A Study of Transonic Flow and Airfoils

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Background

- Transonic regime
  - Loosely defined region of flow around sonic speed (free stream velocities $0.8 \leq M_\infty \leq 1.2$)
- Mixed regions of locally subsonic and supersonic flow.
- Unpredictable effect of shockwaves on the control surfaces

Aims

- Expand knowledge of aerodynamics and compressible flow
- Investigate the effects of transonic flow on airfoils
  - Analyze behavior of pressure coefficient, C_p

Theory: Conservation Equations

Euler's Equations:

Continuity	$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho \bar{V} \right) = 0$
Momentum	$\rho \frac{D\bar{V}}{Dt} = -\nabla p$
Energy	$\rho \frac{Dh_o}{Dt} = \frac{\partial \rho}{\partial t}$

Theory: Irrotational Flow

- Vorticity $\nabla \times \vec{V}$
  - Vorticity = 0 for irrotational flow
  - Define: velocity potential Φ such that

$$\vec{V} \equiv \nabla \Phi$$

- Why?
  - Simplifies conservation equations into one governing equation [1]:

$$\left(1-\frac{\Phi_x^2}{a^2}\right)\Phi_{xx}+\left(1-\frac{\Phi_y^2}{a^2}\right)\Phi_{yy}+\left(1-\frac{\Phi_z^2}{a^2}\right)\Phi_{zz}$$

$$-\frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy}-\frac{2\Phi_x\Phi_z}{a^2}\Phi_{xz}-\frac{2\Phi_y\Phi_z}{a^2}\Phi_{yz}=0$$

Theory: Irrotational Flow

- Validity for transonic flow:
  - Entropy across shock [1]

$$\frac{s_2 - s_1}{R} \approx \frac{2\gamma}{3(\gamma+1)^2}(M_1 - 1)^3$$

  - For transonic, $M_1 - 1 \rightarrow 0$
  - Flow can be assumed as isentropic, and therefore irrotational!

Theory: Irrotational Flow

- Introduce perturbation velocity potential:

$$\Phi = V_\infty x + \phi$$

- Governing equation simplifies to:

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz}$$

$$= M_\infty^2 \left[(\gamma + 1) \frac{\phi_x}{V_\infty} \right] \phi_{xx}$$

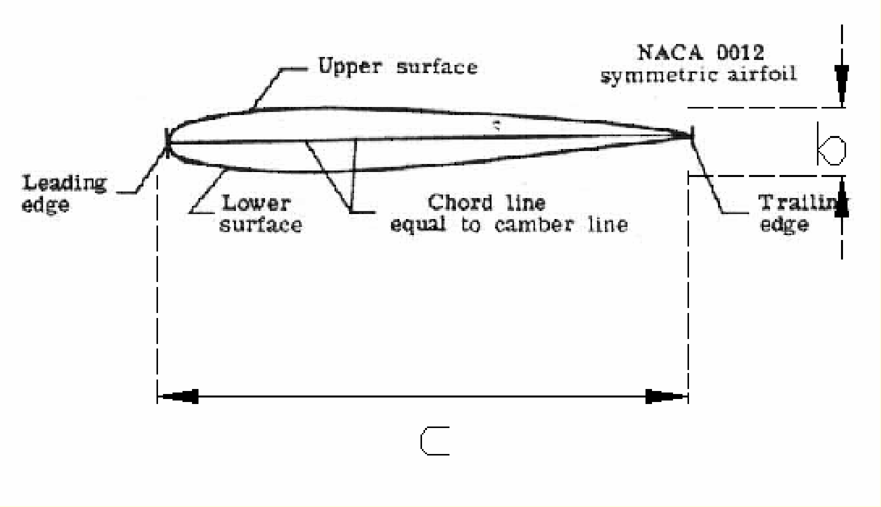
  - Note: RHS drops out for subsonic or supersonic flow, resulting in linearized PDE.

Theory: Self-Similarity

- Recall governing equation:

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz}=M_\infty^2\left[(\gamma+1)\frac{\phi_x}{V_\infty}\right]\phi_{xx}$$

- Introduce slenderness ratio τ = b / c



Theory: Self-Similarity

- Self-similar variables:

$$\bar{x} = \frac{x}{c} \qquad \bar{y} = \frac{y\tau^{1/3}}{c} \qquad \bar{z} = \frac{z\tau^{1/3}}{c}$$

- Nondimensionalize:

$$\bar{\phi} = \frac{\phi}{cV_\infty \tau^{2/3}}$$

- Transonic similarity equation:

$$\left[K - (\gamma+1)\bar{\phi}_{\bar{x}}\right]\bar{\phi}_{\bar{x}\bar{x}} + \bar{\phi}_{\bar{y}\bar{y}} + \bar{\phi}_{\bar{z}\bar{z}} = 0$$

where *K* = transonic similarity parameter:

$$K = \frac{1 - M_\infty^2}{\tau^{2/3}}$$

Theory: Characteristics

- Recall governing equation (2D):

$$\left(1-\frac{\Phi_x^2}{a^2}\right)\Phi_{xx}+\left(1-\frac{\Phi_y^2}{a^2}\right)\Phi_{yy}-\frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy}=0$$

- From midterm:

$$\left(1-\frac{u^2}{a^2}\right)\left(\frac{dy}{dx}\right)^2+\left(\frac{2uv}{a^2}\right)\left(\frac{dy}{dx}\right)+\left(1-\frac{v^2}{a^2}\right)=0$$

Theory: Characteristics

- Solving,

$$\frac{dy}{dx} = \frac{-uv \pm a\sqrt{u^2 + v^2 - a^2}}{\left(a^2 - u^2\right)}$$

subsonic → elliptic

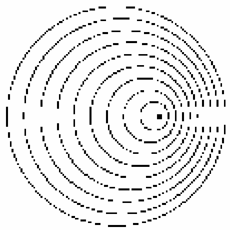
sonic → parabolic

supersonic → hyperbolic

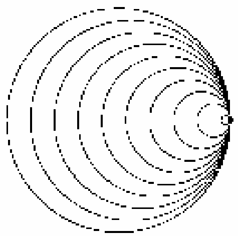
- Characteristic Slopes?
  - Interpretation: The Mach Cone

Theory: Characteristics

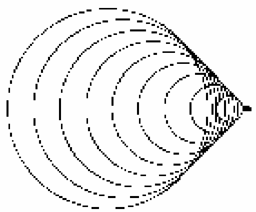
- From NASAexplore’s website:



Subsonic (below Mach 1) the pressure waves radiate in front of, as well as behind, the airplane. The plane is traveling below the speed of sound.



Transonic (at Mach 1) the airplane catches up with its own pressure waves, which build up into a shock wave.



Supersonic (above Mach 1) the shock waves form a cone. This causes a sonic boom when it hits the ground. Mach is another way of referring to the speed of sound. Flying at Mach 2, for instance, means you're flying at twice the speed of sound.

Numerical Modeling

- Pressure coefficient

$$C_P = \frac{p_o - p}{\frac{1}{2}\rho v^2}$$

- National Advisory Committee for Aeronautics (NACA) – Data for foils
- Panel Methods

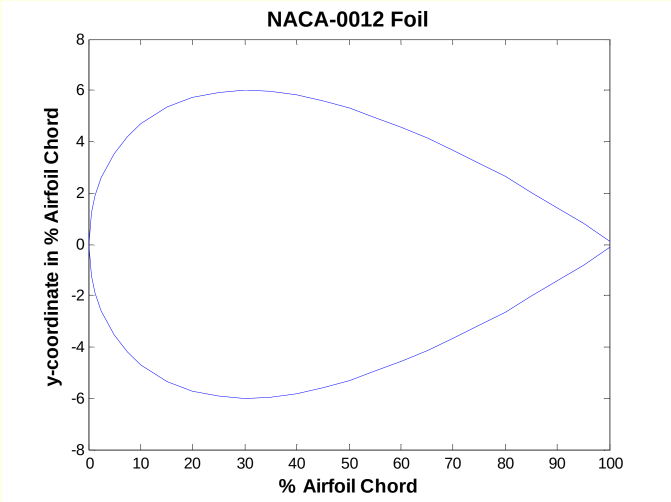
Panel Methods

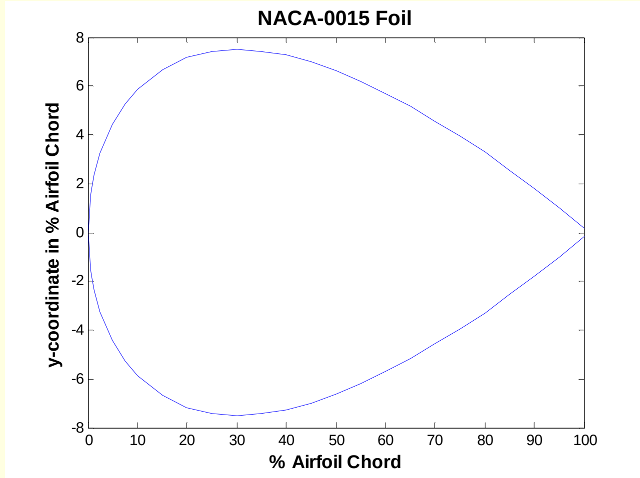
- Basic principle: Superposition
- Boundary element method: Panels
- Sources/Sinks (simple solution)
- Vortices
- The Kutta Condition:
  - Pressure above and below trailing edge must be equal

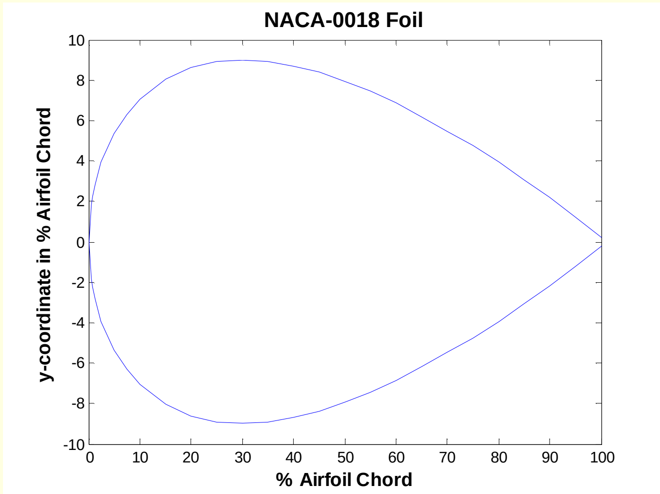
Strategy

- Attempt Vortex Panel method for three symmetric airfoils for linearized full potential equation (FPE)
  - Help: ME163 website
- Extend to transonic modeling

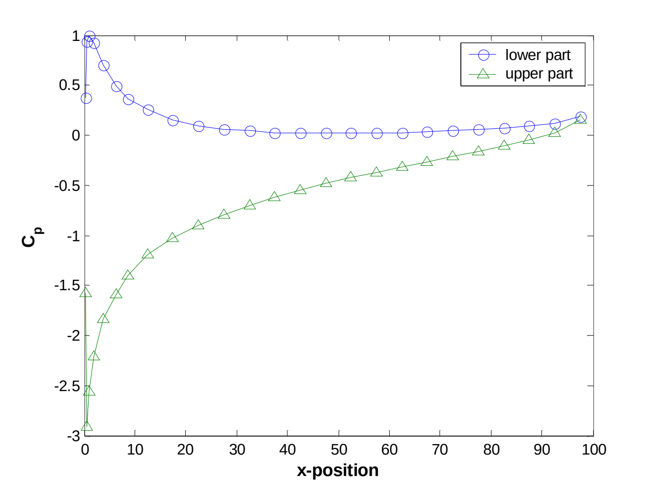
Results: Symmetric Airfoils

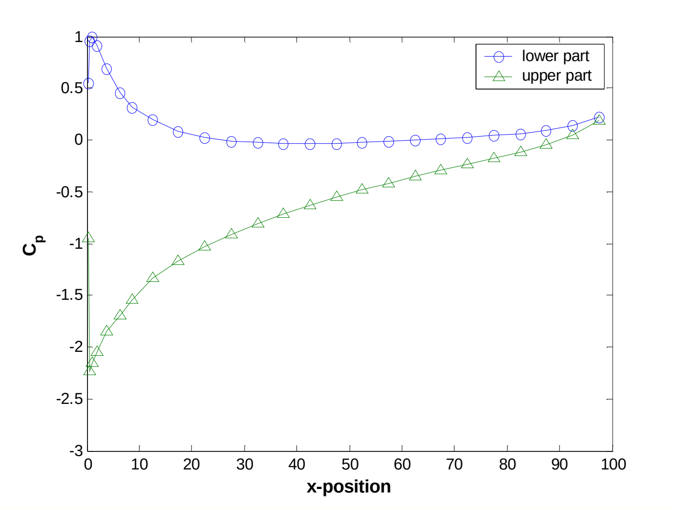


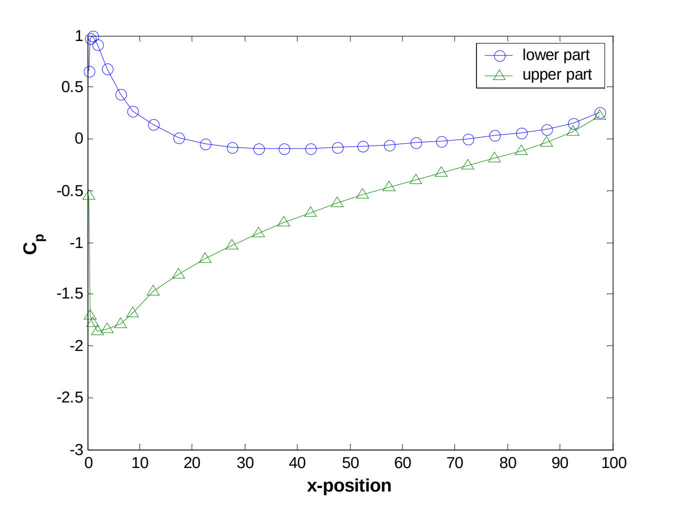




Results: Linearized FPE





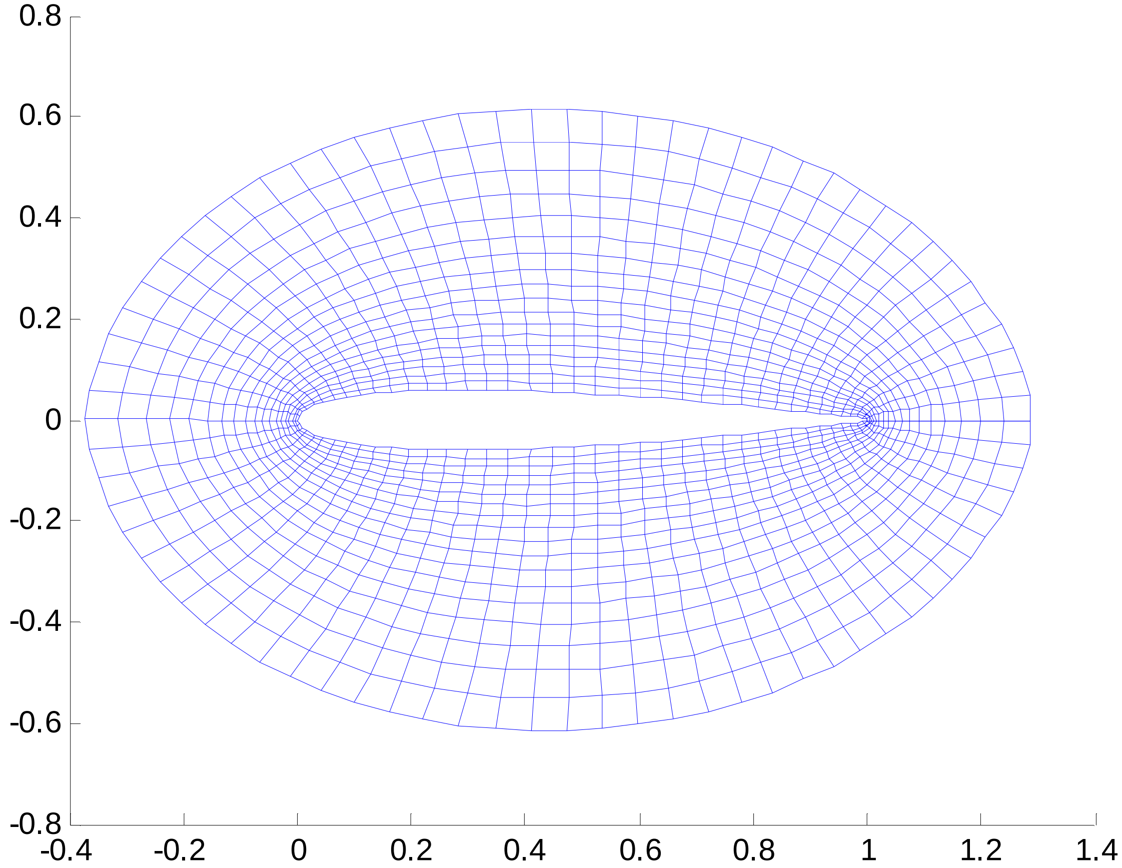


Transonic Modeling

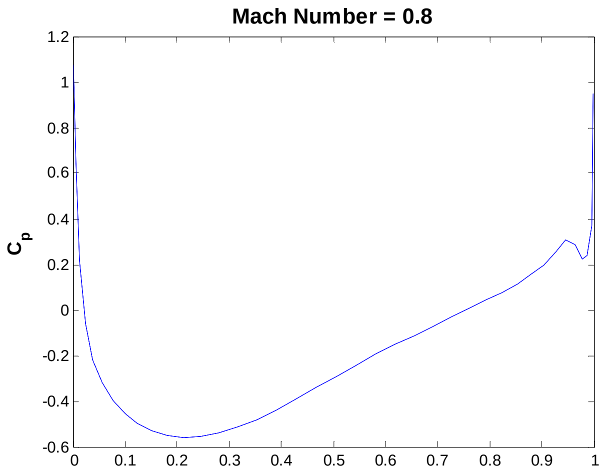
- Numerical solution is exponentially harder to obtain because of nonlinearity
- Make use of characteristics
- Further steps needed:
  - Grid Generation: Solve FPE at nodes
  - Discretization of the PDE
  - Iterative solution

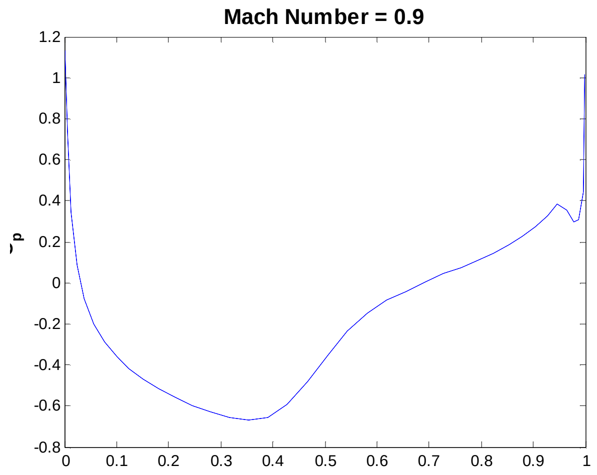
Sample Grid for NACA-0012

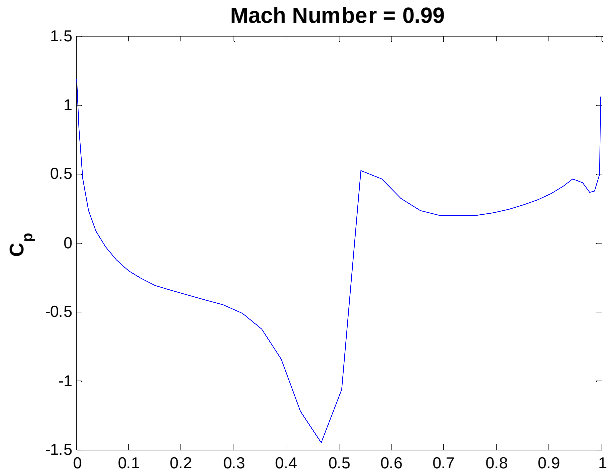
- 100 x 20 field panels (from GA Tech)

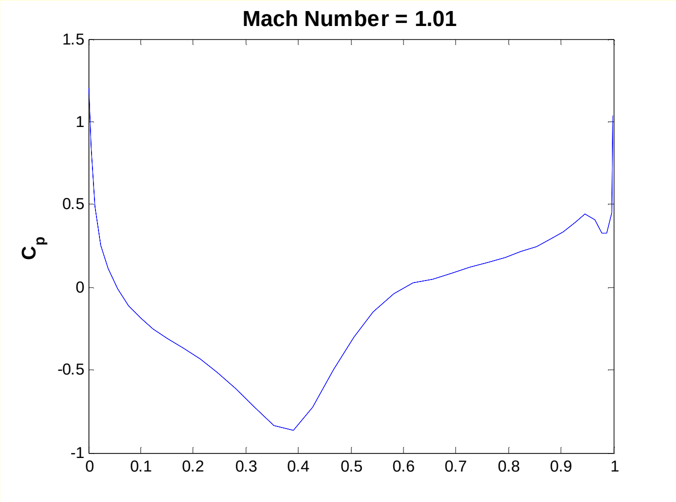


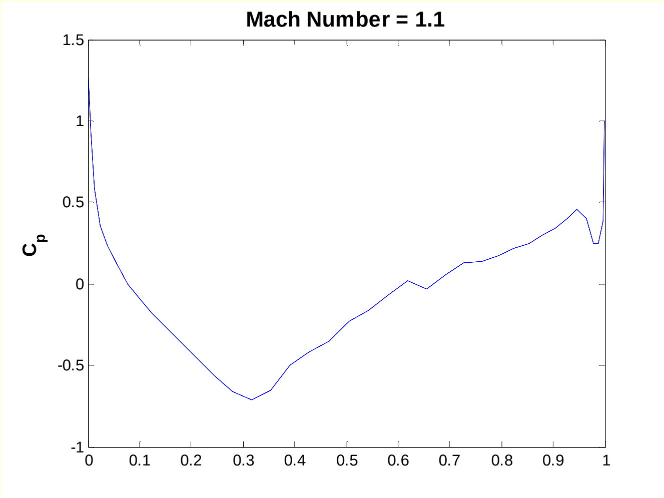
Results


Mach Number = 0.8
Cp
1.2 1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Mach Number = 0.9
1.2 1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6 -0.8
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Mach Number = 0.99
Cp
1.5 1 0.5 0 -0.5 -1 -1.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Mach Number = 1.01
Cp
1.5 1 0.5 0 -0.5 -1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Mach Number = 1.1
Cp
1.5 1 0.5 0 -0.5 -1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Future Work

- Generate one case for nonlinear, transonic flow, and solve iteratively
- Validate with results from Oskam's article [5]: "Transonic Panel Method for the Full Potential Equation Applied to Multicomponent Airfoils"

Conclusion

- Better understanding of aerodynamics
- Application of mathematical methods for modeling
- Numerical modeling for nonlinear PDEs is significantly tougher than linearized PDEs
  - Simplify PDEs whenever possible!

References

1. Anderson, J.D. “Modern Compressible Flow”
2. Houghton E.L. and Carpenter, P.W. “Aerodynamics for Engineering Students”
3. Ferrari, C. and Tricomi F.G. “Transonic Aerodynamics”
4. **AE 3903/4903 Airfoil Design** http://www.ae.gatech.edu/people/lsankar/AE3903/
5. Oskam, B. “Transonic Panel Method for the Full Potential Equation Applied to Multicomponent Airfoils”
6. ME163 Fall 2006 Project 2 “Vortex Panel Method” http://me.berkeley.edu/ME163/project_2.pdf