A General Phase Transition Model

Applications for vehicular traffic

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Outline

- Background and Motivation
- The Phase Transition Model
- Empirical Data Analysis
- Conclusions

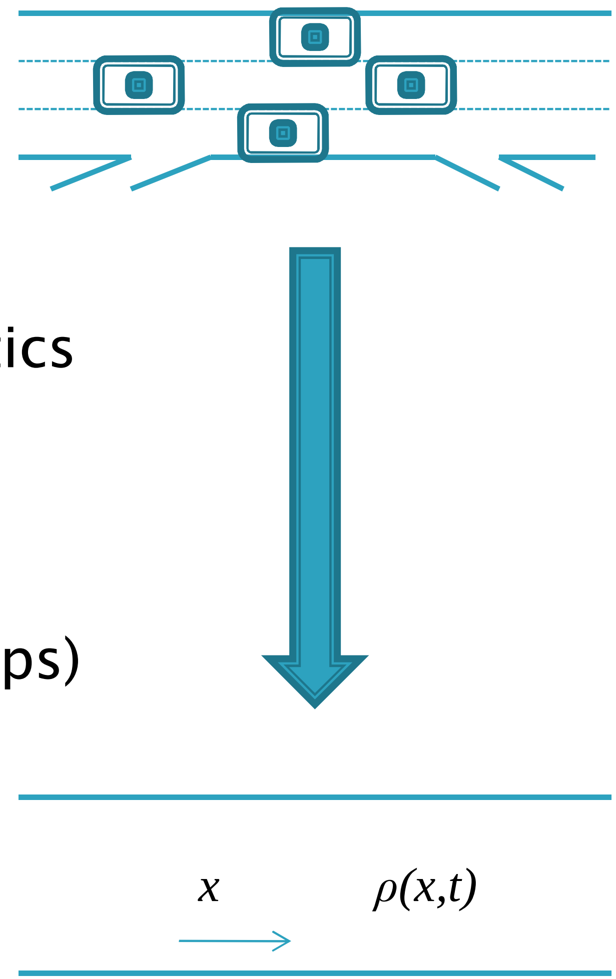
Background and Motivation

- Traffic models:
  - Level of detail

Model Type	Traffic Flow Representation	Traffic Dynamics
MACro	Aggregate	Continum Models
MESo	Individual Vehicles	Continum Models
MICro	Individual Vehicles	Car-Following

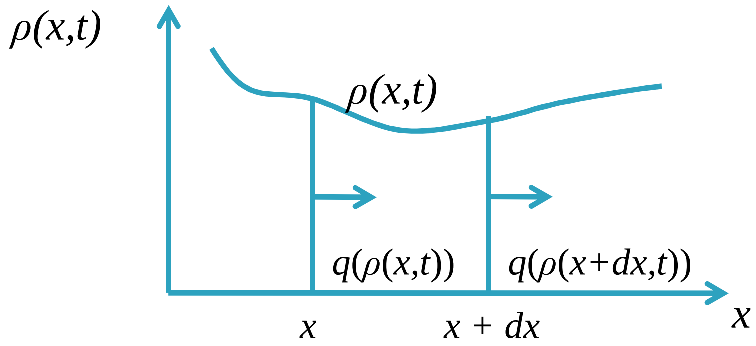
Background and Motivation

- Macroscopic models
  - Assumptions:
    - Average traffic stream caracteristics
    - Continuum approximation
    - Isolated segment (no on/off-ramps)



Background and Motivation

- Macroscopic models
  - Conservation of mass (LWR equation):



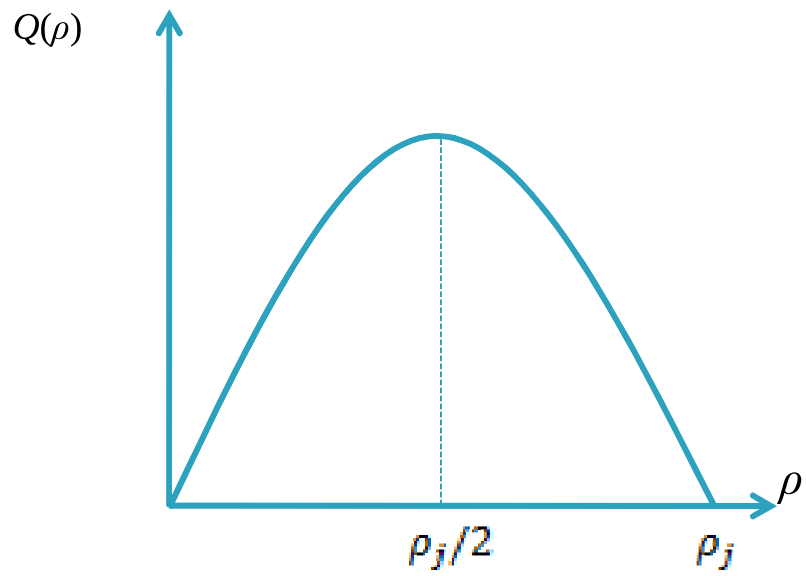
$$\rho(x,t+dt)dx = \rho(x,t)dx + \Big(q\big(\rho(x,t)\big) - q\big(\rho(x+dx,t)\big)\Big)dt$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\frac{\partial q(\rho)}{\partial x}}$$

Background and Motivation

- Fundamental Diagram
  - Relationship between density and flow:
  - Examples:
    - Greenshields

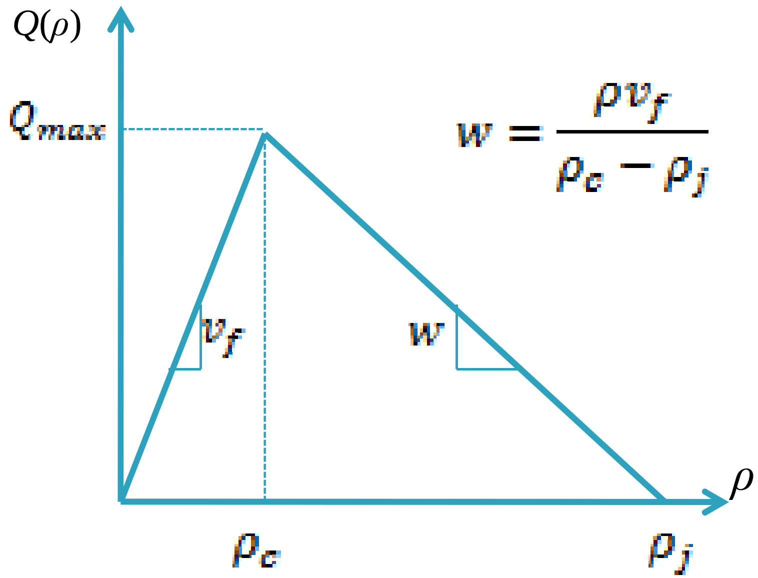
$$Q(\rho) = v_f \rho \left(1 - \frac{\rho}{\rho_j}\right)$$



Background and Motivation

- Fundamental Diagram
  - Examples:
    - Newell-Daganzo

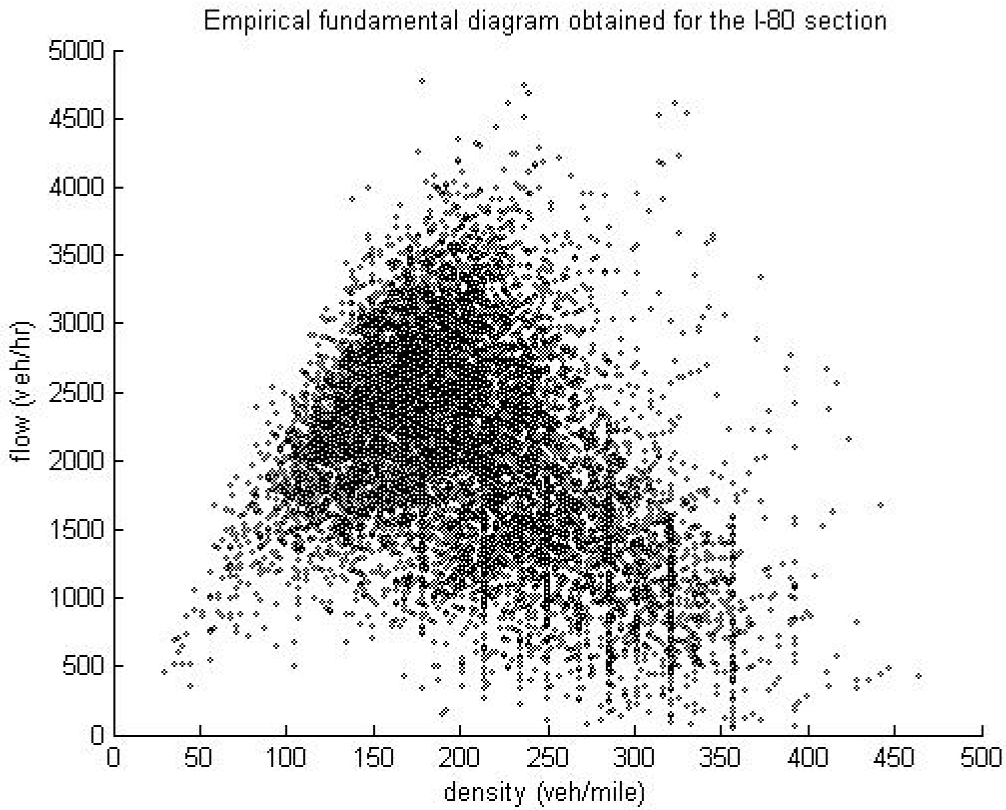
$$Q(\rho) = \begin{cases} v_f \rho & \text{if } \rho \le \rho_c \\ \dfrac{\rho v_f}{\rho_c - \rho_j}(\rho - \rho_j) & \text{if } \rho > \rho_c \end{cases}$$



    - Others: $Q(\rho)$

Background and Motivation

- Fundamental Diagram
  - However, empirical flow density plot:

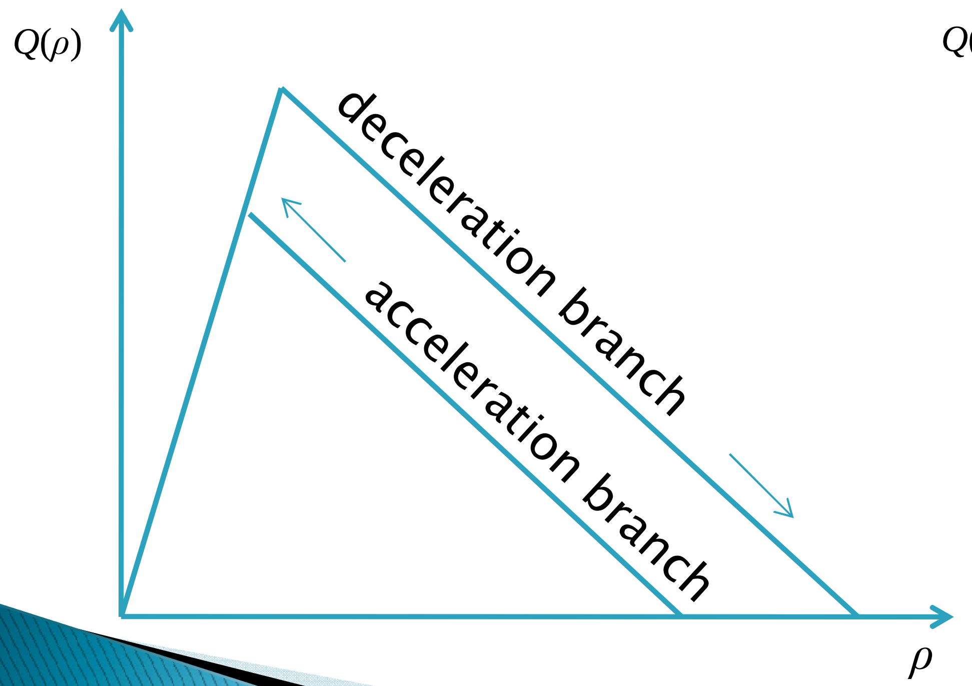


- Hysteresis in the congested branch.

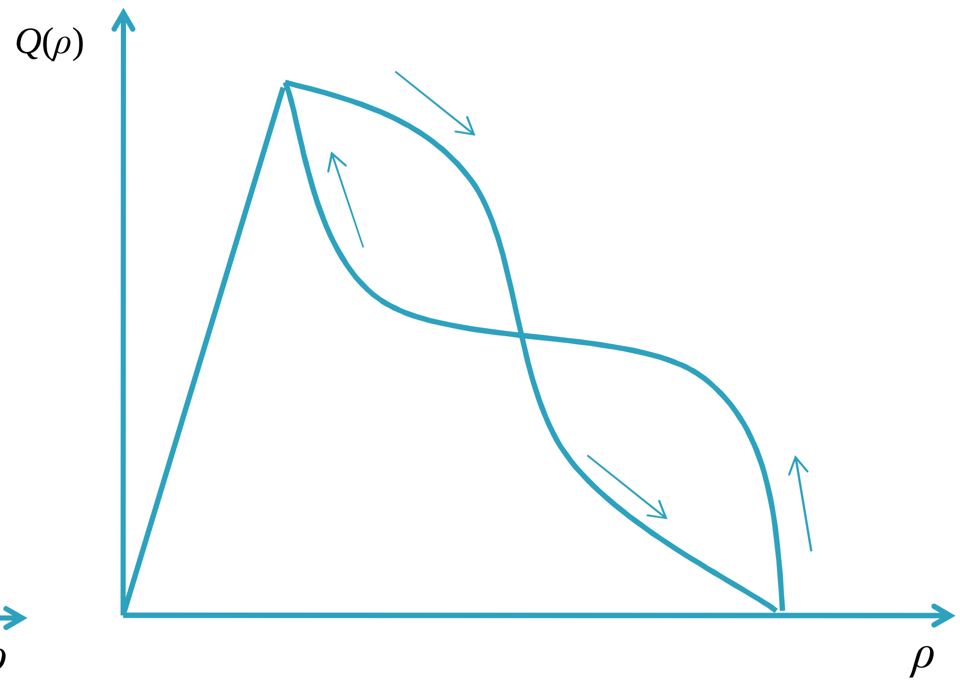
- There's not a single value flow for a given density.

Background and Motivation

- Previous theories:
  - Newell (1962):



  - Zhang (1999):



The Phase Transition Model

- Based on the Colombo 2x2 phase transition model.
- In free flow: constant speed
- In congestion: introduction of perturbation q

The Phase Transition Model

- Formulation:
  - Conservation equations:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 & \text{if free-flow } (\Omega_f) \\ \begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t \rho + \partial_x(q v) = 0 \end{cases} & \text{if congestion } (\Omega_c) \end{cases}$$

  - Velocity:

$$\begin{cases} v_f(\rho) = V & \text{if free-flow } (\Omega_f) \\ v_c(\rho, q) = v_c(\rho, 0)(1+q) & \text{if congestion } (\Omega_c) \end{cases}$$

Equilibrium at $q = 0$

The Phase Transition Model

- Constraints:
  - Positivity of speed: $1 + q \geq 0$
  - Strict hyperbolicity of the congested state:

$$\forall\,(\rho, q) \in \Omega_c \quad \rho\,\partial_\rho v_c(\rho, 0) + q\,(v_c(\rho, 0) + \rho\,\partial_\rho v_c(\rho, 0)) \neq 0$$

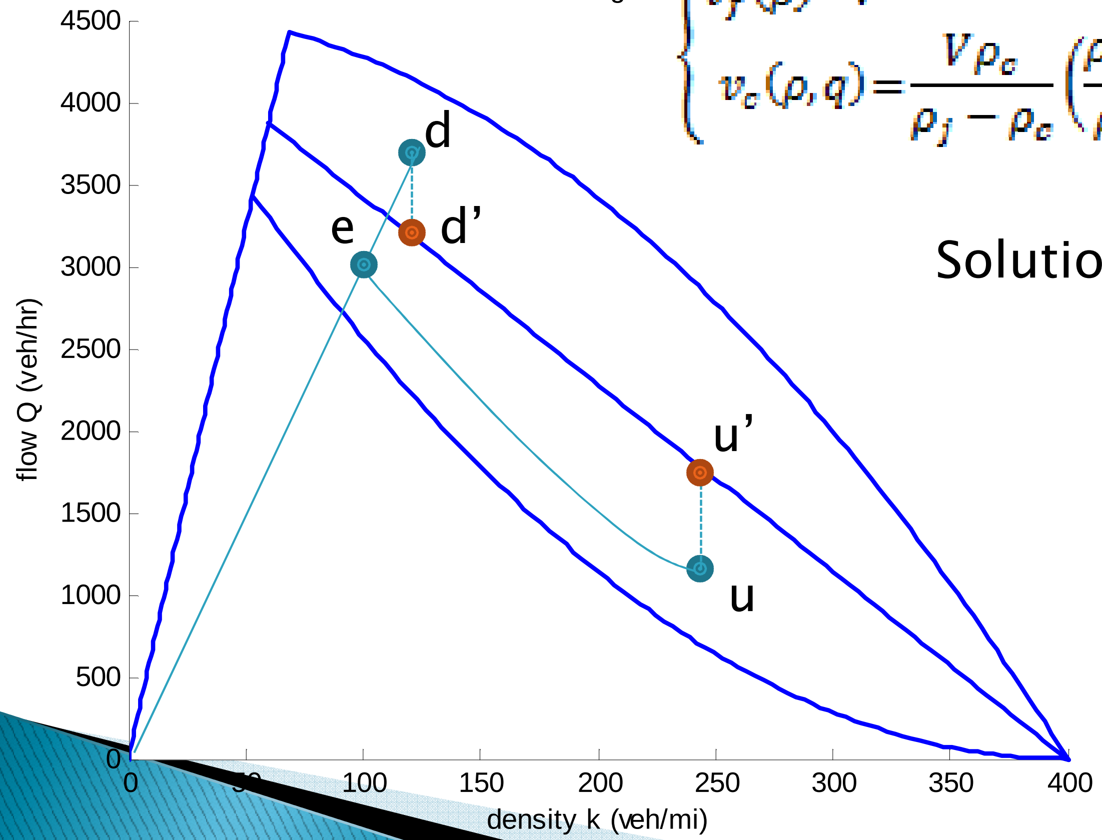
  - Shape of Lax curves

$$\rho\,(2\,\partial_\rho v_c(\rho, 0) + \rho\,\partial^2_{\rho\rho} v_c(\rho)) \\ + q\,(2\,v_c(\rho, 0) + 4\,\rho\,\partial_\rho v_c(\rho, 0) + \rho^2\,\partial^2_{\rho\rho} v_c(\rho, 0))$$

The Phase Transition Model

- Implementation: Newell-Daganzo

$$\begin{cases} v_f(\rho) = V & \text{if free-flow } (\Omega_f) \\ v_c(\rho, q) = \dfrac{V\rho_c}{\rho_j - \rho_c}\left(\dfrac{\rho_j}{\rho} - 1\right)(1+q) & \text{if congestion } (\Omega_c) \end{cases}$$

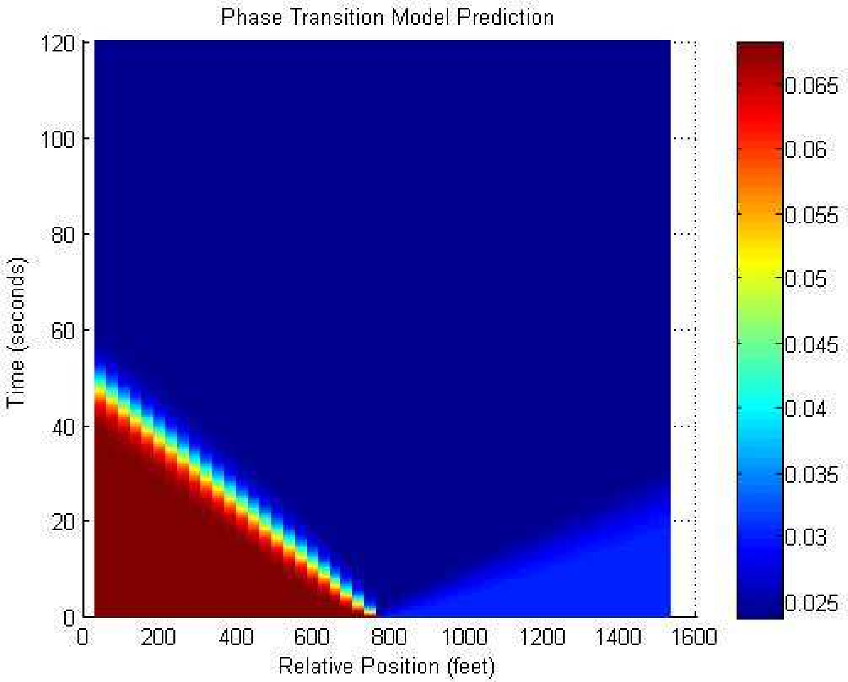


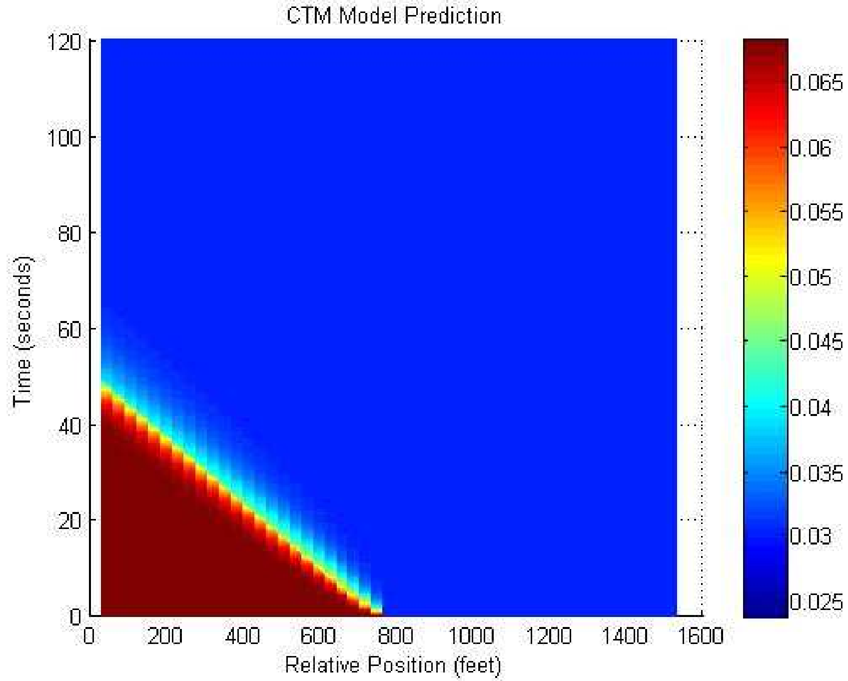
Solution of the Riemann problem

$$\begin{cases} \dfrac{q_e}{\rho_e} = \dfrac{q_u}{\rho_u} \\ v_c(\rho_e, q_e) = v_c(\rho_d, q_d) \end{cases}$$

The Phase Transition Model

- Time space diagram





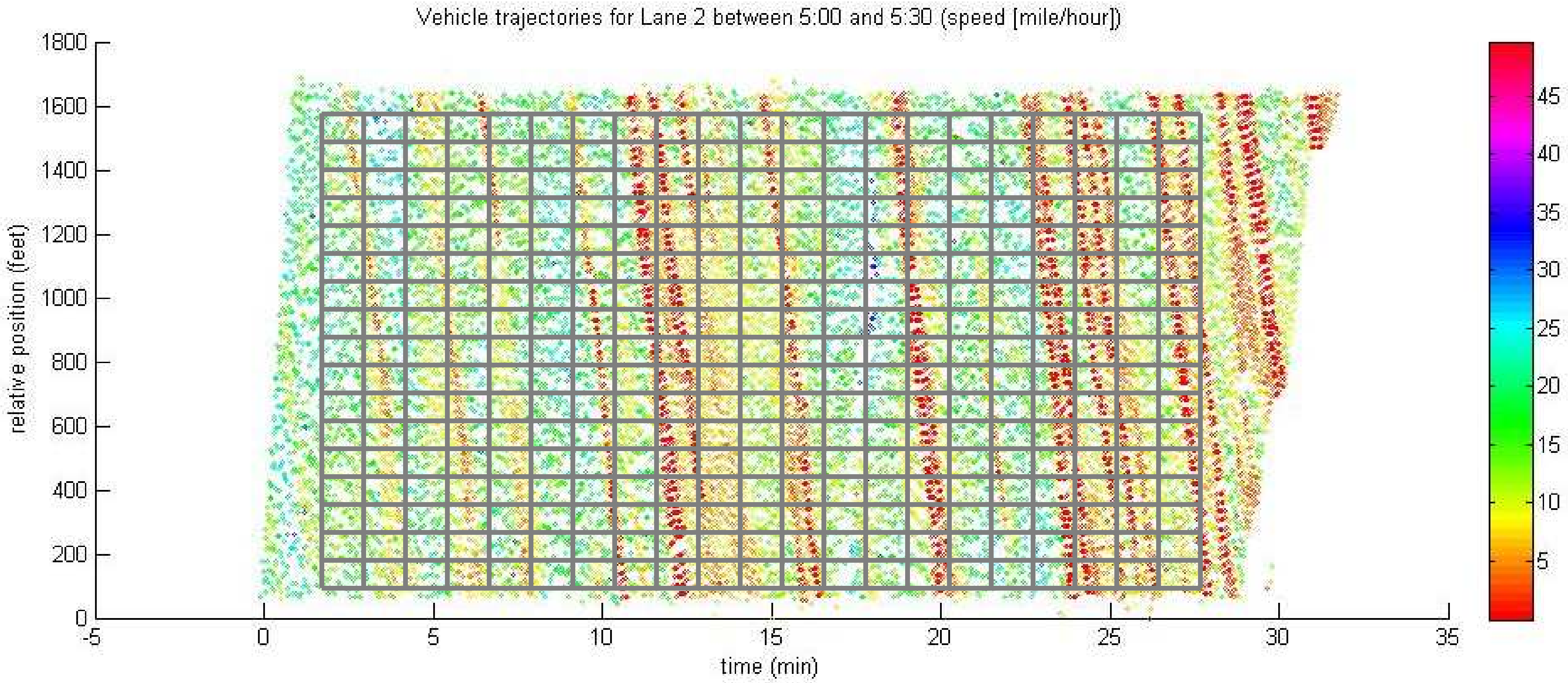
The Phase Transition Model

Empirical Data Analysis

- NGSIM dataset:
  - Recorded in I-80 (video image)
    - 7 lanes (HOV+merging)
  - 30 minutes long (in congestion)
  - 0.3 miles

Empirical Data Analysis

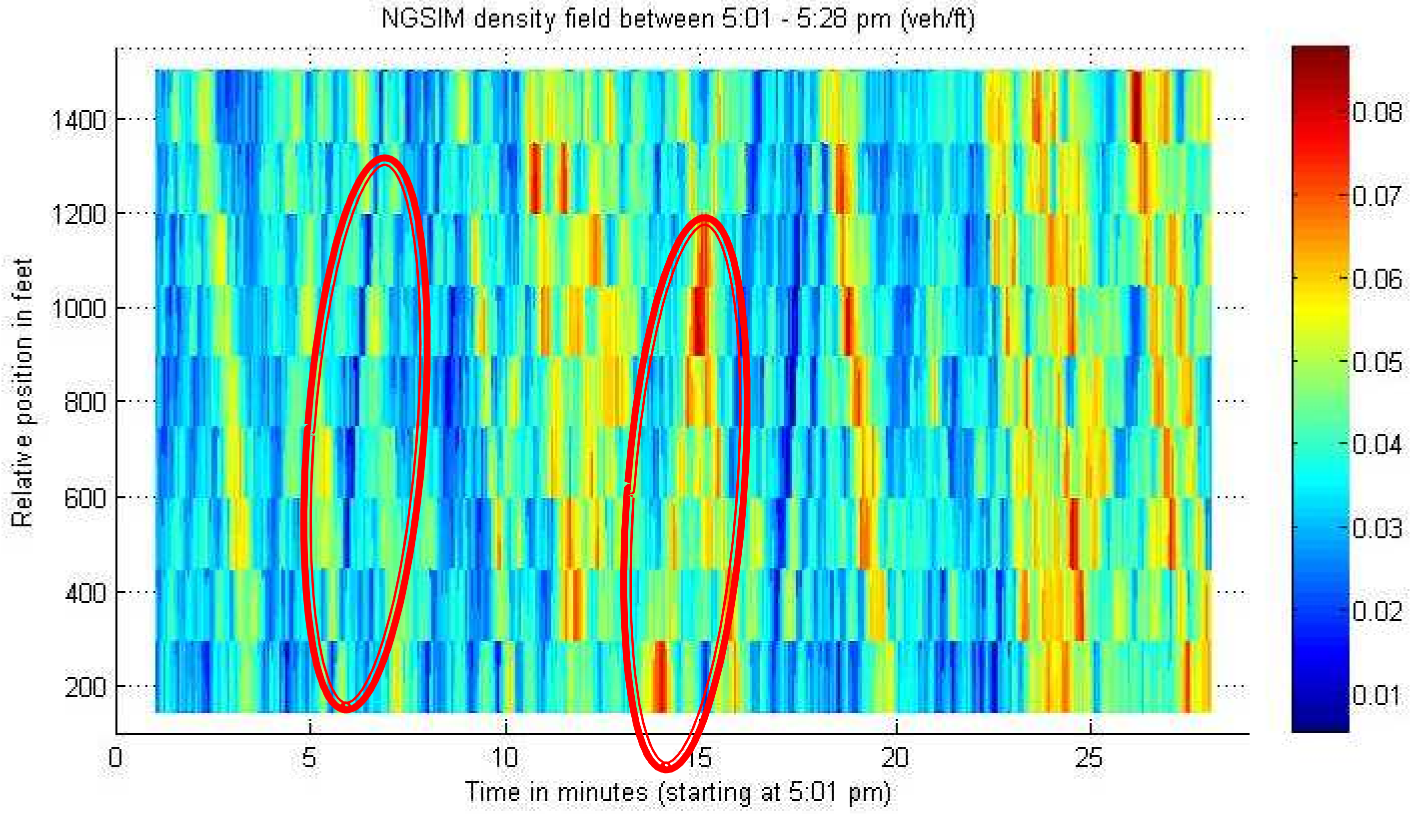
- Trajectory data:



Eddie's generalized definitions of flow, density and speed

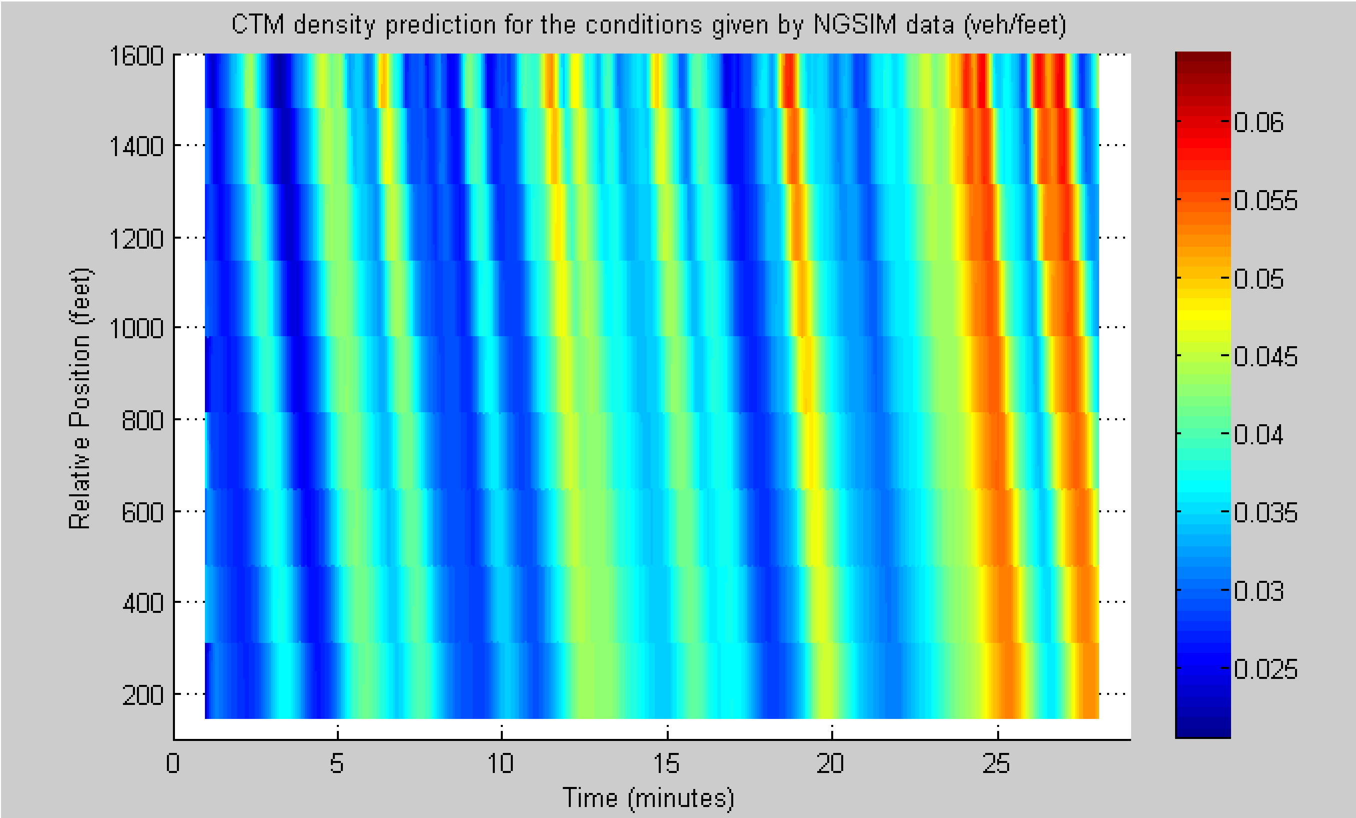
Empirical Data Analysis

- Density field:



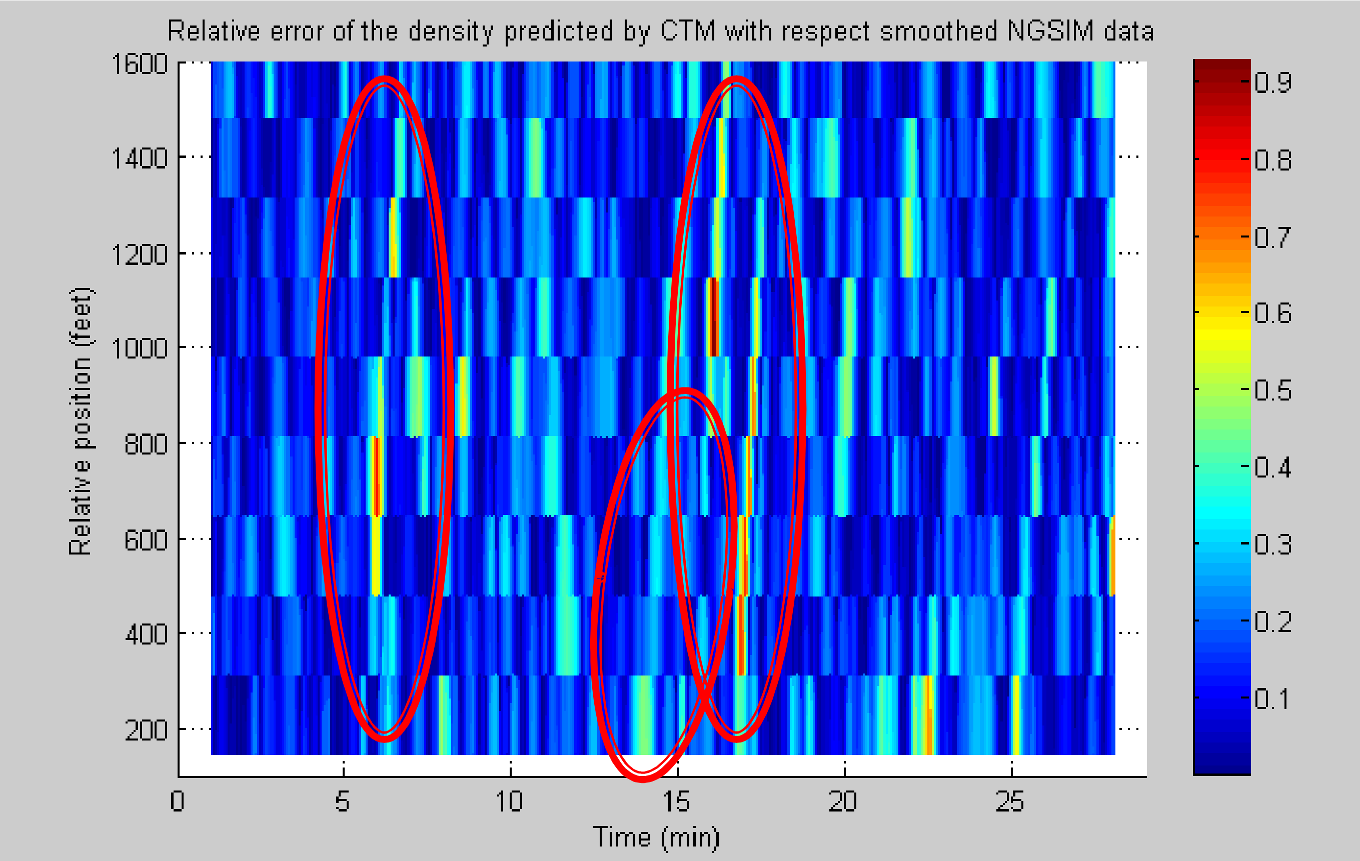
Empirical Data Analysis

- CTM prediction:



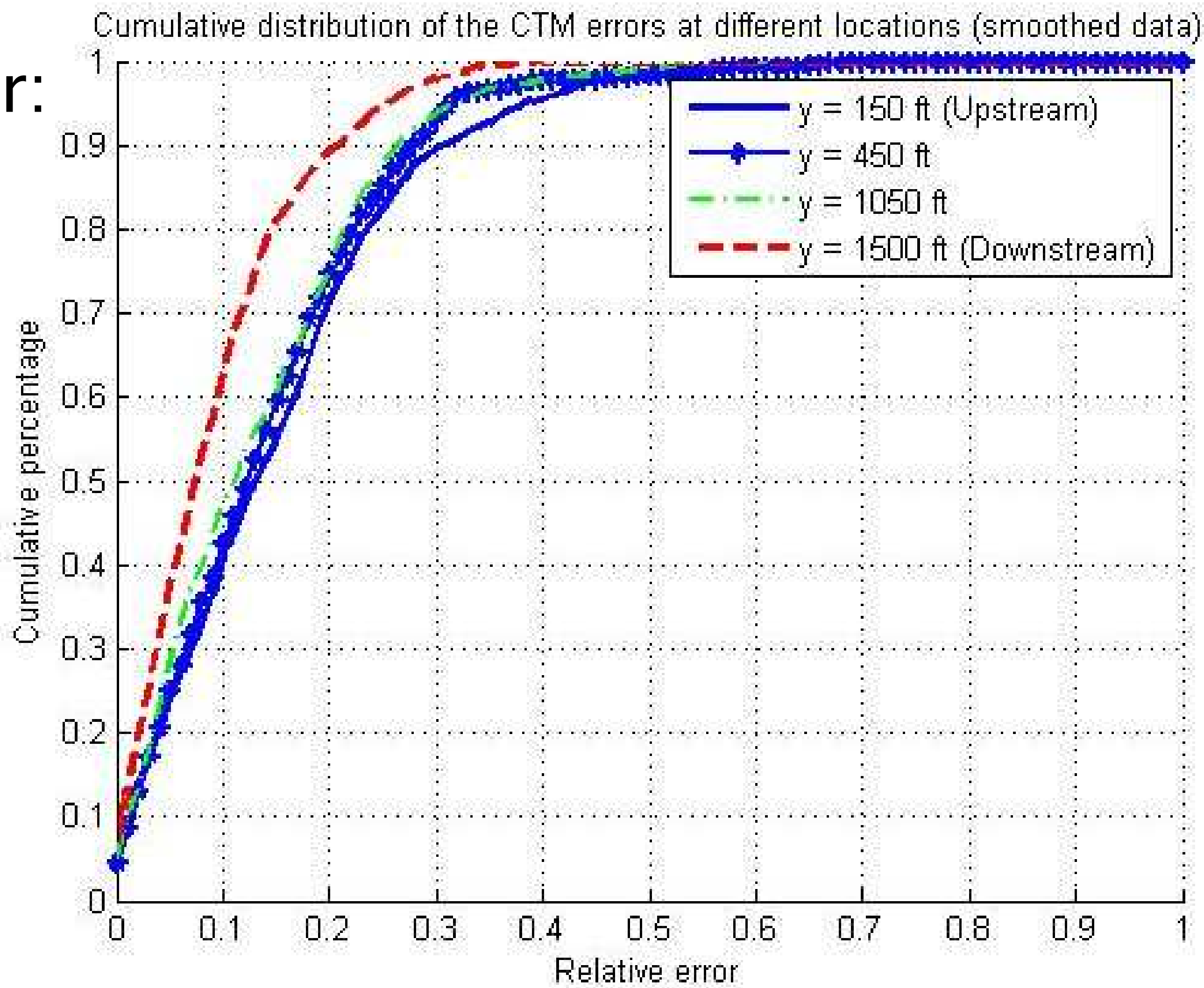
Empirical Data Analysis

- CTM prediction (error field):



Empirical Data Analysis

- CTM error:



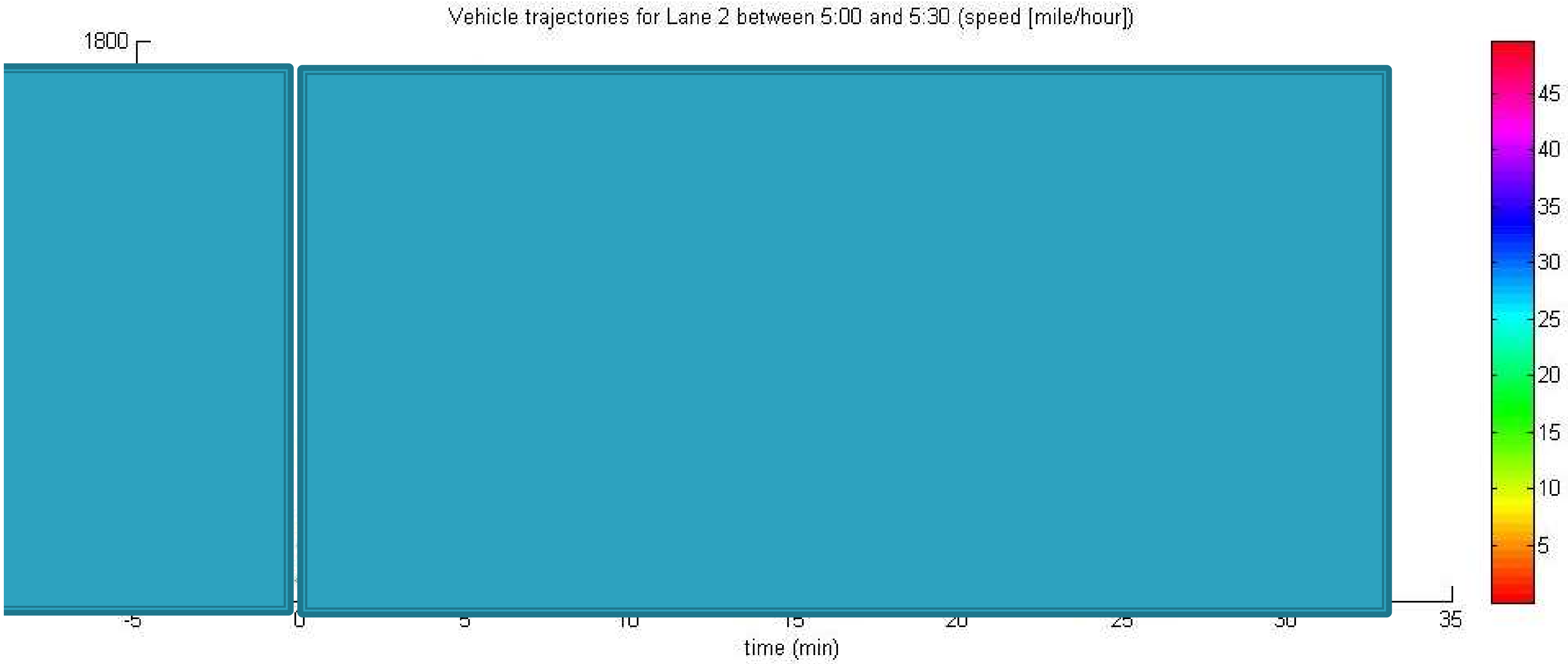
Conclusions

- The presented model accounts for the fact that the relation $Q(\rho)$ is not single-valued.
- Possibility of account for stop and go phenomena.
- Riemann problem solved by two different waves.
- Makes it possible to integrate density and speed measurements
- What's next: Application to NGSIM dataset.

Any questions?

Empirical Data Analysis

- Trajectory data:



Eddie's generalized definitions of flow, density and speed