Cancellation of Acoustic Waves

CE 291 F | Control and Optimization of Distributed Parameters Systems

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Outline

- Introduction
- Project steps
  - 1D wave cancellation
  - 2D wave cancellation
- Upcoming work
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Introduction

- Interference: when two waves meet
- Destructive interference: when two waves meet and cancel each other
- Why reduce, or cancel, acoustic waves?
  - Reduce unwanted sound (background noise, mufflers, machines)
  - Reduce transmission of vibration energy, decrease wear on machine components
  - Health and psychological effects (hearing loss, loss of sleep, stress, etc.)

Project Steps

- 1D wave equation cancellation
- 2D wave cancellation approaches
  - Analytical
  - Differential flatness
- 2D wave cancellation scenarios
  - Equidistant input and control
  - Input and control are not equidistant from the desired point of noise cancellation


[1]

1D wave cancellation

- 1D wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$$

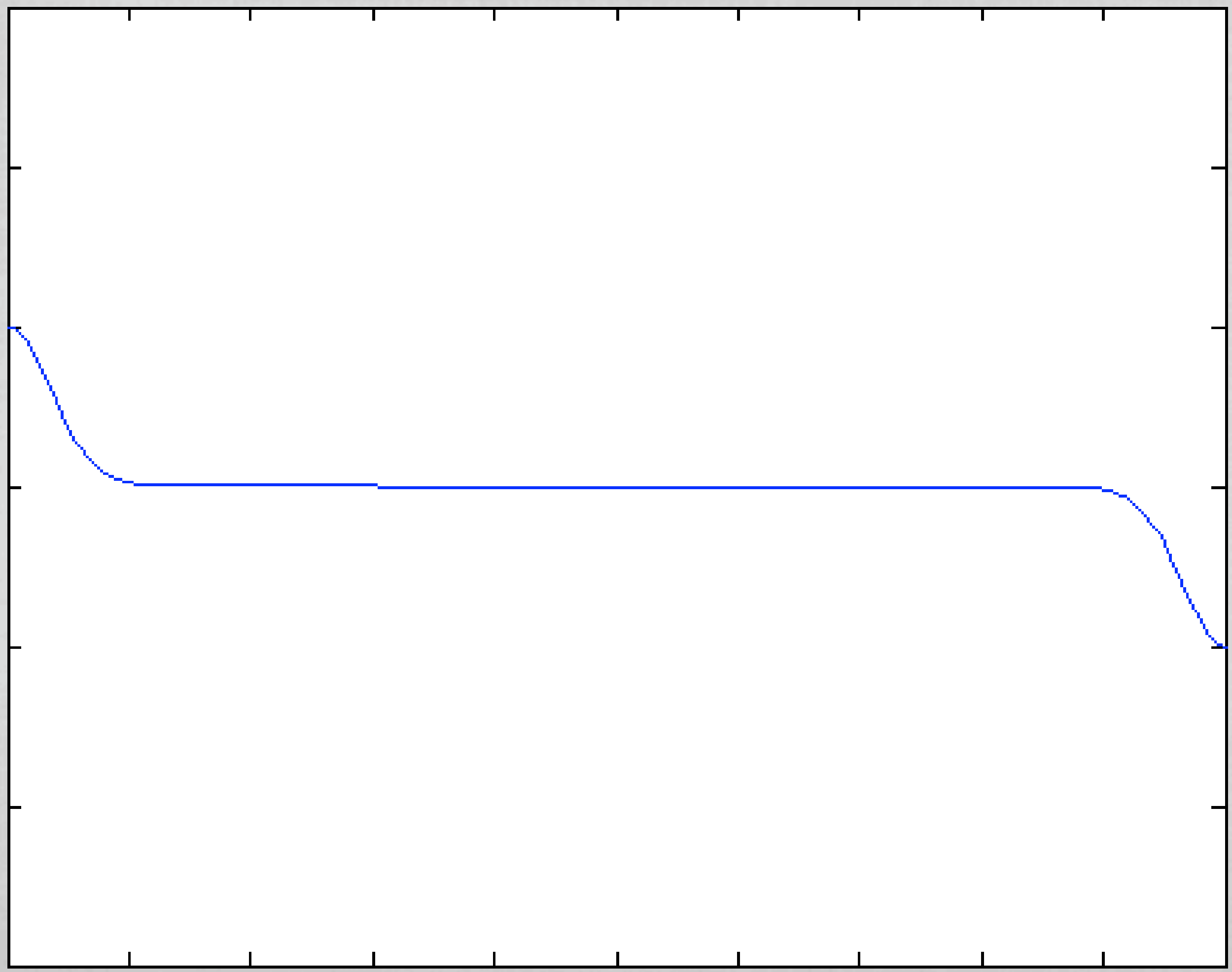
- Solve using method of characteristics:

$$u(x,t) = F(x + ct) + G(x - ct)$$

- Solution for canceling wave is intuitive:

$$u(x,t) = -F(L - x + ct) - G(L - x - ct)$$

1D wave cancellation



2D wave cancellation

- 2D wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$$

- Transform into cylindrical coordinates and solve using separation of variables:

$$u(r,\theta,t) = BJ_\alpha\left(\frac{\omega r}{c}\right)\cos(\alpha\theta)\cos(\omega t)$$

where B is an unknown constant, ω is frequency, and α is an integer

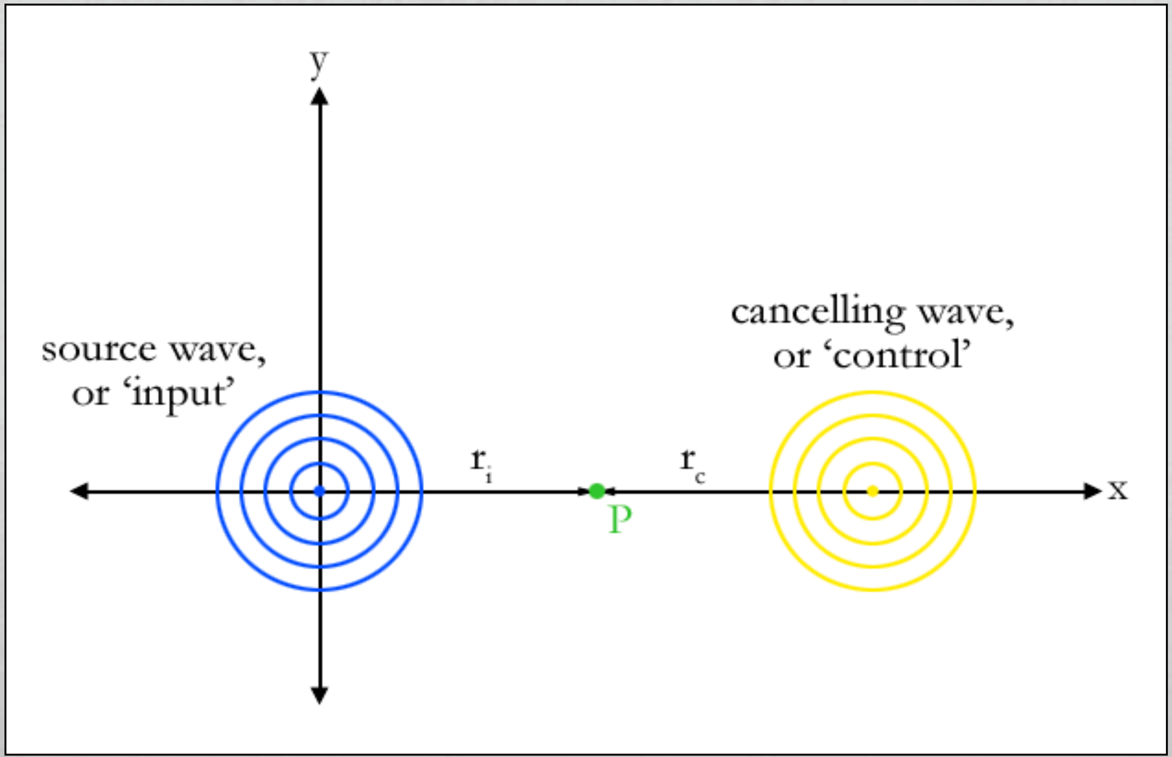
BC:

$$\lim_{r\to\infty} u(r,\theta,t) = 0$$

2D wave cancellation

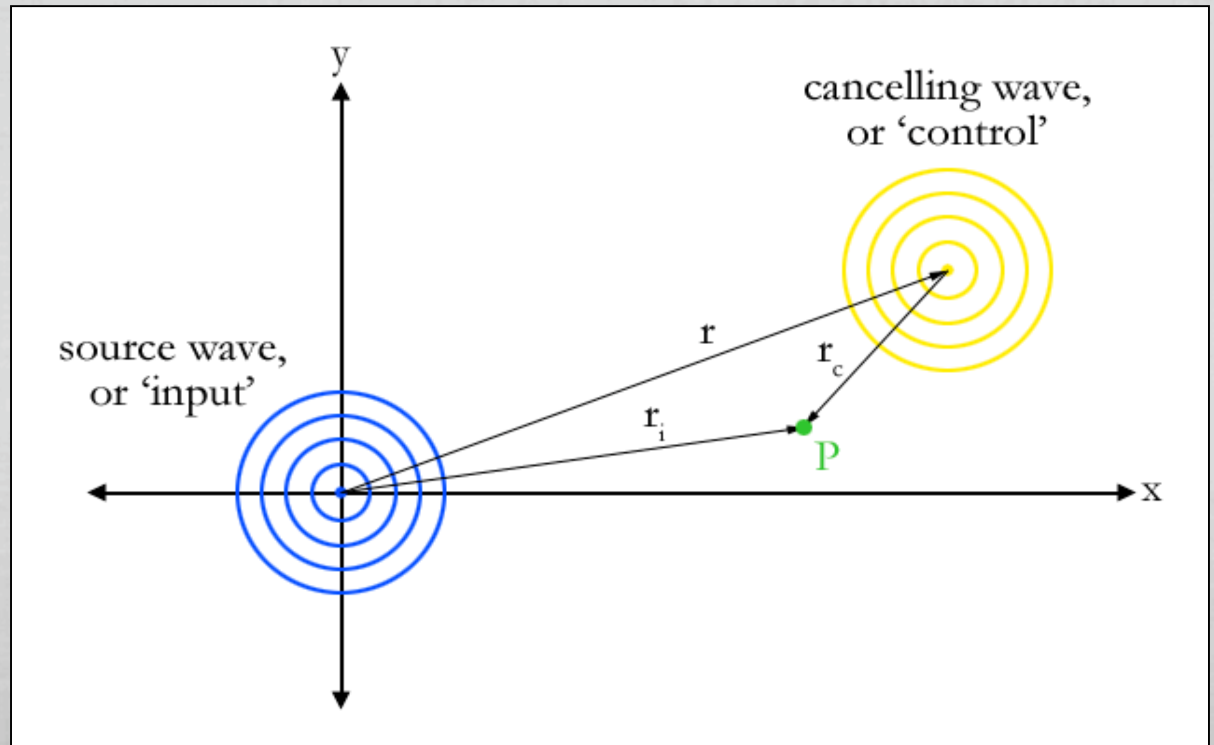
- For equidistant input and control, solution is intuitive, i.e.

$$u(r_c, \theta_c, t) = -u(r_i, \theta_i, t)$$



Upcoming work

- Solve 2D wave cancellation problem for sources not equidistant



Upcoming work

- Solve for the 'controlling' wave
- Will use solution in the following form:

$$u(r,\theta,t) = \sum_{i=0}^{\infty} \alpha_i J_i\left(\frac{\omega r}{c}\right)\cos(i\theta)\cos(\omega t)$$

- Would like the following condition to be true:

$$u(r,\theta,t) = u(r_i,\theta_i,t) + u(r_c,\theta_c,t) = 0$$

- Calculate eigenfunction coefficients, eigensolutions, plot the results

Upcoming work

- Canceling a human wave

[2]

References

[1] www.bonluxat.com/a/jurgen-bey-ear-chair.html

[2] http://cache.viewimages.com/xc/495122.jpg?v=1&c=ViewImages&k=2&d=17A4AD9FDB9CF193CEF9A2866663FA43C6CD391E83164853284831B75F48EF45

[3] Reynolds, W.C. Course Reader for ME 200B. Solution of Partial Differential Equations for Engineers. Stanford University. Preliminary Ed. – Winter 1999 version. Chapters 3, 6.

[4] Bayen, Alexandre. Course Reader for EE 291c/ME 236/CE 291F. Control and optimization of distributed systems and partial differential equations. Fall 2007 version. Chapters 3, 7.

Questions & Comments?