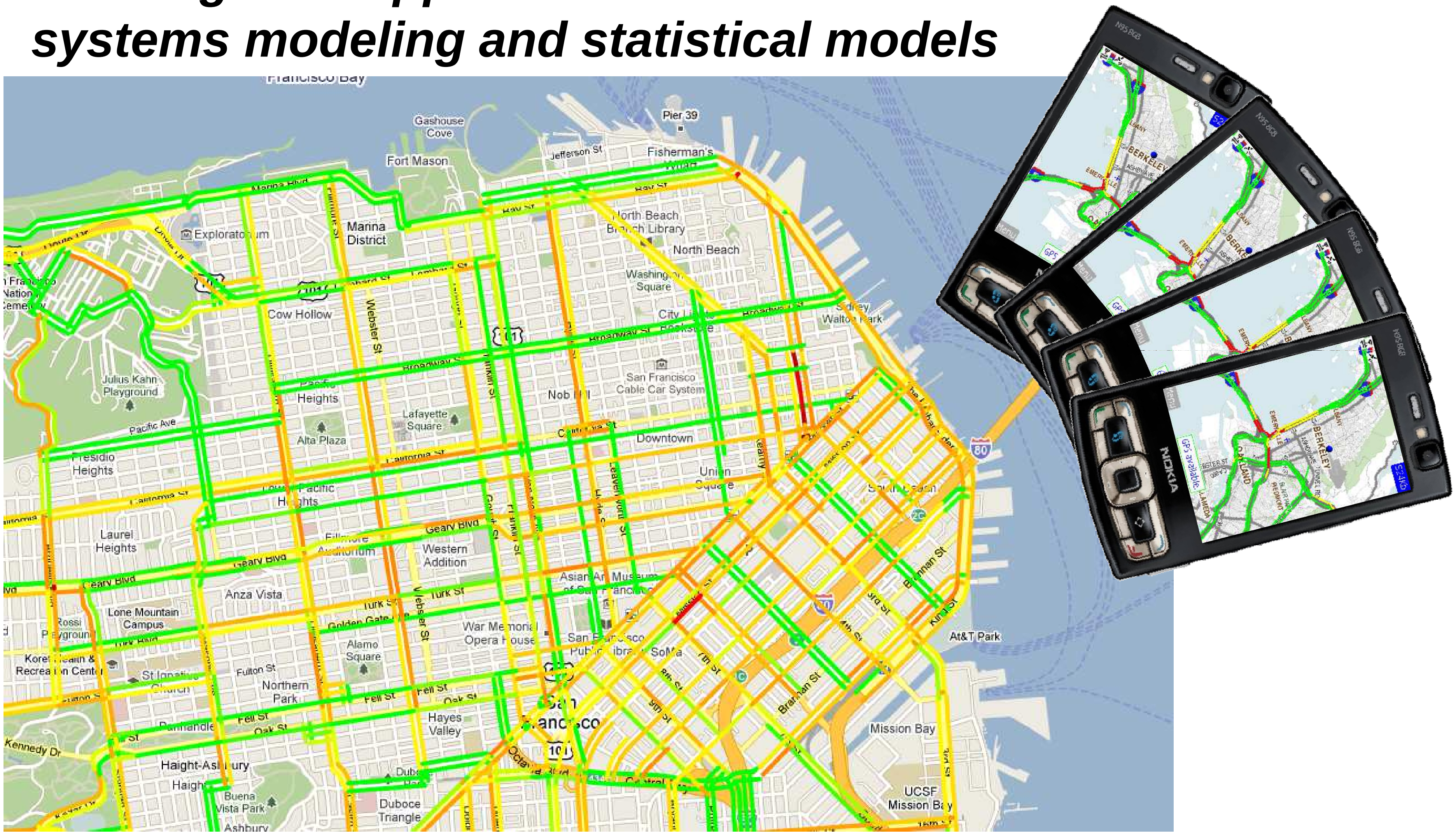
Arterial traffic

An integrated approach of distributed systems modeling and statistical models

http://traffic.berkeley.edu



EE 291 (Spring 2010) – Aude Hofleitner

Recall: distributed systems modelling

Lighthill-Whitham-Richards partial differential equation

- Nonlinear first order hyperbolic scalar conservation law
- Concave flux function (empirical fundamental diagram)
- Weak boundary conditions

[Lighthill-Whitham, 1955; Richards, 1956]

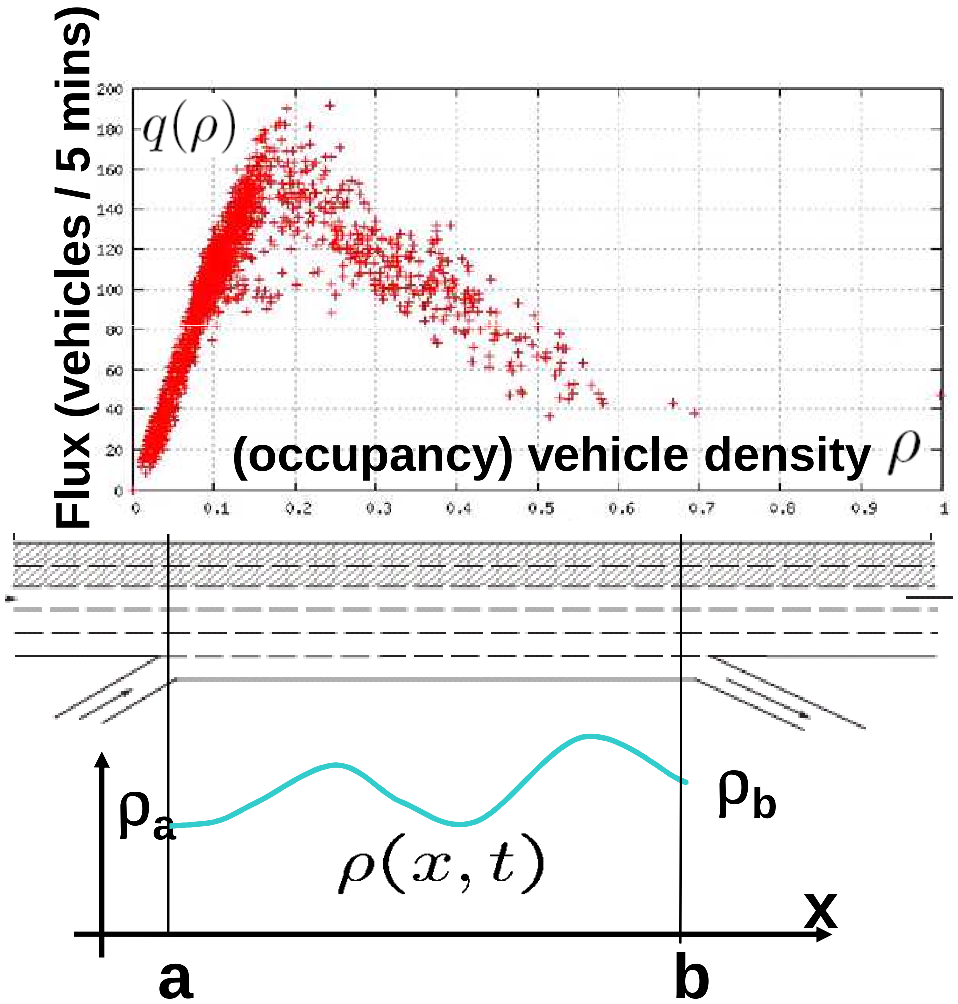
$\rho(x,t)$ **is the vehicle density.**

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

$$\rho(a,t) = \rho_a(t)$$

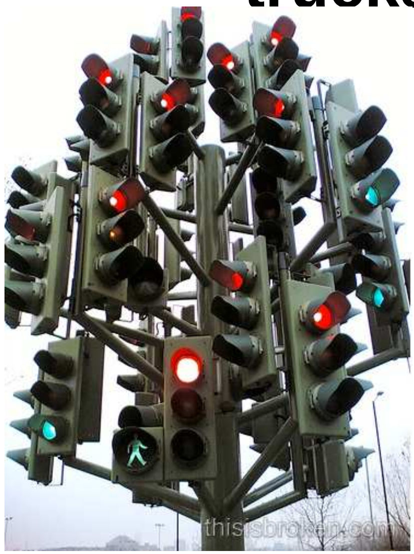
$$\rho(b,t) = \rho_b(t)$$

$$\rho(x,0) = \rho_0(x)$$



In the case of arterial traffic

- A lot of **unknown** and highly **variable** parameters
  - **Traffic lights**
  - **Pedestrians**
  - **Bad parking**
  - **Delivery trucks…**
  - **Capacity of the road**
  - **Different traffic flows (bikes, trucks…)**



An integrated approach of distributed systems and statistical models

1. **Model the dynamics of the distributed system (assumptions, LWR equation…)**

2. **Define a set of independent parameters P describing the model (often have physical interpretations)**

3. **Derive the probability distributions of the state variables (density, velocity…) parameterized by P**

4. **When you observe data, estimate the parameters by maximizing the probability to observe the data: maximize the likelihood**

   If you observe a lot of long travel times, long delays, the road is likely congested…

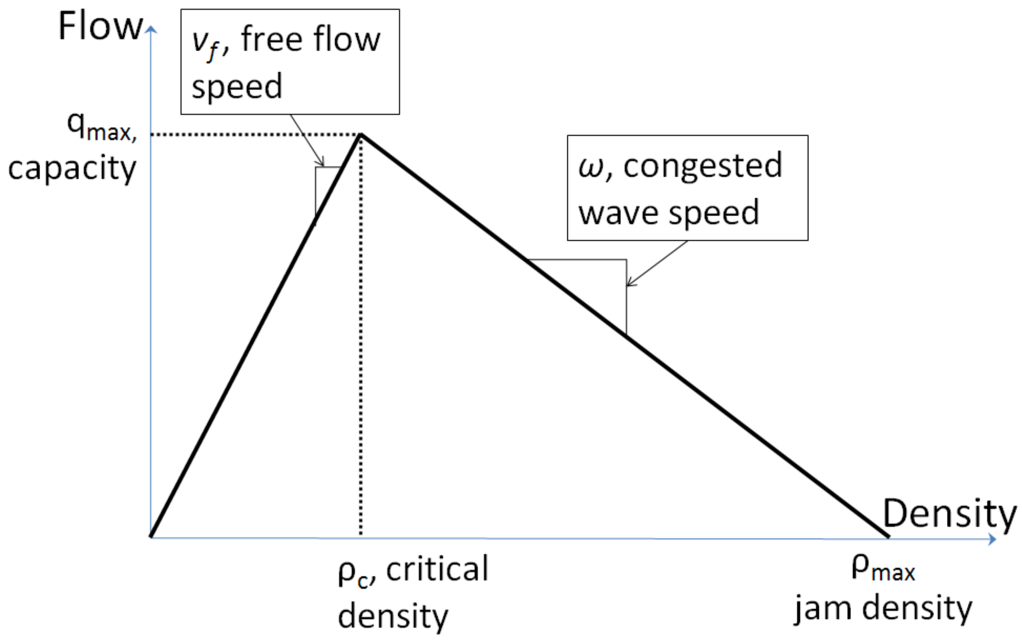
Step 1: Assumptions and model dynamics

- Neglect overtaking
- Stationarity of traffic conditions (constant arrival rate, fixed cycle timing, no constant increase or decrease of queue lengths)

- Triangular fundamental diagram
- Conservation of vehicles

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

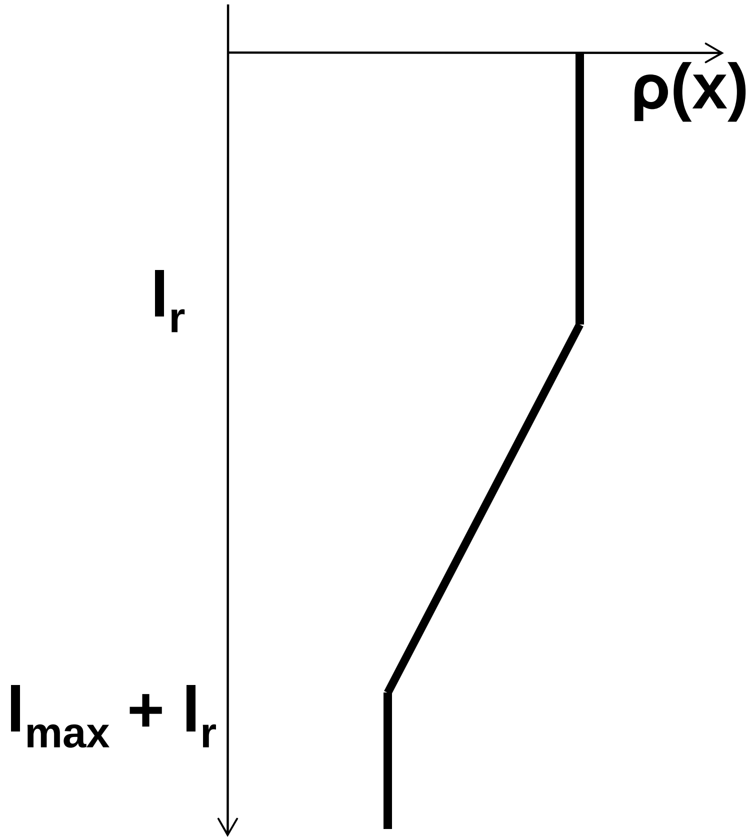
$$\rho(a,t) = \rho_a(t)$$
$$\rho(b,t) = \rho_b(t)$$
$$\rho(x,0) = \rho_0(x)$$

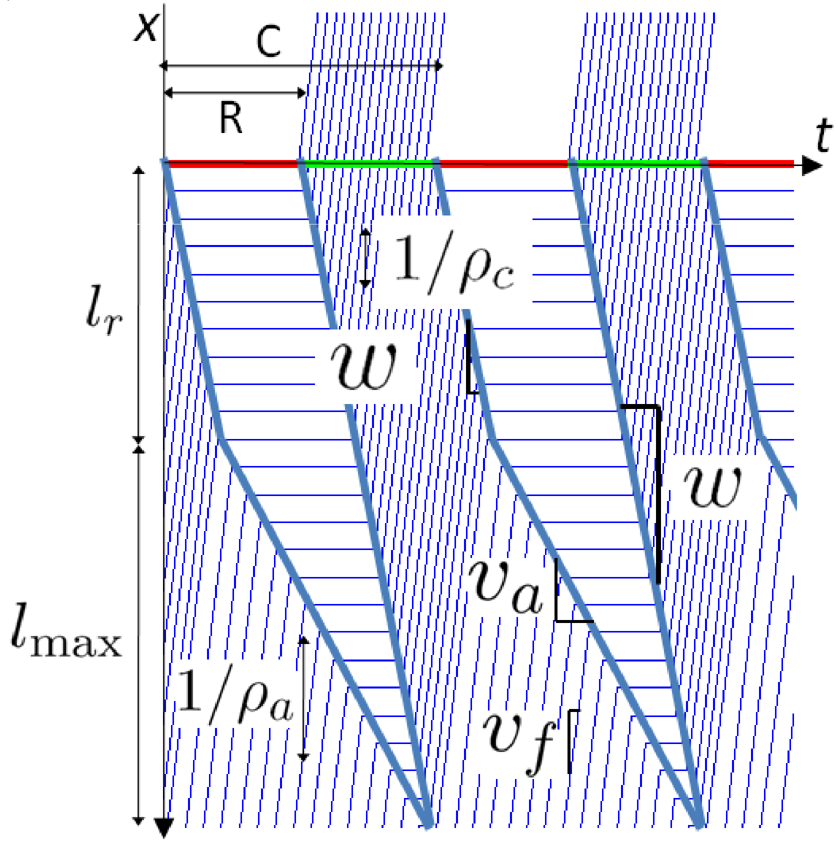


Step 2: Define the relevant parameters of the model

- Cycle timing (red and cycle time)
- Traffic conditions (arrival rate, queue length, clearing time)
- Driving behavior (free flow speed, distribution of free flow speed)

- Undersaturated regime
- Congested regime





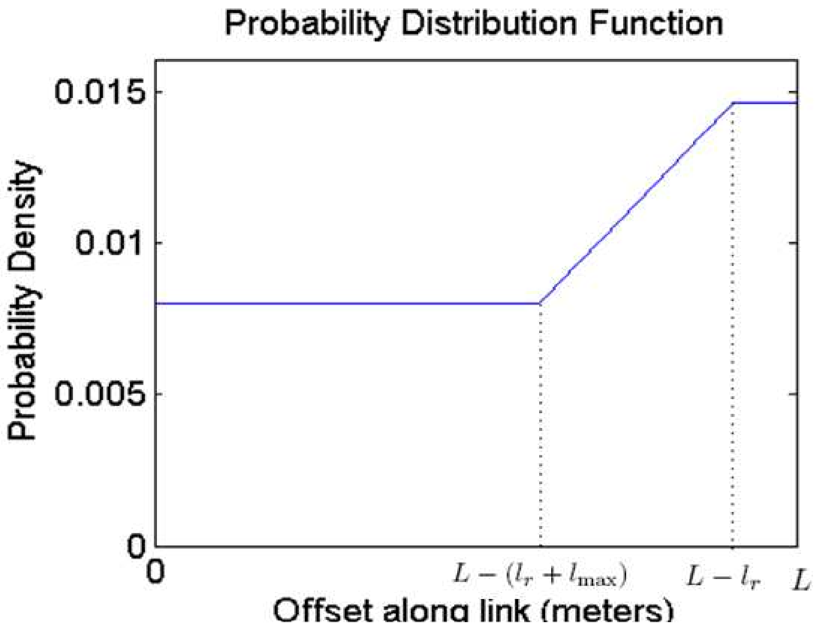
Step 3: Model the distribution of state variables

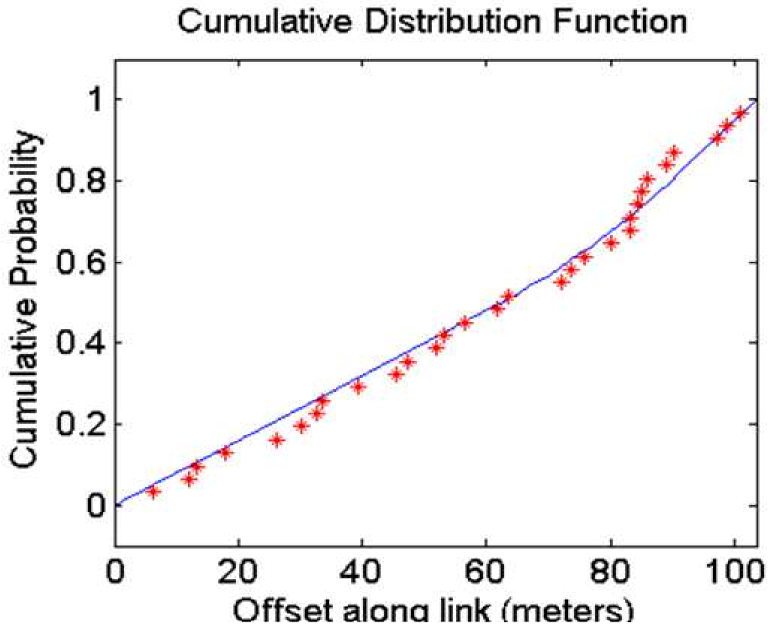
Variable of interest: **density** **at location x, averaged over time**

➔ **Probability distribution of observing a car at location x is proportional to the average density (normalizing constant)**

➔ **Spatial heterogeneity of the travel time on arterials**

➔ **Depends on the network and model parameters**





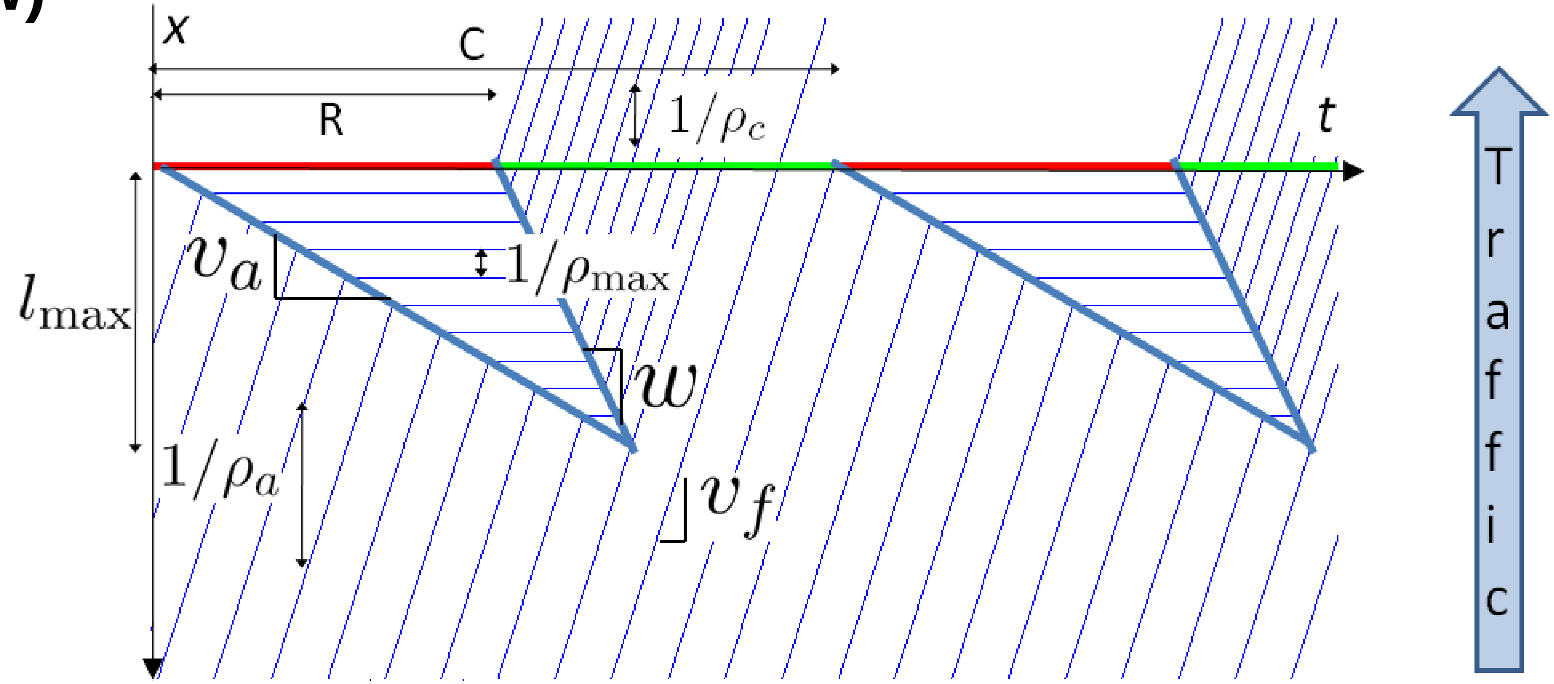
Step 3: Model the distribution of state variable

***Variable of interest*: travel time. Depends on:**

- **the delay experienced,**
- **the *driving behavior***

➔ **Conditional probability distribution of travel time for a given free flow speed for a given set of parmaters**

➔ **Integrate over the free flow speeds (total probability law)**



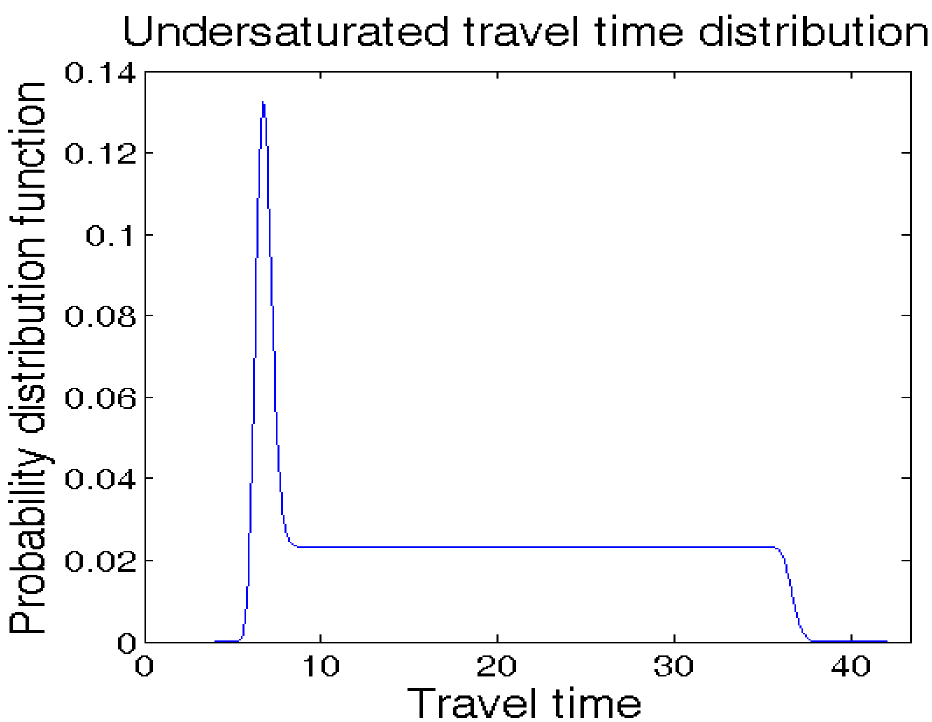
Step 3: Model the distribution of travel times

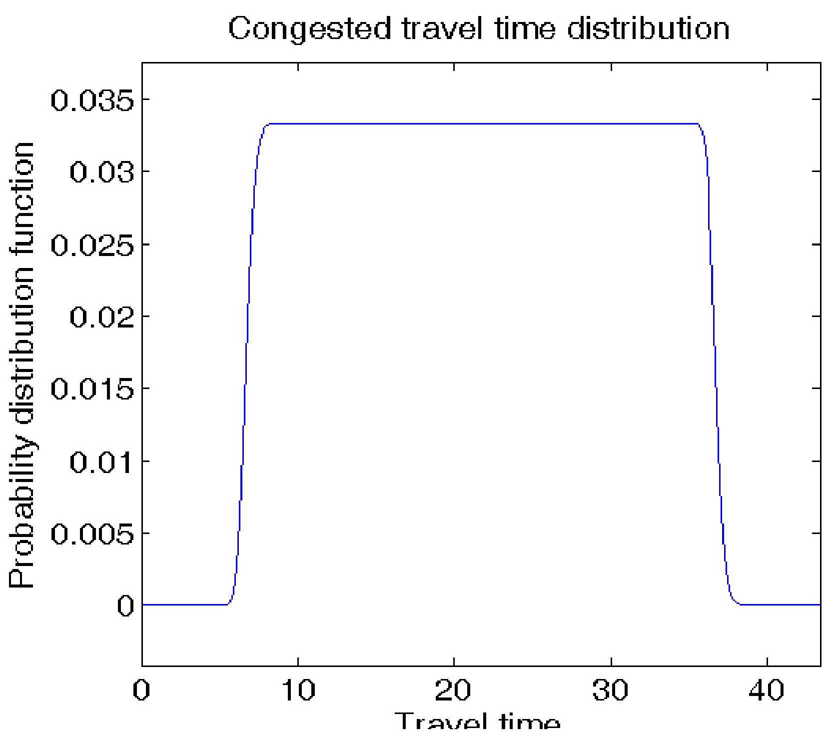
Parametric travel time distributions, depends on

- **Origin a and destination b**
- **Parameters of the model**

The distributions are quasi-concave (sublevel sets are convex).

Proof not detailed here





Step 4: estimate the parameters with data

The likelihood function:

- **Probability of the observations conditioned on the value of the parameters,**

Maximize the log-likelihood:

- **Optimization problem**
- **Might have constraints dictated by the physics of the problem**

  e.g. bounds on the parameters, constraint on signals sharing an intersection, stationarity assumptions…

➔ **Best estimate of the parameters, given the observed data**

Step 4: estimate the parameters with data

- **Solve the constrained optimization problem**

$$\underset{\rho_a,\rho_{\max},\rho_c,\ \bar{v}_f, R, C, l_r, \alpha, \sigma^2}{\text{maximize}} \quad \sum_i \ln(f(x_i))$$

$$\text{s.t} \quad \rho_a \leq \rho_c(1 - R/C)$$
$$l_r + v_f(C - R)\rho_c/\rho_{\max} \leq L$$

The constraints are given by the physics of the model (stationarity, cycle timing, queue length…)

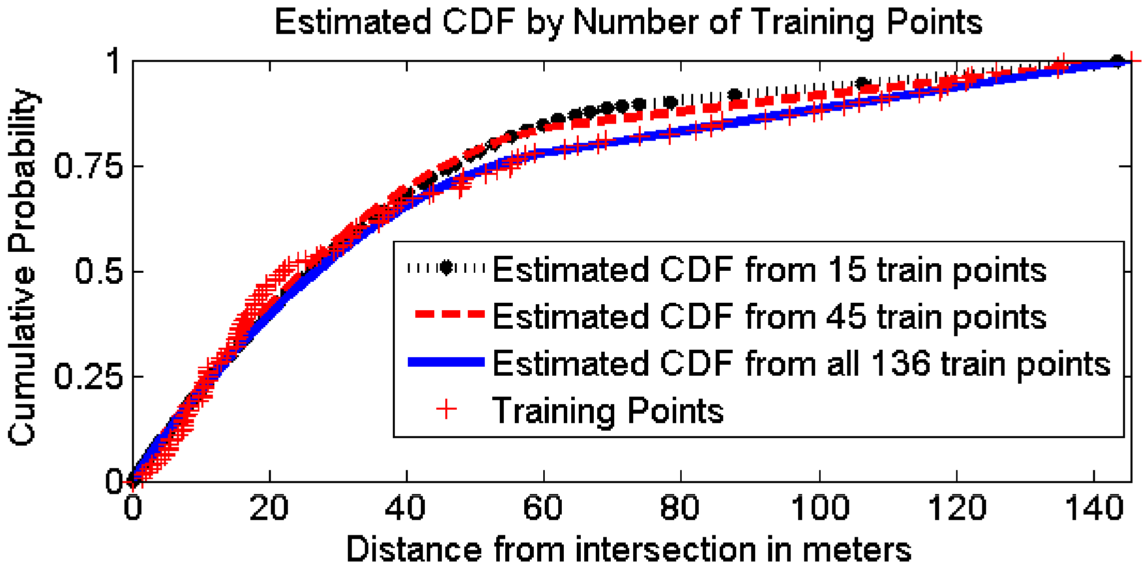
➔Estimation of the road parameters

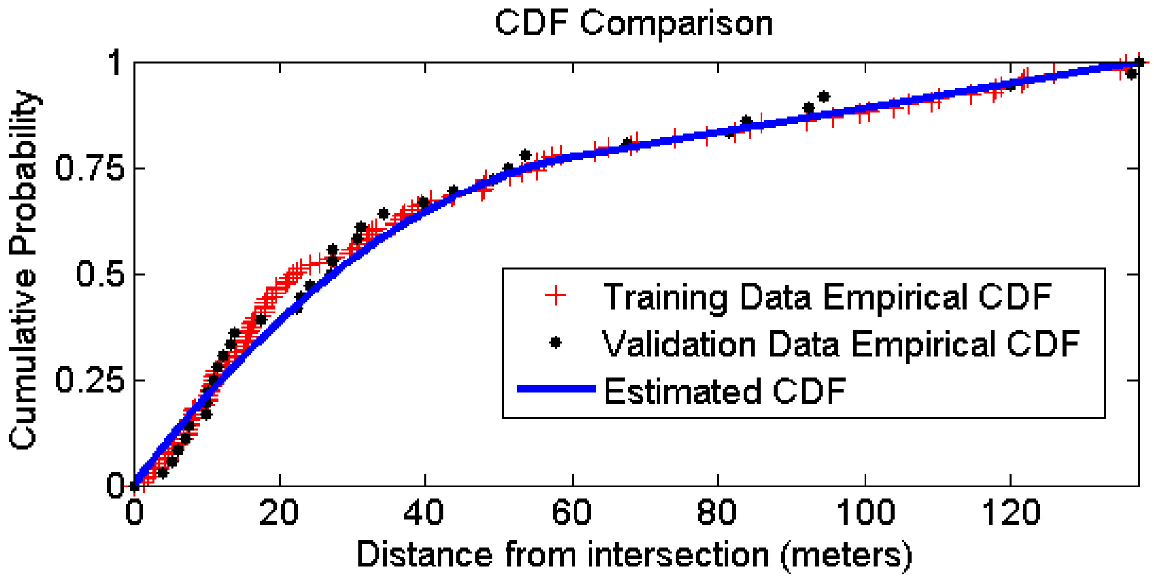
➔ Spatial distribution of vehicles on a link: models the spatial heterogeneity of travel times

➔ Travel time distribution estimation and prediction

Results

- **Needs little data**
- **Does not overfit**
- **Physical interpretation**
- **Estimation and prediction of travel times**
- **Improve accuracy (compare to baseline model)**





Future work

- **For this project: finalize the results for the travel time estimation and prediction**

- **Ongoing work:**
  - **Spatio temporal evolution of traffic conditions: transition probability matrices on the evolution of parameters (Hidden Markov Model)**
    ➔ Summer 2010
  - **Hierarchical model (bayesian analysis)**
    ➔ Currently testing
  - **Joint distribution of travel times on links with multiple intersections (flow synchronization)**

Questions?

Thank you for your attention

http://traffic.berkeley.edu