V \neq 0$ the CO_2 pump is turned on. Bottom: The input V to the model (1)–(4) from Experiment II modeling the input concentration of CO_2 from the humans.

model (1)–(4) with parameters shown in Table I, initial condition $u(x, 0) = 400$ ppm, and input V that is shown in Fig. 6 (bottom), with which we emulate the behavior of the CO_2 produced by humans.

IV. ESTIMATION OF THE HUMANS' EFFECT

We construct an observer for the plant (1)–(4) assuming measurements of $u(1, t)$ and $U(t)$. We assume that the parameters of the model are known since they can be manually identified (as in Section III).

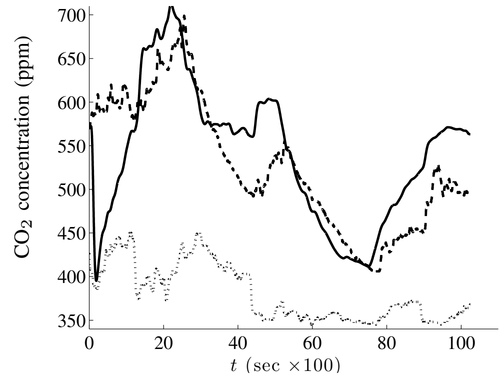

Fig. 7. Solid: The simulated concentration of the CO_2 at the air return $u(1, t)$ given by the model (1)–(4) for Experiment II. Dashed: The concentration of the CO_2 at the air return measured by the CO_2 sensor. Dotted: The concentration of CO_2 at the air supply measured by the CO_2 sensor.

A. Observer Design

Our observer is a copy of the plant plus output injection

$$\hat{u}_t(x,t) = b\hat{u}_x(x,t) + b_X\hat{X}(t) + p(x)\left(u(1,t) - \hat{u}(1,t)\right) \quad (5)$$

$$\hat{u}(0,t) = -U(t) + 2U_\mathrm{e} \quad (6)$$

$$\dot{\hat{X}}(t) = -a\hat{X}(t) + \hat{V}(t) + L_1\left(u(1,t) - \hat{u}(1,t)\right) \quad (7)$$

$$\dot{\hat{V}}(t) = L_2\left(u(1,t) - \hat{u}(1,t)\right). \quad (8)$$

The following corollary follows from Theorem 2 in [4].

Corollary 1: Consider system (1)–(4) and the observer (5)–(8) with

$$p(x) = L_1\gamma_1(x) + L_2\gamma_2(x) \quad (9)$$

$$\gamma_1(x) = \frac{b_X}{a}\left(e^{-\frac{a}{b}x} - 1\right) \quad (10)$$

$$\gamma_2(x) = -\frac{b_X}{ba}x + \frac{b_X}{a^2}\left(1 - e^{-\frac{a}{b}x}\right). \quad (11)$$

Let $b_X \neq 0$ and choose L_1, L_2 such that $A - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} C$, where $A = \begin{bmatrix} -a & 1 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} \gamma_1(1) & \gamma_2(1) \end{bmatrix}$, is Hurwitz. Then for any $u(x,0), \hat{u}(x,0) \in L_2(0,1)$, $X(0), \hat{X}(0), V(0), \hat{V}(0) \in \mathbb{R}$, there exist positive constants κ and λ such that the following holds for all $t \geq 0$

$$\Omega(t) \leq \kappa\Omega(0)e^{-\lambda t} \quad (12)$$

$$\Omega(t) = \int_0^1 \left(u(x,t) - \hat{u}(x,t)\right)^2 dx + \left(X(t) - \hat{X}(t)\right)^2 + \left(V(t) - \hat{V}(t)\right)^2. \quad (13)$$

B. Simulations

We test our observer design for the model (1)–(4). We apply the input U that is measured from the sensor and the input V which is shown in Fig. 6 for each of the two experiments. We choose $\hat{u}(x, 0) = 400$, for all $x \in [0, 1]$, $\hat{X}(0) = \hat{V}(0) = 0$, and $L_1 = 9.5$, $L_2 = 4$, such that the matrix $A - \begin{bmatrix} L_1 & L_2 \end{bmatrix}^T C$ has two eigenvalues at -1, for the parameters of the model of Experiment I, and at $-0.8012 + 0.3976i$ for Experiment II. In Fig. 8 we show the

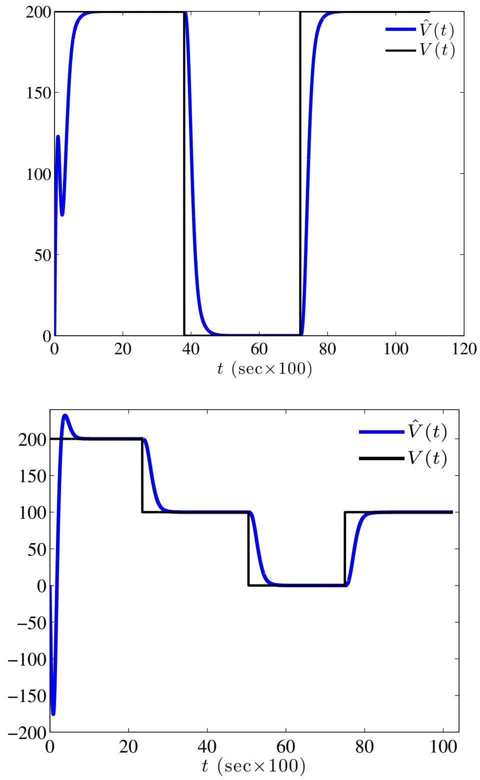

Fig. 8. Top: Estimation $\hat{V}$ (blue line) of the pump input V (black line) in Fig. 6 for Experiment I. Bottom: The estimation $\hat{V}$ (blue line) of the input V (black line) in Fig. 6 produced from the people for Experiment II.

estimation of the input V from the pump and produced by the people, which converge to the true value of V.

V. Conclusions

Future work will address the problem of estimation of the actual human occupancy level using CO_2 measurements. This is a highly non-trivial problem because humans' CO_2 generation rates vary widely between different persons depending on current activity, diet, and body size [24].

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References

[1] S. Amin, X. Litrico, S. Sastry, and A. Bayen, "Cyber security of water SCADA systems: (II) Attack detection using enhanced hydrodynamic models," *IEEE Transactions on Control Systems Technology* , vol. 21, pp. 1679–1693, 2012.

[2] A. Bastani, F. Haghighat, J. A. Kozinski, "Contaminant source identification within a building: toward design of immune buildings,", *Building and Environment*, vol. 51, pp. 320–329, 2012.

[3] A. Baughman, A. Gadgil, and W. Nazaroff, "Mixing of a point source pollutant by natural convection flow within a room," *Indoor Air*, vol. 4, pp. 114–122, 1994.

[4] N. Bekiaris-Liberis and M. Krstic, "Lyapunov stability of linear predictor feedback for distributed input delays", *IEEE Transactions on Automatic Control*, vol. 56, pp. 655–660, 2011.

[5] J. S. Bendat, A. G. Piersol, *Engineering Applications of Correlation and Spectral Analysis*, Wiley, 1980.

[6] H. Cai, X. Li, Z. Chen, L. Kong, "Multiple indoor constant contaminant sources by ideal sensors: A Theoretical model and numerical validation," *Indoor and Built Environment*, vol. 22, pp. 897–909, 2013.

[7] C. Chao and J. Hu, "Development of a dual-mode demand control ventilation strategy for indoor air quality control and energy saving", *Building and Environment*, vol. 39, pp. 385–397, 2004.

[8] Q. Chen, "Ventilation performance prediction for buildings: A method overview and recent applications", *Building and Environment*, vol. 44, pp. 848–858, 2009.

[9] F. Di Meglio, R. Vazquez, M. Krstic, "Stabilization of a system of $n+1$ coupled first-order hyperbolic linear PDEs with a single boundary input," *IEEE Trans. Autom. Contr.*, vol. 58, pp. 3097–3111, 2013.

[10] F. Di Meglio, D. Bresch-Pietri and U. J. F. Aarsnes, "An adaptive observer for hyperbolic systems with application to underbalanced drilling," *IFAC World Congress*, Cape Town, 2014.

[11] V. L. Erickson, M. Carreira-Perpinan, and A. E. Cerpa, "Observe: Occupancy-based system for efficient reduction of HVAC energy", *IEEE Conf. on Information Processing in Sensor Networks*, 2011.

[12] M. Krstic and A. Smyshlyaev, *Boundary Control of PDEs: A Course on Backstepping Designs*, SIAM, 2008.

[13] M. Krstic, A. Smyshlyaev, "Backstepping boundary control for first order hyperbolic PDEs and application to systems with actuator and sensor delays," *Systems & Control Letters*, vol. 57, pp. 750–758, 2008.

[14] K-30 10,000 ppm CO_2 sensor, 2013. http://co2meters.com/Documentation/Datasheets/DS30-01%20-%20K30.pdf.

[15] K. P. Lam, M. Hoynck, B. Dong, B. Andrews, Y. Chiou, R. Zhang, D. Benitez, J. Choi, "Occupancy detection through an extensive environmental sensor network in an open-plan office building", *IBPSA Building Simulation*, pp. 1452–1459, 2009.

[16] Y. Li and P. V. Nilsen, "Commemorating 20 years of Indoor Air: CFD and ventilation research", *Indoor Air*, vol. 21, pp. 442–453, 2011.

[17] X. Liu and Z.-J. Zhai, "Protecting a whole building from critical indoor contamination with optimal sensor network design and source identification methods", *Building and Environment*, vol. 44, pp. 2276–2283, 2009.

[18] A. C. Megri and F. Haghighat, "Zonal modeling for simulating indoor environment of buildings: Review, recent developments, and applications," *HVAC & R Research*, vol. 13, pp. 887–905, 2007.

[19] L. Mora, A. Gadgil, and E. Wurtz, "Comparing zonal and CFD model predictions of isothermal indoor airflows to experimental data," *Indoor Air*, vol. 13, pp. 77–85, 2003.

[20] S. J. Moura, N. A. Chaturvedi, and M. Krstic, "PDE Estimation Techniques for Advanced Battery Management Systems - Part I: SOC Estimation," *American Control Conference*, 2012.

[21] S. Moura, M. Krstic, and N. Chaturvedi, "Adaptive PDE observer for battery SOC/SOH estimation via an electrochemical model," *ASME J. Dynamic Syst., Measurem., and Contr.*, vol. 136, paper 011015, 2014.

[22] W. W. Nazaroff and G. R. Cass, "Mathematical modeling of indoor aerosol dynamics", *Environmental Science & Technology*, vol. 23, pp. 157–166, 1989.

[23] S.-T. Parker and V. Bowman, "State-space methods for calculating concentration dynamics inmultizone buildings", *Building and Environment*, vol. 46, pp. 1567–1577, 2011.

[24] A. K. Persily, "Evaluating building IAQ and ventilation with indoor carbon dioxide," *Transactions of American Society of Heating Refrigerating and Air Conditioning Engineers*, vol. 103, pp. 193–204, 1997.

[25] O. Seppanen, W. Fisk, M. Mendell, "Association of ventilation rates and CO_2 concentrations with health and other responses in commercial and institutional buildings," *Indoor Air*, vol. 9, pp. 226–252, 1999.

[26] M. D. Sohn, P. Reynolds, N. Singh, A. J. Gadgil, "Rapidly locating and characterizing pollutant releases in buildings," *Journal of the Air and Waste Management Association*, vol. 52, pp. 1422–1432, 2002.

[27] A. Smyshlyaev, M. Krstic, "Backstepping observers for a class of parabolic PDEs," *Systems & Contr. Letters*, vol. 54, pp. 613–625, 2005.

[28] S. Wang and X. Jin, "CO_2-based occupancy detection for on-line outdoor air flow control", *Indoor and Built Environment*, vol. 7, pp. 165–181, 1998.

[29] K. Weekly, N. Bekiaris-Liberis, A. M. Bayen, "Modeling and estimation of the humans' effect on the CO_2 dynamics inside a conference room," 2014. http://arxiv.org/abs/1403.5085.

[30] T. F. Zhang and Q. Chen, "Identification of contaminant sources in enclosed environments by inverse CFD modeling", *Indoor Air*, vol. 17, pp. 167–177, 2007.