CE191: Civil and Environmental Engineering Systems Analysis

Lab 2: Energy Portfolio Optimization

Due: Wednesday 10/07/09 1:00pm

Professor A. Bayen *Fall 09*

1 Lab overview

In this lab, you will learn to formulate a quadratic program to find a mix of energy supplies which minimizes the variability of the price of energy. Section 2 provides a description of the full problem and the constraints. In Section 3, you are asked to form the quadratic program, and implement it in MATLAB. Finally, in Section 4, you will modify the quadratic program to analyze various scenarios. **Please remember to submit your MATLAB code (.m files in one ZIP file), and explain in the report how to run the code.**

2 Energy portfolio selection problem

The country of Ecotopia is charged with strategically planning its energy generation over the next several years to meet its growing energy demand. Currently, Ecotopia generates energy from a combination of eight sources. This collection, or mix, of energy sources is known as an *energy portfolio*. The contributions of the eight sources to Ecotopia's current portfolio is listed in Table 1.

Source	Wind	Solar	Coal	Gas	Geo	Nuclear	Bio	Hydro
2009 energy mix (%)	2.39	0.24	18.21	45.74	4.46	14.44	2.08	12.44

Table 1: Current sources of energy to Ecotopia, and percent contribution to the current energy mix.

By the year 2030, Ecotopia will be able to adjust their current energy portfolio to meet their growing demand for energy. The expected price in 2030 of each source in dollars per megawatt hour ($/MWh) is given in Table 2.

Source	Wind	Solar	Coal	Gas	Geo	Nuclear	Bio	Hydro
Expected Price ($/MWh)	67	200	115	102	222	74	103	120

Table 2: Expected price of energy in Ecotopia in the year 2030.

Through technology improvements, Ecotopia will be able to expand the supply of each of the energy sources as much as required to meet their future demands. However, due to uncertain fuel costs, the cost of technology development, and unknown environmental regulations, the future price of each energy source is uncertain. The standard deviation σ of the expected price for each source is given in Table 3[1].

Source	Wind	Solar	Coal	Gas	Geo	Nuclear	Bio	Hydro
σ ($/MWh)	9.38	13.40	40.99	43.66	39.18	29.64	22.71	21.80

Table 3: Standard deviation of the energy source prices in the year 2030.

The variance[2] of the price of a good (for example 1 MWh of electricity) can be used as a measure of the risk of that good. Portfolio theory assumes that for a given level of risk, planners prefer lower costs to higher ones. Similarly, for a given expected cost, planners prefer less risk to more risk. By combining various goods in a portfolio, it is possible to create a portfolio with lower risk than any of the goods individually. This is known as *diversification*.

To remain industrially competitive, Ecotopia must keep the expected cost of its energy portfolio under $100 per MWh. Therefore, Ecotopia would like to determine an optimal energy portfolio, that is, one in which there is no other portfolio with less risk (price variance) and the same expected cost.

3 Implementation

Question 3.1 Formulate a Quadratic Program (QP) that Ecotopia can use to minimize the risk of obtaining its energy in 2030, while satisfying the maximum expected cost of

[1] For simplicity, we assume the prices are uncorrelated.

[2] Recall the variance (var) is given as: $\text{var} = \sigma^2$

energy constraint described above. Define your notation carefully and precisely.

Question 3.2 Solve the QP that you have formulated using MATLAB. Provide the value of the objective function and the value of the decision variables at optimum. Compute the standard deviation of the optimal energy portfolio. Describe qualitatively the optimal solution.

4 Additional analysis

The purpose of the following questions is to study modifications to the original quadratic program solved in the previous question. Each question is independent from the other ones, i.e. the changes are not cumulative. Where applicable, provide the value of the objective function as well as the value of the decision variables.

Question 4.1 A concerned citizen of Ecotopia, who is not familiar with portfolio optimization, suggests the safest plan is to apply the 2009 energy portfolio mix to the year 2030. What is the expected risk and expected cost of this portfolio? Is there a safer (in the sense of lower variability) portfolio with at least as small expected cost? Explain.

Question 4.2 The Secretary of Energy would like to know the minimal risk of each energy profile ranging in price \$67 to \$222 per MWh. Compute the minimal risk for each energy profile in this price range.

1. Summarize your results in a plot with portfolio price on the y axis and portfolio variance on the x axis. Describe qualitatively the results.
2. Describe qualitatively the mix of the lowest risk energy portfolio in this price range. How much does it cost?

Question 4.3 By the year 2030, Ecotopia must generate 345 *terawatt hours* (TWh) to meet its growing demand for energy. A new report suggests that, due to technology and resource limitations, the maximum energy supply for each source in 2030 will be constrained. The energy supply limits are given in Table 4. Modify the QP in Question 3.1 to include this constraint.

Source	Wind	Solar	Coal	Gas	Geo	Nuclear	Bio	Hydro
2030 Max Supply (TWh)	92	43	97	204	27	52	16	15

Table 4: 2030 Energy supply limits in (TWh).

1. Formulate the appropriate constraints imposed by these limits, using the notation of Question 3.1. Describe any additional notation introduced.

2. Solve the QP from Question 3.2 with these additional constraints. How does the new solution compare with your results from Question 3.2? Which supply limits are binding?